# Extrapolative Expectations and House Prices in Canada

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#### Abstract

This paper investigates whether house prices in Canada are in line with fundamentals. A simple asset pricing model solved under rational expectations is used to derive a fundamental, long-run equilibrium value for the price-rent ratio. The rational expectation model explains the sample average of the data but it does not generate the persistence and the large fluctuations observed in the actual series. The deviations of the actual series from fundamentals are attributed to the mechanism used by agents to form their expectations. Then, extrapolative expectations triggering an explosive behaviour of the price-imputed rent series are consistent with the evolution of the price-imputed rent ratio for the period 2001Q1-2006Q4. A comparison with the US experience shows that although the Canadian price-rent ratio appears high relative to the model implied value the current misalignment is less severe than the misalignment in the US before the recent crisis.

**Keywords**: House Prices, Rational Expectation, Extrapolative Expectations, Lucas' Asset Pricing Model.

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### 1 Introduction

Many economies have experienced a substantial run up in house prices in the past decade. Some as the US and Ireland have also seen a subsequent severe downturn of the housing market, while in others like New Zealand and Norway prices are still well above their pre-crisis level. Canada belongs to this second group of countries: after a decade of stagnation in the housing sector house prices have increased by almost 70% in real term and almost doubled in nominal terms in the period 2000-2008. A moderate correction during the financial crisis was immediately followed by a rapid rebound leading to historically high price levels in both nominal and real-terms by 2010Q4 (see Figure 1). Then, a relevant question is whether the observed boom in house prices is in line with the path implied by some fundamentals or whether the housing market is overvalued. If prices are not in line with fundamentals then we could expect corrections that might be harmful on real activity: previous episodes of decline in house prices were followed by economic downturns, see for example the years 1982-1983 and 1990-1993. To answer the question of whether prices are in line with fundamentals it is first necessary to define fundamental values to which we can compare the actuals. In this paper we determine the equilibrium not in terms of house prices per se but in terms of the ratio between house prices and imputed rents, i.e. the stream of consumption and services that derives from owning a house. Treating houses purely as financial assets and abstracting from financing decisions we use a standard asset pricing model to derive a fundamental value for the price-rent ratio. In this model the asset (house) provides with an exogenous streams of dividends (imputed rents) that households use for consumption. Solving the model under rational expectations we obtain a time-varying fundamental value for the price-rent ratio that matches the sample average over the sample 1988-2010. However the price-rent series exhibits high persistence which is not accounted for in the solution under rational expectation. Then we abandon the assumption of rational expectations and we explore the implications of a different expectation formation mechanism. We assume that agents form expectations in an extrapolative fashion and this extrapolative behaviour is the only driver of the deviations of the price-rent ratio from its unconditional mean. When agents form expectations in an extrapolative way they base their conditional expectations of future values on past realizations of the variable to forecast. Expectations are related to past realizations through an extrapolation coefficient which defines the weight that agents put on past observations to generate their expectations. This mechanism introduces persistence in the equilibrium price-dividend ratio and amplifies the fluctuations of the price-rent ratio around the mean. However when conducting a historical counterfactual exercise we find that the price-dividend ratio predicted by the model solved under extrapolative expectations with a constant extrapolation parameter fails to account for the increase of the past decade. Finally we compute the value of the extrapolation coefficient that would generate the observed price-rent ratio given the dividend growth process for each quarter in the sample. The implied evolution of the extrapolation coefficient shows that extrapolative expectations triggering an explosive behaviour of the price-imputed rent series are consistent with the evolution of the price-imputed rent ratio for the period 2001Q1-2006Q4 and 2009Q3-2010Q3.

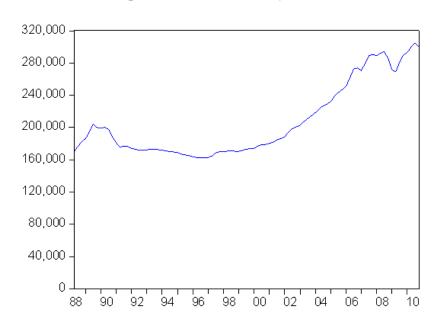


Figure 1. Real House Prices, Canada

Given its unique positioning with respect to Canada, the US is used as a benchmark to evaluate the conditions of the Canadian housing market. We offer a simple way to compare the US and Canadian experience that takes into account the fundamental price-rent valuation of both countries. Our findings indicate that in the sample 1988-2010 the misalignments in the US price-rent ratio were consistently larger than the misalignments for the Canadian price-rent ratio with the exception of the years 2000-2003 and 2007-2010. Also, the current overvaluation for Canada is about half the US overvaluation before the crisis. The larger misalignments in the US are accompanied by larger values of the extrapolation parameter in the period 1997-2006. An extrapolation parameter consistent with a reduction in the US price-dividend series is associated with the house price downturn of the years 2007-2010.

In this paper we follow the methodology developed in Lansing (2006 and 2010) which investigate different ways to form expectations and their ability to explain some characteristics observed in the US asset prices such as excess volatility and bubbles. This approach has the advantage to deliver a micro-funded fundamental value for the price-dividend ratio which depends on few structural parameters. However it requires to treat houses as a financial asset, despite the different liquidity of the housing and stock market and the difficulty in correctly measuring 'dividends' in the housing sector. Also, this simple framework abstracts from the financing decisions of the households and therefore it is silent about the link between monetary policy and the developments in the housing sector. Nevertheless we hope this study might provide with some insights on the conditions of the Canadian housing market and on the role played by expectations in determining the evolution of house prices. Many studies are emphasizing the role of expectations in the dynamics of booms and bust of asset prices (Lansing 2006 and 2010, Adam et al. 2009, Fuster et al. 2011) and more recently of housing prices (Burnside et al. 2011, Adam et al. 2011). Piazzesi and Schneider (2009) using data on expectations from the Michigan Survey of Consumers studies household beliefs during the recent US housing boom and provides with micro evidence that expectations of future increase in prices strengthened with the increase in prices, consistently with an extrapolative behaviour analyzed in this study.

The paper is organized as follows: Section 2 derives the fundamental price-rent ratio under rational expectations, Section 3 analyzes the implication of extrapolative expectations, Section 4 provides with a comparison with the US experience, Section 5 concludes.

# 2 Implications of Rational Expectations

We treat houses as an asset that delivers an exogenous stream of consumption (imputed rents) and abstract from function of house as store of value or collateral and from financing decisions. We use a Lucas tree type model<sup>1</sup> with a risky asset to obtain a fundamental value for the price-dividend ratio  $(p_t/d_t)$ . We think of the dividend as the imputed rents, the stream of consumption and services that derives from owning<sup>2</sup> a house; we will use the terms dividends and imputed rents interchangeably. In the Lucas model, which is an endowment economy, the representative agent chooses sequences of consumption and equity (shares of the house) to maximize the expected present value of her lifetime utility. In particular

<sup>&</sup>lt;sup>1</sup>See Lucas 1978.

<sup>&</sup>lt;sup>2</sup>Note that renting a house does not provide with utility in this model; this is model of home-owners only.

the risk-averse representative agent solves the following intertemporal utility maximization problem:

$$\max_{c_t, s_t} \hat{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t\right)$$

s.t.

$$c_t + p_t s_t = (p_t + d_t) s_{t-1}$$
 with  $c_t > 0$ ,  $s_t > 0$ 

where  $c_t$  is consumption in period t,  $\beta$  is the discount factor,  $s_t$  is the equity share purchased at time t,  $d_t$  is the stochastic dividend paid by the share,  $p_t$  is the price of the share.  $\hat{E}_0$  denotes the agent's subjective expectations.

This maximization problem yields the well-know first order condition:

$$p_{t} = \beta \hat{E}_{t} \left[ \frac{U'(c_{t+1})}{U'(c_{t})} \left( p_{t+1} + d_{t+1} \right) \right]. \tag{1}$$

Because there is no technology to store the dividends and houses are available in fixed supply, for simplicity  $s_t = 1$ , consumption will equal to the dividend at each period,  $c_t = d_t$   $\forall t$ . Substituting this equilibrium condition in (1) and assuming CRRA utility function Lansing (2006) shows that the price-dividend ratio can be rewritten as:

$$\frac{p_t}{d_t} = \hat{E}_t \left[ \beta \exp\left( (1 - \alpha) x_{t+1} \right) \left( \frac{p_{t+1}}{d_{t+1}} + 1 \right) \right]$$
 (2)

where  $\alpha$  is the coefficient of relative risk aversion and  $x_t$  is defined as the growth rate of dividends:  $x_t = \log(d_t/d_{t-1})$ ; therefore, the current price-dividend ratio is the conditional forecast of a composite variable, function of the future price-dividend ratio and the future realization of the growth rate of the dividends.

To solve the model it is necessary to specify a stochastic process for the rate of growth of dividends which is assumed to be a stationary autoregressive process of order one with mean  $\bar{x}$  and variance  $\sigma^2 = \sigma_{\varepsilon}^2/(1-\rho^2)$ :

$$x_t \equiv \bar{x} + \rho \left( x_t - \bar{x} \right) + \varepsilon_t \quad |\rho| < 1, \quad \varepsilon_t \sim N \left( 0, \sigma_{\varepsilon}^2 \right)$$
 (3)

Solving the model under rational expectations (with  $\hat{E}_t$  representing the mathematical expectation operator) and following the approach in Lansing (2010) the fundamental pricedividend ratio is obtained as a function of the structural parameters of the economy, i.e. the coefficient of relative risk aversion,  $\alpha$ , the discount factor,  $\beta$ , and the parameters governing the stochastic growth rate of the exogenous process for the dividends:

$$\frac{p_t}{d_t} = \exp(a_0 + a_1 \rho \left(x_t - \bar{x}\right) + \frac{1}{2} a_1^2 \sigma_{\varepsilon}^2) \tag{4}$$

where

$$a_1 = \frac{1 - \alpha}{1 - \rho\beta \exp\left[\left(1 - \alpha\right)\bar{x} + \frac{1}{2}a_1^2\sigma_{\varepsilon}^2\right]} \quad a_0 = \log\left[\frac{\beta \exp\left(\left(1 - \alpha\right)\bar{x}\right)}{1 - \rho\beta \exp\left[\left(1 - \alpha\right)\bar{x} + \frac{1}{2}a_1^2\sigma_{\varepsilon}^2\right]}\right].$$

Then the rational expectation solution to this model implies that the price dividend ratio is time-varying<sup>3</sup> and it depends on the deviation of the current realization of the growth of dividends from its mean.

Table 1: Calibration, Canada

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Parameter	Description	Calibrated to match:	Value	
$\bar{x}$	mean of the growth rate	mean of the growth rate of		
	of dividends	the imputed rents (Q to Q)	0.42	
ho	autocorrelation	autocorr growth rate of		
	growth rate of dividends	the imputed rents (Q to Q)	0.868	
$\sigma_{arepsilon}^2$	variance of the errors of growth	variance of the growth rate of		
	rate of the dividend process	the imputed rents (Q to Q)	0.0928	
$\alpha$	coefficient of relative risk aversion	match price/dividend ratio		
		for stock prices	2	
$\beta$	discount factor	match real rate of $4\%$	0.9902	

Note: this table shows the calibration exercise for the asset prices model described in Section 1. In particular it provides with the calibrated values for the parameters in equation (4). The parameters  $\bar{x}$ ,  $\rho$  and  $\sigma_{\varepsilon}^2$  are the sample mean, autororrelation and variance of the imputed rent series over the sample 1988Q2-2010Q4,  $\beta$  is set to 0.9902 for quarterly data consistently with the literature and the value of  $\alpha$  is chosen such that the price-dividend ratio implied by equation (4) matches the sample average of the price-dividend ratio for stock prices in Canada. Specifically we solve for  $\alpha$  such that  $\sum_{t=1}^{T} p_t^A/d_t^A = \frac{0.9902 \exp\left((1-\alpha)\bar{x} + (1-\alpha)^2\sigma_{\varepsilon}^2/2\right)}{1-0.9902 \exp\left((1-\alpha)\bar{x} + (1-\alpha)^2\sigma_{\varepsilon}^2/2\right)} \text{ where } \sum_{t=1}^{T} p_t^A/d_t^A \text{ is the sample average of the price-dividend ratio, } \bar{x} \text{ and } \sigma_{\varepsilon}^2$  are the sample mean and variance of the real per capita consumption growth rate. We repeat the exercise for three samples (1988Q2-2010Q4, 1961Q2-2010Q4, 1992Q4-2010Q4) and the value of  $\alpha$  obtained equals 2 for all samples.

<sup>&</sup>lt;sup>3</sup>Assuming no autocorrelation in the dividend growth process would lead to a constant price-dividend ratio (see Lansing 2006).

The fundamental price-rent ratio can be obtained once we assign values to the parameters in (4). Given the frequency of our data we interpret the length of each period as being a quarter. Under the calibration in Table 1 the parameters  $\bar{x}$ ,  $\rho$  and  $\sigma_{\varepsilon}^2$  are estimated from the process for the growth rate of the imputed rent series in (3),  $\beta$  is set to 0.9902 a common value for the discount factor for data at the quarterly frequency and the value of  $\alpha$  is chosen such that the price-dividend ratio implied by equation (4) matches the sample average of the price-dividend ratio for stock prices in Canada<sup>4</sup>. We simulate the model given the parameterization in Table 1 and obtain that the price-rent ratio should slightly fluctuate around 70 in every period. Is the prediction of the model consistent with the data? Figure 2 plots the simulated data from the model (left panel) and the actual price-rent ratio for Canada. In computing the actual price-dividend ratio the price series is based on the Teranet house price index<sup>5</sup>, while the dividend series is obtained as the average imputed rent<sup>6</sup> of the owned and occupied housing stock. Figure 2 shows that the actual exhibits strong persistence and it fluctuates substantially throughout the sample while from equation (4) the model delivers the prediction that the price-dividend ratio should be fairly stable around the unconditional mean across time despite the high autocorrelation in the dividend growth process.

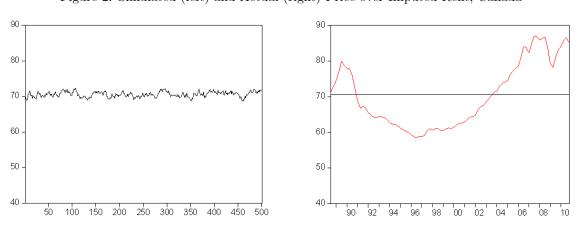


Figure 2. Simulated (left) and Actual (right) Price over Imputed Rent, Canada

Note: this figure shows the simulated (left panel) and actual (right panel) price-rent ratio for Canada over the sample 1988Q2-2010Q4. The simulated data are generated from equation (4) and (3) under the calibration in Table 1.

<sup>&</sup>lt;sup>4</sup>When computing the price-dividend ratio for the stock market,  $\bar{x}$  and  $\sigma_{\varepsilon}^2$  were calibrated on the growth rate of real per-capita consumption.

<sup>&</sup>lt;sup>5</sup>The Teranet National Composite House Price Index is available from the year 2000. Data prior to 2000 is constructed using the Royal LePage House Price Survey.

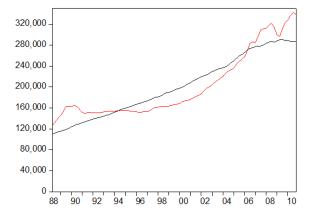
<sup>&</sup>lt;sup>6</sup>The imputed rents series available from the National Accounts is constructed as the paid rent adjusted by a coefficient of quality to take into account the higher quality of owner-occupied dwellings.

However the model can match the first moment of the data: the sample average of the price-rent ratio is 70 which coincides with the prediction from the model. Note that this is not by construction as the coefficient of relative risk aversion is calibrated to match the sample average of the price-dividend ratio for the stock market rather than the house-market.

Next we conduct a counterfactual historical simulation for the price-rent ratio and the price series by feeding the exogenous dividend process into the model. Figure 3 plots the actual and simulated price-rent ratio (left panel) and prices (right panel). From figure 3 it is possible to identify the changes in actual house prices necessary to equalize the actual price-rent to the price-rent value implied by the model and therefore to compute the current misalignment in prices. As of 2010Q4, assuming no change in the imputed rent series, the correction in nominal prices necessary to bring the price-rent ratio to its long-run value is about 17%, slightly down from the 20% required in 2007Q3. For comparison, the correction would have been 13% in the last house prices appreciation at the end of the 80s.

90 80 - 70 - 60 - 50 - 90 92 94 96 98 00 02 04 06 08 10

Figure 3. Fundamental and Actual Price-Rent (left) and Price Series (right), Canada



## 3 Implications of Extrapolative Expectations

We have seen in the previous section that a rational expectation model with a sensible parametrization can match the first sample moment of the price-dividend ratio in Canada, but it cannot generate the large and persistent fluctuations observed in the data. We are interested in identifying models that can generate these features of the data so we examine the role of different expectation formation mechanisms in driving house prices. In particular we abandon the assumption of rational expectation and assume that agents form expectations in an extrapolative fashion, i.e. they rely on past realizations to form expectations about the

variables of interest. Departing from rational expectations is not unusual in the literature that analyses and explains momentum and bubbles in asset prices (Lansing 2006 and 2010, Adams et al. 2009, Fuster et al. 2011). Other factors, above all credit conditions, might play a role in determining the evolution of the series, nevertheless the framework adopted in this paper attributes the movements of the series away from the fundamental values exclusively to the extrapolative behavior of the agents. We follow the approach in Lansing (2006) and we assume that agents form expectations in an extrapolative fashion so that:

$$\hat{E}_t z_{t+1} = H z_{t-1} \qquad H \in (0, H^{\text{max}})$$
 (5)

where  $z_t \equiv \beta \exp\left((1-\alpha)x_t\right) \left(\frac{p_t}{d_t}+1\right)$  is a composite variable that depends on the pricerent ratio and on the growth rate of dividends<sup>7</sup>. The extrapolation parameter H represents the weight that agents put on past values of observables to form expectations on future values. Then by substituting (5) in (2) the price-rent ratio can be rewritten as:

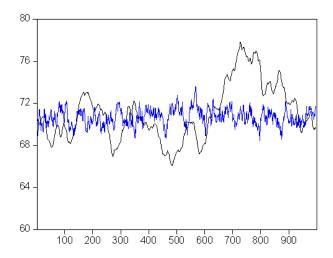
$$\frac{p_t}{d_t} = E_t \left[ z_{t+1} \right] = H z_{t-1} = H \beta \exp\left( (1 - \alpha) x_{t-1} \right) \left( \frac{p_{t-1}}{d_{t-1}} + 1 \right)$$
 (6)

so that the price-dividend ratio is a function of past values of itself and of the past realizations of the dividend growth process. Therefore extrapolative expectations add persistence in the model.

Figure 4 shows the simulated price-rent ratio implied by the model under rational expectations (blue) or under extrapolative expectations (black) under the calibration in Table 1 and for H=1. For a given sequence of shocks generated from (3) the simulated values under rational expectations and under extrapolative expectations are obtained from equation (4) and (6) respectively. From Figure 4 it emerges that the model under extrapolative expectations still matches the sample average, it amplifies the variations around the mean and can generate persistent deviations from the mean.

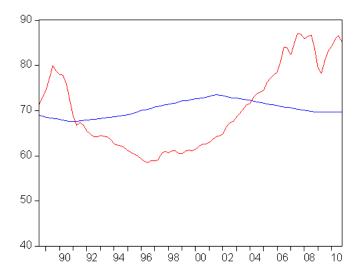
<sup>&</sup>lt;sup>7</sup>The expectation at time t+1 depends on the past realization (at t-1) rather than the current realization (at t). This is a common assumption in the learning literature.

Figure 4. Simulated Price-Rent Ratio under Rational and under Extrapolative Expectations, Canada



A question of interest is whether the model can explain not only some moments of the data but also the realizations of the price-rent ratio for the sample 88Q2 to 10Q4. Then similarly to the previous section we carry out a counterfactual historical simulation for the price-rent ratio by substituting the observed exogenous dividend process into the model solved under extrapolative expectations and plot the counterfactual price-rent series in Figure 5.

Figure 5. Counterfactual Price-Rent Ratio under Extrapolative Expectations, Canada



Although Figure 4 suggests that the model under extrapolative expectations is more successfull than the model under rational expectations in generating the persistence and

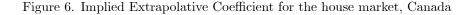
large deviation observed in the data, Figure 5 shows that the counterfactual price-rent ratio is not consistent with the evolution of the data in the particular sample at hand. The counterfactual data for the extrapolative expectation model are generated assuming the extrapolation parameter constant and equal to one.

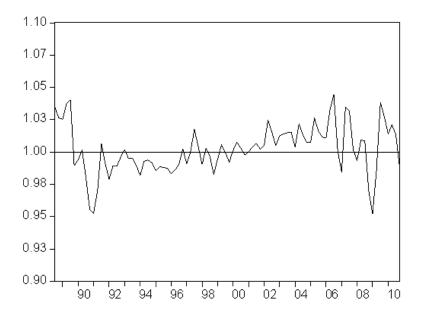
So we ask what is the value of the parameter H that would result from the model at each date t given the realization of the price-dividend ratio and of the dividend growth process over the sample 1988Q2-2010Q4. Therefore, instead of solving the model under extrapolative expectations for a fixed H, we are interested in the evolution of the extrapolative coefficient over time. For any given time t the value of  $H_t$  can be backed out from (6) as:

$$H_t = \left(\frac{p_t}{d_t}\right) \left[\beta \exp\left((1-\alpha)x_{t-1}\right) \left(\frac{p_{t-1}}{d_{t-1}} + 1\right)\right]^{-1}$$
(7)

Because at time t all variables that appear on the left hand side of equation (7) are observable, we can compute  $H_t$  directly from the data once the values for the discount factor and the coefficient of relative risk aversion are selected.

In Figure 6 we plot the time series of  $H_t$  for the sample 1998Q2 to 2010Q4 when  $\beta = 0.9902$  and  $\alpha = 2$ .



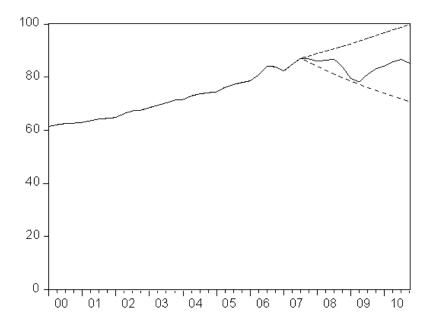


A value of the extrapolative coefficient greater than one is consistent with explosive behavior of the series, while when the extrapolative coefficient is lower than one the series will decrease over time. Also when H>1 (H<1) agents believe future realizations will be greater (lower) than past realizations. From Figure 6 it emerges that the extrapolative time-varying weight in the house market is centered around one but it is volatile and it is persistently above one in the period 2001Q1-2006Q4 consistently with the run up in the house prices. To illustrate the impact of the magnitude of the extrapolative coefficient we run two counterfactual exercises by first fixing  $H_t$  to its average conditional on being larger than one,  $\bar{H}^H = 1.014$ , then to its average conditional on being lower than one,  $\bar{H}^L = 0.9863$  and counterfactual data for both values of  $\bar{H}$  are generated according to:

$$\frac{p_t}{d_t} = \bar{H}^i \left[ \beta \exp((1 - \alpha)x_{t-1}) \left( \frac{p_{t-1}}{d_{t-1}} + 1 \right) \right] \quad i = \{L, H\}$$

The starting point for the generation of the simulated series is 2008Q1, the quarter following 2007Q4 which marks the peak of the price-imputed rent ratio before the crisis. The actual and simulated series for the remaining of the sample are shown in Figure 7. If H had been larger than one for the last 12 quarters in the sample the price-dividend ratio would have surged to 99 by the end of 2010Q4, while if it had been lower than one it would have plunged to the model implied fundamental value.

Figure 7. Actual and Simulated Price-Rent Series, Canada



# 4 A Comparison to the US Experience

The protracted downturn in the US housing market is triggering concerns that the Canadian house market might experience a significant and sudden correction in the house prices similar to that of the US.

To illustrate the similar dynamics of the price series for the two countries Figure 8 plots the nominal house prices for the US (blue) and Canada (red) where the prices for the US are expressed in Canadian dollars. The figure shows that the Canadian house prices in 2010Q4 were approximately the same as the US prices when they reached their peak (in Canadian dollar terms) in 2007Q1 suggesting that Canadian housing prices might have surged to an alarming level. However looking at the row price series might be misleading as we should compare the evolution of price series relative to its fundamental value. We propose a simple way to compare the US and Canadian experience that takes into account the price-rent ratio fundamentals in each country.<sup>8</sup>

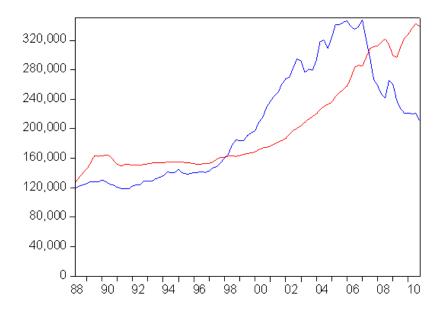


Figure 8. Nominal House Prices, Canada and US

Note: Nominal house prices for Canada (red) and US (blue). Prices for the US series are expressed in Canadian dollars.

To carry out the comparison we repeat the analysis of Section 2 and Section 3 for the US

<sup>&</sup>lt;sup>8</sup>Again, our simple analysis abstracts from banking and fiscal regulations such as the fiscal traetment of the mortgage interest and mortgage securizations, which differ across the US and Canada and are important determinants of the housing market.

housing market. First the fundamental price-rent ratio is computed from equation (4) and under the calibration in Table 2.

Table 2: Calibration, US

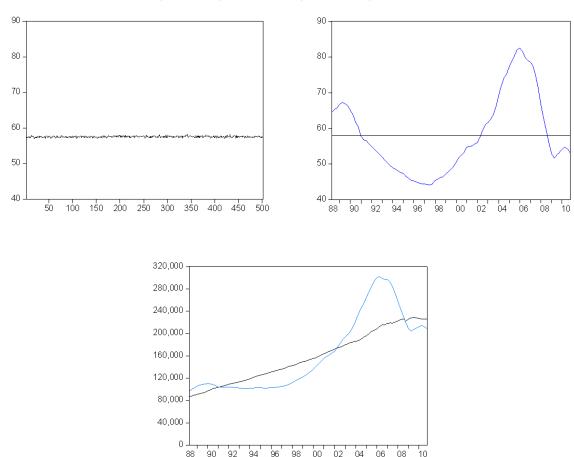
Table 2. Cambration, CD				
Parameter	Description	Calibrated to match:	Value	
$\bar{x}$	mean of the growth rate of dividends	mean of the growth rate of the imputed rents (Q to Q)	0.36	
ρ	autocorrelation growth rate of dividends	autocorr growth rate of the imputed rents (Q to Q)	0.212	
$\sigma_{arepsilon}^2$	variance of the errors of growth rate of the dividend process	variance of the growth rate of the imputed rents (Q to Q)	0.0928	
$\alpha$	coefficient of relative risk aversion	match price/dividend ratio for stock prices	2	
$\beta$	discount factor	match real rate of $4\%$	0.9902	

Note: this table shows the calibration exercise for the asset prices model described in Section 1. In particular it provides with the calibrated values for the parameters in equation (4). The parameters  $\bar{x}$ ,  $\rho$  and  $\sigma_{\varepsilon}^2$  are the sample mean and autocorrelation of the US imputed rent series and variance over the sample 1988Q2-2010Q4,  $\beta$  is set to 0.9902 for quarterly data consistently with the literature and the value of  $\alpha$  is chosen such that the price-dividend ratio implied by equation (4) matches the sample average of the price-dividend ratio for stock prices in the US. Specifically we solve for  $\alpha$  such that  $\sum_{t=1}^{T} p_t^A/d_t^A = \frac{0.9902 \exp\left((1-\alpha)\bar{x} + (1-\alpha)^2\sigma_{\varepsilon}^2/2\right)}{1-0.9902 \exp\left((1-\alpha)\bar{x} + (1-\alpha)^2\sigma_{\varepsilon}^2/2\right)} \text{ where } \sum_{t=1}^{T} p_t^A/d_t^A \text{ is the sample average of the price-dividend ratio, } \bar{x} \text{ and } \sigma_{\varepsilon}^2$  are the sample mean and variance of the real per capita consumption growth rate over the sample 1988Q2-2010Q4.

Figure 9 plots the fundamental price-dividend ratio (upper left) and the price series (lower panel) under rational expectations and the evolution of the US house price-imputed rent ratio (upper right) over the sample 1988Q2-2010Q4. The house price series is based on the S&P Case Shiller index<sup>9</sup>.

<sup>&</sup>lt;sup>9</sup>The Teranet and S&P Case Shiller are both measuring the average resale price of existing homes over major cities in Canada and the US respectively. The major difference between the two indices is that the Case Shiller index does not include condominiums in its computation.

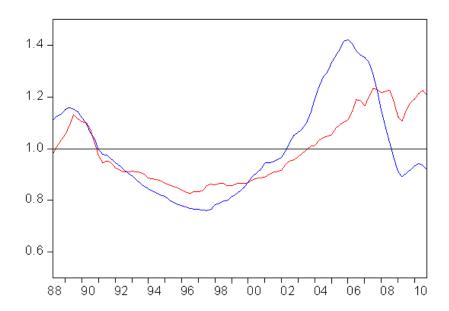
Figure 9. Simulated (upper left) and Actual (upper right) Price over Imputed Rent, US



For  $\alpha=3$  the price-imputed rent ratio predicted by the model solved under rational expectations is almost constant around 58, matching exactly the sample average. As it was the case for Canada, this simple model can capture the long run dynamics of the data. The current price dividend ratio lies below the fundamental value and given the current price imputed rent ratio an adjustment in the nominal price level of 9% would be required to bring the price-rent ratio back up to its fundamental value if there were no variation in the nominal imputed rents. The fundamental value predicted by the model for the US price-rent ratio is substantially lower than the fundamental value predicted for Canada. This is due to a higher calibrated coefficient of relative risk aversion for the US.

We compare the price-imputed rent ratio for Canada and the US in Figure 10. Because the fundamental values for the two series differ substantially, instead of plotting the crude series, we plot the value of the series relative to their fundamental  $\left(\frac{p_t}{d_t}/\frac{p_t^*}{d_t^*}\right)$ . Therefore, when the series is greater (smaller) than one, the price over imputed rent lies above (below) its model-implied value.

Figure 10. Price to Rent Ratio: Actual over Fundamental Canada and US



The figure shows that the fluctuations of the price-rent ratio for the US are larger than for Canada. For example, at the price-rent ratio peak occurring in 2006Q1 the ratio for the US was 30% higher than the fundamental values, while in Section 2 we computed a misalignment of 20% in the pick quarter for Canada. However during the financial crisis the price-rent ratio in the US quickly reverted back close to fundamentals, while for Canada the misalignment has persisted through time. The two series are highly correlated: for the full sample 1988Q2-2010Q4 the correlation is 70% but for the sub-sample before the crisis it reaches 93%.

The left panel of Figure 11 plots the implied extrapolative coefficient for the US house market  $(H_t)$  which is obtained as described in Section 3 for Canada. The Canadian house market extrapolative coefficient has been constantly above one for a sustained period of time from 2001Q1 to 2006Q4. A similar pattern can be observed for the US with an increase above one of the extrapolative coefficient from 1997Q3 to 2006Q2. Comparing  $H_t$  for the US and Canada (see Figure 11 right panel), the two series move together till the end of 2005 (their correlation coefficient is 55%) whereas the two series differ greatly during the last recession when the extrapolative expectations parameter dropped sharply for the US (and the correlation coefficient for the full sample decreased to 30%) instead for Canada it dropped for two quarters in 2008Q4 and 2009Q1 but it bounced immediately back to values higher than one. The larger misalignments in the price-dividend ratio observed for the US

are accompanied by larger values of the extrapolation parameter in the period 1997-2006. An extrapolation parameter consistent with a reduction in the US price-dividend series is associated with the house price downturn of the past three years.

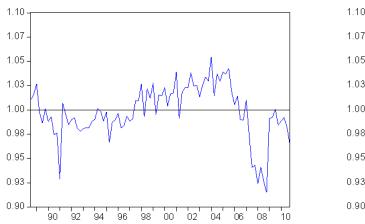
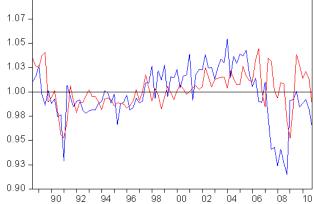


Figure 11. Implied Extrapolative Coefficient, US (left) Canada and US (right)



Note: the left panel plots the values of the extrapolation parameter for the US; the right panel plots the values of the extrapolation parameter for the US in blue and for Canada in red.

## 5 Conclusion

By treating houses simply as a financial asset this paper uses a Lucas' tree model to derive a fundamental value for the price-rent ratio in Canada. This value matches the historical average of the price to rent series. We compute the misalignment in prices as the correction that prices should take in order to bring the price-dividend ratio down to its fundamental value, assuming no change in the value for the rent. However the model solved under rational expectations does not generate persistent and substantial deviations from the mean and it does explain the protracted surge in prices of the last decade. In order to capture these features of the data we abandon the assumption of rational expectations solve the model by assuming that agents form their expectations in an extrapolative fashion, basing their expectations on past realizations of the data. This introduces persistence in the model. However, although the model under extrapolative expectations is more successfull than the model under rational expectations in generating persistence and large deviation of the price-rent ratio a historical counterfactual simulation shows that the extrapolative is not consistent with the evolution of the data in the particular sample at hand as it fails to capture the substantial build up in prices. The counterfactual data for the extrapolative expectation

model are generated assuming the extrapolation parameter constant and equal to one. So one promising extension of this paper would be to model the dynamics of the extrapolation parameter H.

A further counterfactual exercise aimed at backing out the value of the extrapolation parameter consistent with the realization of the price-rent and dividend series over the sample 1988Q2-2010Q4 shows that extrapolative expectations triggering an explosive behaviour of the price-imputed rent series are consistent with the evolution of the price-imputed rent ratio for the period 2001Q1-2006Q4.

We suggest to conduct a comparison with the US housing market by looking at deviations of the countries price-dividend ratio from its fundamental. Then, although the Canadian price-rent ratio appears high relative to the model implied value the current misalignment is less severe than the misalignment reached by the US before the last crisis.

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