CBDC, Monetary Policy Implementation, and The Interbank Market*

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PRELIMINARY AND INCOMPLETE — COMMENTS ARE WELCOME

Abstract

We analyze the impact of banks intermediating CBDC on the interbank market. When banks increase their CBDC accounts by $1 they need $1 of reserves. Increasing CBDC usage drains reserves and may increase the interbank rate. The effect of CBDC remuneration is unclear: It makes CBDC a more attractive means to pay, thus reducing funding costs (reducing the drain in reserves) and encouraging investment (increasing the drain in reserves). A cap on CBDC holdings reduces interbank and commercial deposit rates, as banks require fewer deposits to buy reserves. Tiered remuneration does not provide additional benefits over a single (lower) rate.

Keywords: Central bank digital currency (CBDC), monetary policy, interbank markets, payments

JEL-Codes: E42, E58, G21

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1 Introduction

Central bank digital currencies are high on the agenda of central banks. Some central banks already launched pilots to understand the implications of different CBDC designs.[1] While the effects on the economy and the financial systems have been extensively studied, fewer studies seek to understand the effects of CBDC on the market for bank reserves and its implications for the implementation of monetary policy.[2] This paper seeks to fill that gap by analyzing the effects of CBDC on the demand for reserves in a model of the interbank market featuring uncertainty in the form of a Poole (1968) shock.

Specifically, we embed the environment of Chiu et al. (2022) in a Poole (1968) model. We simplify the analysis by assuming that banks have no market power when issuing deposits. In an initial round of investment, banks finance some entrepreneurs with deposits. Banks are subject to a reserve requirement as a function of the amount of deposits they issue. Banks can acquire these reserves in two ways: They can issue deposits, or they can borrow reserves on the interbank market. After the market for reserves closes, banks face a refinancing shock: they learn that a (stochastic) fraction of entrepreneurs they fund needs to purchase more inputs, either with deposits or with CBDC. This is equivalent to the Poole (1968) shock in our model. At this stage, banks only have access to the central bank's deposit and lending facilities. We assume that CBDC can be bought by the banks with reserves directly at the central bank. Since CBDC has a 100% reserve requirement in our model, a bank may have to borrow at the lending facility an amount of reserves equal to the full amount of CBDC needed by the entrepreneurs to purchase more input goods.

In this context, we study the effects of increasing the market share of CBDC (the chance that entrepreneurs need CBDC), as well as the effects of the remuneration rate of CBDC on the interbank market rate and the level of banks' investment. We also study the effects of a tiered remuneration system as well as limits on CBDC holdings.

We obtain several results, some of which are surprising. First, as expected, an increase in the market share of CBDC imposes a drain on reserves. As a result, the interbank market rate tends to increase and banks tend to more often access the lending facility. Since reserves become more expensive, everything else constant, there will be disintermediation in the sense of lower investment. However, there will

[1] These include for example the Bank of China's digital Renminbi or the Sand dollar issued by the Central Bank of The Bahamas.

[2] A good reference is the special issue of the Journal of Economic Dynamics and Control (2022), as well as the CEPR report edited by (editor).

not necessarily be disintermediation in the sense of a reduction of banks' liabilities because banks will seek to "attract" deposits in order to increase their reserve holdings by raising their deposit rates. Of course, the central bank could limit the drain in reserves and, therefore, the impact of CBDC on the money markets by supplying more reserves. Hence, CBDC demand acts like autonomous liquidity factors and its detrimental effects on the interbank market can be undone by lengthening the balance sheet of the central bank.[3]

Second, and more surprisingly, we find that the effect of a higher CBDC remuneration rate on the money market is ambiguous, because it relies on two effects. By paying a higher interest rate on CBDC, the central bank increases its purchasing power, thus making it a better payment instrument: the same amount of CBDC can buy more (investment) goods. Therefore, the same level of activity can be sustained with less CBDC, which plays to reduce the demand for reserves. This is the funding effect. However, there is a counteracting investment effect: since it is cheaper to fund entrepreneurs, banks invest more, which in turn puts pressure on the demand for reserves — the investment effect. Which effect dominates depends on the model parameters; specifically on how costly it is for banks to invest with productive entrepreneurs. Also, we leave aside any discussion on the fiscal consequences of the CBDC remuneration rate.[4]

In a next step, we investigate the effect of quantitative limitations on the amount of CBDC holdings — a CBDC design feature that is actively discussed at central banks. The idea of the cap is to limit the conversion of deposits into CBDC. However, it also means that only so much can be purchased with CBDC. As a consequence, entrepreneurs who need CBDC at the refinancing stage will have to liquidate some of their initial investment whenever their refinancing needs exceed the CBDC limit. We find that for a given market share of CBDC or a given CBDC rate, a cap will reduce the demand for reserves and, therefore, the interbank market rate. However, by reducing the effective productivity level, a cap could also decrease banks' investment and deposits. We show that the central bank can reverse this detrimental effect when it combines a tighter cap with a higher CBDC rate.

Finally, we add to our model a two-tiered CBDC remuneration, as this is another design feature in current policy discussions (see, e.g. Bindseil (2020)). Specifically, CBDC holdings up to a certain threshold are remunerated at a higher rate than

[3]Autonomous liquidity factors can be defined as the items in the consolidated balance sheet of the central bank, apart from monetary policy operations, that provide or withdraw liquidity and thus affect the current accounts which credit institutions hold with the central bank, mostly to fulfill their minimum reserve requirements.

[4]See Williamson (2022) for a discussion on this important topic.

holdings above this threshold. We find that a tiered remuneration of CBDC is equivalent to a decrease in the CBDC rate when there is no two-tiered system in place.

The literature on CBDC is growing rapidly (a structured overview is provided by Ahnert et al. (2022a)). Major studies include Andolfatto (2020), Chiu et al. (2022), Keister and Sanches (2022), as well as Williamson (2022). On the effect of CBDC on the fragility of the financial system, see, for example, Keister and Monnet (2022) featuring bank runs, or Assenmacher et al. (2022) who describe a channel through which CBDC could stabilize the economy. Brunnermeier and Niepelt (2019) show that, under certain conditions, the introducion of CBDC does not pose a threat for financial stability. Niepelt (2022) generalizes their equivalence result in a model with reserves. He finds that CBDC can raise banks' funding costs and that the optimal CBDC rate differs form the optimal rate on reserves. On the use of CBDC for online transactions see Garratt and van Oordt (2021) and Ahnert et al. (2022b). For a discussion of the effects of different design features of CBDC on bank disintermediation and welfare, see Assenmacher et al. (2021).

There are only few studies that focus on the demand for bank reserves in the presence of CBDC. Malloy et al. (2022) illustrate through stylized balance sheet analyses that CBDC could decrease aggregate reserves, putting upward pressure on the policy rate. Fegatelli (2021) points out that CBDC could improve banks' profitability by reducing the amount of potentially expensive excess reserves. We add to this literature by developing a dynamic general equilibrium model with uncertainty regarding the amount of reserves needed. Through the lens of our model, we can analyze the effects of an increasing adoption of CBDC, as well as its remuneration rate, on the money market, bank deposits, and investment. Furthermore, we implement in our model different CBDC design features that are currently discussed at central banks, such as a holding limit or a tiered CBDC remuneration.

2 Environment

Our model combines elements of the money market model of Berentsen and Monnet (2008) and the CBDC model of Chiu et al. (2022). Time $t = 1, 2, ...$ is discrete and continues forever. The common discount factor is β. There are four types of agents: a measure one of buyers, sellers, and bankers, and a large measure (greater than one) of entrepreneurs. In addition, there is a central bank that manages the supply of reserves and CBDC, and offers a lending and a borrowing facility. In each period, two (goods) markets open sequentially: goods market 1 (M1) and goods market 2 (M2). Both markets are Walrasian. An interbank market for central bank reserves opens at the same time as goods market 1.

Buyers and sellers Buyers and sellers are infinitely lived. Buyers have utility $u(Y)$ from consuming Y units of market 1 goods. $u(.)$ is increasing, strictly concave, and $u(0) = 0$. Sellers have a linear disutility $-Y$ of producing Y units of market 1 goods. As a result, the efficient level of market 1 good consumption is Y^* such that $u'(Y^*) = 1$. Buyers cannot commit and promises cannot be enforced. Therefore, buyers need a means of payment to pay sellers on market 1. As will be clear below, they will use CBDC, that is digital cash issued by the central bank, and/or bank deposits.

In market 2, buyers and sellers work and consume X units of the consumption good. Their labor h is transformed into market 2 goods, one-for-one. The utility of consumption is $U(X)$, increasing, and strictly concave. All in all, buyers' and sellers' instant utility function for any period, given an allocation (Y, X, h), is respectively

$$\begin{aligned} U^B(Y, X, h) &= u(Y) + U(X) - h \\ U^S(Y, X, h) &= -Y + U(X) - h. \end{aligned}$$

Entrepreneurs Entrepreneurs are born young in market 2 of period t, become old in $t + 1$ and die in market 2 of period $t + 1$. So they live for 1 and 1/2 period. Entrepreneurs cannot work in market 2 and they consume only when old. However, young entrepreneurs are endowed with a technology $F(X, Y) : \{0, 1\}^2 \to \mathbb{R}_+$. The technology has two different returns depending on the state of nature,

$$F(X, Y) = \begin{cases} AX & \text{in state 1} \\ AXY & \text{in state 2} \end{cases},$$

where $X, Y \in \{0, 1\}$. In words, young entrepreneurs make an initial investment $X \in \{0, 1\}$ of market 2 goods. In state 1 they produce AX in the next market 2 when they are old. In state 2, entrepreneurs who have made an initial investment of $X = 1$ need to purchase $Y = 1$ of market 1 goods in order to bring their project to fruition. We interpret state 2 as a re-financing state. It will become clear that the probability of each state is only important for bankers and we will specify it later. In state 2, entrepreneurs will purchase goods from sellers in market 1. However, there is no credit arrangement possible between entrepreneurs and sellers, and the former need a means to pay the latter. As will be clear below, they will use CBDC and/or bank deposits.

Bankers Bankers have the same life span as entrepreneurs: they are born young in market 2 of period t, become old in period $t+1$, and die in market 2 of period $t+1$. Bankers only consume market 2 goods when old. They have a commitment technology to repay their liabilities, therefore, bank deposits can be used as a means to pay. Also, bankers have a technology to enforce repayment from entrepreneurs and so they are willing to lend to entrepreneurs. There is free entry into the business of lending to entrepreneurs. A banker who lends to n entrepreneurs will suffer a utility cost $c(n)$. This can be equally interpreted as a search cost, a screening cost, or a monitoring cost. What is important is that $c(n)$ is increasing and convex. Bankers face uncertainty regarding the refinancing shock of entrepreneurs: the fraction of state 2 entrepreneurs in a banker's portfolio is $\gamma \in [0,1]$ and γ is distributed according to a distribution function with cdf $G(\cdot)$. A banker does not know his own γ when choosing n and only learns it in market 1, and importantly, *after* the interbank market has closed.

A banker finances its loans in market 2 at date t and market 1 at date $t+1$ by issuing checkable deposits to buyers and entrepreneurs, and possibly IOUs to other banks and the central bank. A banker's checkable deposits can be used as a medium of exchange in market 1 between sellers and buyers or entrepreneurs who need reinvestment. Finally, bankers are subject to a reserve requirement: they must hold in reserves at least a fraction $\chi \in [0,1]$ of the amount of checkable deposits they issued. Cash holdings qualify as reserves. Required reserves are remunerated at rate i_r, which can be negative.

Young bankers can acquire reserves/cash by issuing deposits to buyers or old bankers (who will then use those deposits to purchase goods in market 2). Young bankers can also borrow reserves in the interbank market or from the central bank lending facility. Banks can purchase CBDC from the central bank by spending reserves.

We stay close to Poole (1968) by assuming that bankers can trade reserves on an interbank market before learning their shock γ. The interbank market rate is R_m. As in Poole (1968), there is no reason why banks would trade reserves on this market (since they are identical and they have the same information), but there is an equilibrium interest rate R_m that will leave them indifferent between borrowing or lending reserves.[5]

[5] A feature of the Poole model is the (uninsurable in the money market) end-of-day uncertainty on the level of reserves that bankers face. We could also assume that the interbank market opens after banks learn γ, but we would depart from the structure of the standard Poole model.

Government/Central bank The central bank issues three forms of liabilities: cash, central bank reserves, and CBDC. Only bankers have access to reserves. CBDC is a digital entry on the central bank's balance sheet that can be used for retail payments, and it pays a net rate i_e, which can be negative. In our model, one can interpret CBDC as a type of commercial bank deposits carrying a 100% reserve requirement. Being ear-marked for CBDC, these reserves are remunerated at the rate i_e (and whoever holds the CBDC can redeem it at the central bank and get the interest).

The central bank operates standing facilities for reserves: It offers a lending facility and stands ready to lend any amount of reserves bankers require at a rate $i_{\ell f}$. Similarly, it offers a deposit facility and remunerates at a rate i_{df} any amount of reserves that bankers would deposit. The central bank also manages the supply of cash in the economy by making lump-sum transfers to buyers. As a result, cash grows at a constant rate $\mu \geq \beta$. Finally, seignorage goes to the government that rebates it through lump-sum transfers to buyers.

Market 1 and payment instruments Sellers are differentiated by the type of payment instruments they accept and they know their types. A fraction $\omega_1 \in [0, 1]$ of sellers only accept deposits, while a fraction $\omega_2 = 1 - \omega_1$ of sellers only accept CBDC. No sellers ever accept cash in our economy. Cearly, one can question the validity of this assumption, but given the declining use of cash in modern economies, we do not see this assumption as being especially strong; even if the interest rate on CBDC is negative, CBDC could be preferred to cash as long as we understand that CBDC has more benefits than cash, e.g. it is useful to trade online.

To keep things simple, we assume that those sellers are located on two separate trading venues, trading venue 1 and 2 (tv-1 and tv-2) and both trading venues are Walrasian markets (with possibly different prices). Buyers and entrepreneurs cannot choose their trading venues and we assume that with probability ω_1 they enter tv-1 and with probability ω_2 they enter tv-2.[6] Since buyers have no use for cash, they will not bring any into market 1. This does not mean, however, that cash will have no value, but it will only be used in market 2: Young bankers will hold it as reserves, old bankers will use it to pay the interest on deposits previously issued and to consume in market 2, sellers will obtain the interest on the deposits they hold when entering market 2 and use the cash to consume from buyers, and buyers will use cash to purchase the young bankers' deposits.

[6]We could have assumed that buyers and entrepreneurs meet randomly sellers, at the cost of having to introduce tools from bargaining. We chose to keep market 1 as simple as possible while using some ideas from models with random matching.

We think of ω_2 as the demand for CBDC. We do not take a stance on the size of ω_2, but it should be related with different design features of CBDC that the model does not capture. For example, a CBDC that does not preserve anonymity may be less likely to be accepted in transactions, thus leading to a lower ω_2.[7]

Timing Figure 1 shows the timing of the economy.

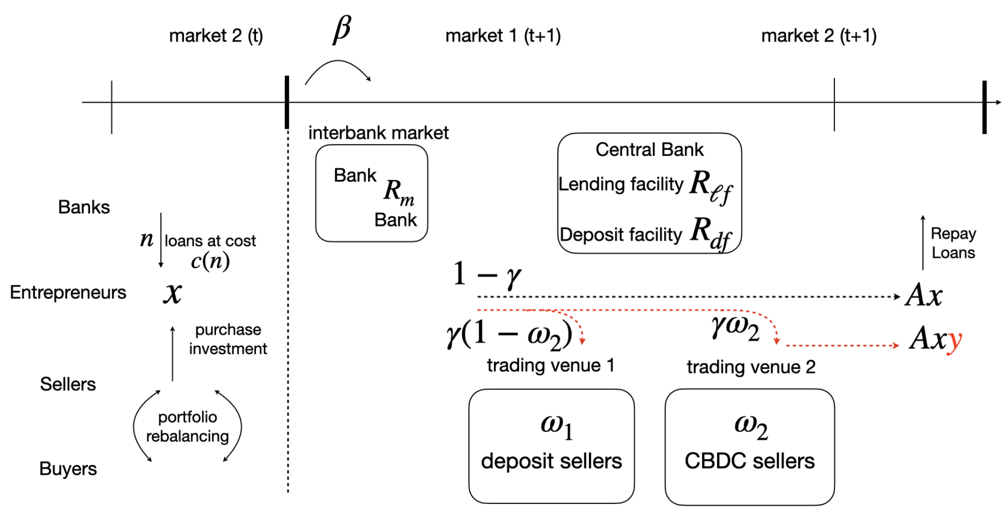

Figure 1: Timeline

We proceed to solve this economy by first looking at the buyers' and sellers' decisions, which are standard. Then we consider the banks' problem. Since entrepreneurs have no bargaining power, the banks' problem is the most interesting to analyze.

3 Agents' problems and market clearing

Buyers' and sellers' problems

The buyers' and sellers' problems are rather standard and follow the same structure as in the basic Lagos and Wright (2005) literature. First, it is clear that a seller will carry no means of payment from market 2 to market 1 since they only produce and

[7]Some features of CBDC that are in the model, such as how the central bank remunerates CBDC holdings, may of course affect the size of ω_2. We choose, however, to separate these effects.

they have no needs to consume in market 1. Therefore, a seller of type m (which refers to the means of payment they accept) solves

$$W_S^m(\mathbf{a}) = \max U(X) - h + \beta V_S^m(\mathbf{0})$$

subject to

$$X = h + \mathbf{R}.\mathbf{a},$$

where $\mathbf{a} = (z, e, d)$ is the vector of real balances (z), real CBDC (e) and real bank deposits (d), while $\mathbf{R}$ is the vector of gross real return $\mathbf{R} = (R_z, R_e, R_d)$ where $R_s = (1 + i_s)/\mu$ with $i_z = 0$.[8] Also

$$V_S^m = \max_Y -Y + p_m Y + W_m^S(0),$$

where the price in trading venue $m = 1, 2$ in terms of market 1 goods is p_m. Hence, the only equilibrium has for both trading venues,

$$p_m = 1.$$

Next we turn to buyers. In market 2, a buyer solves

$$W_B(\mathbf{a}) = \max U(X) - h + \beta V_B(\mathbf{a}')$$

subject to

$$X + \mathbf{1}.\mathbf{a}' = T + h + \mathbf{R}.\mathbf{a},$$

by choosing her net consumption of the market 2 good $x - h$, and her real asset portfolio $\mathbf{a}'$ at current price, given the return on her current portfolio $\mathbf{R}.\mathbf{a}$.

Given the price is $p_m = 1$ in both trading venues $m = 1, 2$, the buyer's value function when entering market 1 is

$$V_B(\mathbf{a}) = \sum_{m=1,2} \omega_m \left[u(Y_m(\mathcal{L}_m)) - Y_m(\mathcal{L}_m) + W_B(\mathbf{a}) \right],$$

where

$$\begin{aligned} \mathcal{L}_1 &= R_d d, \\ \mathcal{L}_2 &= R_e e, \end{aligned}$$

[8]Note that inflation μ will negatively affect R_e but not necessarily R_d since i_e is a policy variable, while the effect on R_d will be determined in equilibrium.

denotes the usable liquidity in trading venue $m = 1, 2$, including the expected return of the asset, and Y_m denotes the amount of good Y that is traded in trading venue m. The buyers' budget constraint in both trading venues is

$$Y_m \leq \mathcal{L}_m \qquad (\lambda(\mathcal{L}_m)).$$

Hence, the first order condition of buyers in trading venue m of market 1 gives

$$u'(Y(\mathcal{L}_m)) = 1 + \lambda(\mathcal{L}_m),$$

with the envelope condition (with some abuse of the vectorial notation)

$$V'_B(\mathbf{a}) \;=\; \sum_{m=1}^{2} \omega_m \left[u'(Y(\mathcal{L}_m)) - 1\right] \mathbf{R}\mathbb{I}_{\{\text{a used in venue } m\}} + \mathbf{R}.$$

The first order condition of buyers in market 2 is

$$\beta V'_B(\mathbf{a}') \leq 1 \quad (= \text{ if } \mathbf{a}' > 0).$$

Combining the last two equations, we obtain the Euler conditions of buyers,

$$\begin{aligned} \omega_1 \left[u'(Y(\mathcal{L}_1)) - 1\right] R_d + R_d &\leq \beta^{-1}, \\ \omega_2 \left[u'(Y(\mathcal{L}_2)) - 1\right] R_e + R_e &\leq \beta^{-1}. \end{aligned}$$

Since buyers do not bring any cash, $R_z < 1/\beta$, and

$$(\beta R_d)^{-1} = \begin{cases} 1 & \text{if } Y^* \leq R_d d \\ \omega_1 u'\left(R_d d\right) + (1 - \omega_1) & \text{otherwise} \end{cases}$$

and

$$(\beta R_e)^{-1} = \begin{cases} 1 & \text{if } Y^* \leq R_e e \\ \omega_2 u'\left(R_e e\right) + (1 - \omega_2) & \text{otherwise} \end{cases}.$$

Notice that since more inflation decreases R_e, inflation affects the amount of CBDC that buyers choose to carry over from market 2 at t to market 1 at $t+1$. Finally, let us stress that sellers have no incentive to acquire deposits (or CBDC) in market 2. Indeed, it would cost them 1 and they would only get R_d next period. The equations above show that $\beta R_d \leq 1$. So, at best, sellers are indifferent between purchasing deposits or CBDC.

Bankers' problem

Young banker j chooses to fund n_j entrepreneurs in market 2 by issuing deposits. Bankers are perfectly competitive in lending and in issuing deposits. In the following market 1, a banker learns that it will have to refinance a fraction γ of these loans, where γ is i.i.d. across bankers. To keep symmetry, we assume that, of those entrepreneurs who need to invest an additional unit, a fraction ω_1 do it in the first trading venue and the rest in the second trading venue. Therefore, a young bank's problem in market 2 is to choose how many entrepreneurs n_j to fund ,[9] how many deposits d_j to issue and how much reserves r_j to hold, to maximize

$$\max_{n_j, r_j, d_j} \quad V\left(n_j, r_j - \chi d_j\right) + \underbrace{R_r \chi d_j}_{\text{int on req. res.}} - R_d d_j \tag{1}$$

subject to

$$\begin{aligned} n_j + \underbrace{r_j}_{\text{cash reserves}} &= d_j, \\ r_j &\geq \chi d_j. \end{aligned}$$

Given the banker's value $V(n, e)$ of holding a portfolio of n loans and excess reserves $e = r - \chi d$, the young banker issues d_j deposits to fund n_j entrepreneurs in market 2, and also to purchase cash reserves r_j from old bankers, buyers and /or sellers. For each deposit it issues, the banker has to pay the equilibrium deposits (real) rate R_d, and it has to set aside a fraction $\chi \in [0, 1]$ of required reserves remunerated at the (real) rate R_r.

Next, in market 1 of period $t+1$, bankers learn the extent to which they have to refinance their loans to entrepreneurs. A banker draws the fraction γ of entrepreneurs it has to refinance from the distribution $G(\gamma)$. Given γ, a banker has to refinance γn entrepreneurs. We assume that entrepreneurs are sufficiently productive such that bankers always refinance entrepreneurs, irrespective of the trading venue they use to purchase the additional unit of investment.[10] Indeed, a fraction ω_1 of those will have to purchase the (investment) good on the first trading venue with deposits, and a fraction ω_2 will need CBDC to purchase the (investment) good on the second trading venue.

[9]Each entrepreneur needs 1 unit of deposits to purchase 1 unit of the investment good in market 2.

[10]A priori, the bank could choose to refinance some but not all entrepreneurs. We assume $A > A^*$ so that the banker chooses to refinance all entrepreneurs on both trading venues.

Therefore, the bank issues $\omega_1\gamma n/R_d$ new deposits to $\omega_1\gamma n$ entrepreneurs so that they can each purchase 1 unit of the investment good with deposits on tv-1. The bank also acquires $\omega_2\gamma n/R_e$ new CBDC from the central bank so that the $\omega_2\gamma n$ entrepreneurs on tv-2 can purchase 1 unit of the investment good with CBDC. At this stage the interbank market is closed, therefore, the bank acquires CBDC by using its excess reserves or borrowing reserves from the central bank.

When new deposits are used as means to pay sellers, the reserve requirement is still $\chi < 1$, while the banker needs the full amount of reserves to purchase CBDC on behalf of entrepreneurs. Therefore, at the refinancing stage, the banker has $r_j - \chi d_j$ reserves that it carried over from the last period's market 2, plus the amount of reserves y_j the banker borrowed on the interbank market, and it needs an amount of reserves equal to $\frac{\omega_1\gamma n_j}{R_d}\chi + \frac{\omega_2\gamma n_j}{R_e}$. Hence, the banker has a reserve shortfall if

$$\frac{\omega_1\gamma n_j}{R_d}\chi + \frac{\omega_2\gamma n_j}{R_e} - (r_j - \chi d_j + y_j) \equiv \sum_{s=1,2}\chi_s\omega_s\gamma n_j - (r_j - \chi d_j + y_j) > 0 \quad (2)$$

with

$$\chi_1 \equiv \chi/R_d \quad \text{and } \chi_2 \equiv 1/R_e.$$

If the inequality in (2) is reversed, the banker has a reserve surplus. It should be clear that inequality (2) holds whenever the refinancing shock is large enough, that is if $\gamma > \bar{\gamma}$, where $\bar{\gamma}$ is

$$\bar{\gamma}(d_j, n_j, y_j) = \frac{(r_j - \chi d_j + y_j)}{\sum_s \omega_s\chi_s n_j}.$$

Since the refinancing shock γ is stochastic and the interbank market closes before bankers learn their γ, we obtain an expression for $V(n, e)$ that is close to the one in Poole (1968):

$$\begin{aligned} V(n_j, r_j - \chi d_j) &= An_j - c(n_j)/\beta \qquad (3)\\ &+ \max_{y_j}\left[\begin{array}{c} \int_{\gamma<\bar{\gamma}(d_j,n_j,y_j)} R_{df}\underbrace{\left((r_j - \chi d_j + y_j) - \sum_s \omega_s\chi_s\gamma n_j\right)}_{\text{long in reserves - can deposit @ DF}} dG(\gamma) \\ -\int_{\gamma\geq\bar{\gamma}(d_j,n_j,y_j)} R_{\ell f}\underbrace{\left(\sum_s \omega_s\chi_s\gamma n_j - (r_j - \chi d_j + y_j)\right)}_{\text{short in reserves - needs to borrow @ LF}} dG(\gamma) - R_m y_j \end{array}\right] \\ &- \int \omega_1\gamma n_j dG(\gamma) + \int \underbrace{\frac{\omega_1\gamma n_j}{R_d}\chi R_r}_{\text{interest on required reserves}} dG(\gamma) \end{aligned}$$

The first line of (3) shows the gain from investing with n_j entrepreneurs, net of the investment cost which occurred at the end of the previous period. The second line of (3) shows the problem of the banker on the interbank market: When the banker faces a low enough refinancing shock ($\gamma < \bar{\gamma}$) it has a reserve surplus after refinancing all entrepreneurs and can deposit that surplus at the deposit facility to earn R_{df}. If the banker faces a high refinancing shock, it will have to cover the reserve deficit at the lending facility and pay $R_{\ell f}$. The first term on the last line refers to the interest payment that the bank has to make when it issued $\omega_1 \gamma n / R_d$ new deposits: then she will have to pay the interest rate cost $R_d\left(\omega_1 \gamma n / R_d\right)$ on it. Finally, the last term of (3) shows the interest rate that the bank obtains on the reserves she is required to hold to back newly issued deposits.

The solution of the banker's problem on the interbank market is standard and the first order condition gives y_j as the solution to

$$R_m = R_{df} G(\bar{\gamma}(d_j, n_j, y_j)) + R_{\ell f}\left[1 - G(\bar{\gamma}(d_j, n_j, y_j))\right]. \tag{4}$$

This expression says that the marginal cost of borrowing more reserves on the interbank market, R_m, has to equal the marginal gain of having more reserves, which is an average of the standing facility rates, weighted by the probability to access those facilities. Accordingly, (4) shows that the effect of a lower R_m is to increase the demand for reserves in the interbank market. Holding more excess reserves, y_j , banks are more likely to end up long in reserves. As a result, they are more likely to have to use the central bank deposit facility than its lending facility, thus putting more weight on R_{df}. This lowers the gain of holding more reserves and preserves the equality (4) following the reduction in R_m.

Figure 2 shows a typical demand for reserves y_j for different levels of ω_2 and R_e for one bank, holding everything else constant, that is excluding all general equilibrium effects. For a given level of the interbank market rate R_m, an increase in the market share of CBDC ω_2 will increase the demand for reserves, and the demand becomes much less elastic. Similarly, given R_m, an increase in the CBDC rate R_e makes CBDC a more sought after payment instrument, which reduces the amount of CBDC needed to conduct the same amount of purchase, and this acts to reduce the demand for reserves. [11]

From there, we derive the marginal payoff of lending to more entrepreneurs, $\frac{\partial V}{\partial n_j}$, as well as the marginal payoff of holding more excess reserves, $\frac{\partial V}{\partial e}$, ahead of the

[11] Note that the lines intersect when $\bar{\gamma} = 0$,that is when $r_j - \chi d_j + y_j = 0$. To visualize the effects of different ω_2 and $R_e$in a more intuitive way, we choose a normal distribution of γ for the graphs with $N \sim (0.5, 0.5)$, implying $G(0) > 0$.

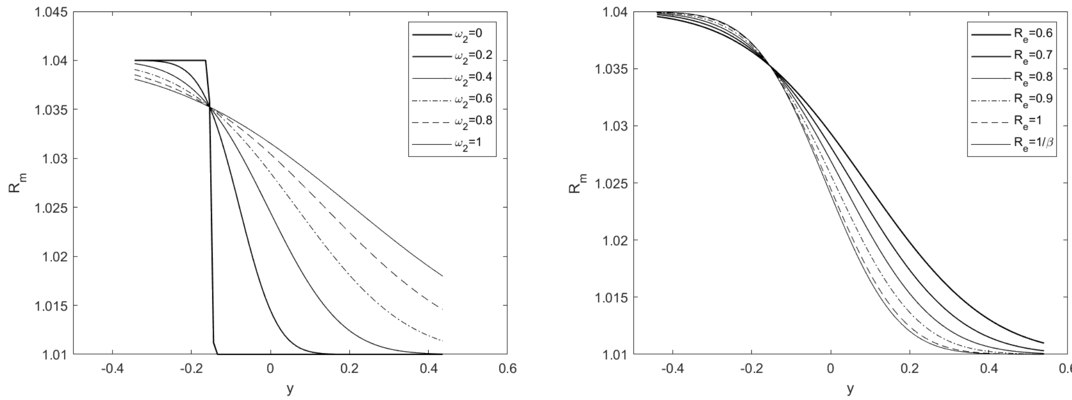

Figure 2: Demand for reserves y_j.

refinancing shock,

$$\begin{aligned}
\frac{\partial V}{\partial n_j} &= A - c'(n_j)/\beta - \int (R_d - \chi R_r)\, \frac{\omega_1 \gamma}{R_d} dG(\gamma) & (5)\\
&\quad - \left[\int_{\gamma < \bar{\gamma}(d_j, n_j, y_j)} R_{df} \left(\sum_s \omega_s \chi_s \gamma \right) dG(\gamma) + \int_{\gamma \geq \bar{\gamma}(d_j, n_j, y_j)} R_{\ell f} \left(\sum_s \omega_s \chi_s \gamma \right) dG(\gamma) \right] \\
\frac{\partial V}{\partial e_j} &= \int_{\gamma < \bar{\gamma}(d_j, n_j, y_j)} R_{df} dG(\gamma) + \int_{\gamma \geq \bar{\gamma}(d_j, n_j, y_j)} R_{\ell f} dG(\gamma) = R_m & (6)
\end{aligned}$$

Notice that we can disregard the effect of n_j and e_j on $\bar{\gamma}$ because the function that is being integrated in (3) is zero at $\bar{\gamma}$. The marginal benefit of lending to entrepreneurs (5) include the net marginal costs of investment $A - c'(n_j)/\beta$, as well as the marginal interest rate margin $(R_d - \chi R_r)$ on the additional $\gamma \omega_1 / R_d$ deposits, minus the expected refinancing costs. That expected refinancing cost is a weighted average of the standing facility rates since the liquidity shocks happen after the interbank market closes and have to be covered by accessing the central bank facilities.

(6) shows the benefit of holding more excess reserves when exiting market 2 and has to equal the interbank market rate because reserves can always be borrowed there. Also (6) shows that if the central bank were to auction reserves in market 2 in the style of a full allotment main refinancing operation, it would have to do it at the expected interbank market rate.

Replacing $r_j = d_j - n_j$ in (1), we then obtain the first order conditions of the banker,

$$n_j \quad : \qquad \frac{\partial V}{\partial n_j} - \left(\frac{\partial V}{\partial e} + \lambda \right) = 0, \tag{7}$$

$$d_j \quad : \qquad R_r \chi - R_d + (1-\chi) \left(\frac{\partial V}{\partial e} + \lambda \right) = 0, \tag{8}$$

where λ is the Lagrange multiplier on the reserve requirement constraint. In a symmetric equilibrium, all bankers finance the same amount of entrepreneurs so that (7) gives the overall level of intermediation in this economy by setting $n_j = N$, and (8) will give us the total amount of (commercial bank) private liabilities, $d_j = D$.

Interbank market clearing

The interbank market clearing condition is

$$\int y_j dj = 0.$$

Since all banks are the same, it must be that $y_j = 0$ for all j. Since $n_j = N$ and $d_j = D$, we obtain the interbank market rate as

$$R_m = R_{df} G(\bar{\gamma}(D, N, 0)) + R_{\ell f} \left[1 - G(\bar{\gamma}(D, N, 0)) \right]. \tag{9}$$

4 Equilibrium

In this section we analyze two equilibrium regimes. One where the reserve constraint binds (so $\lambda > 0$ in (7) and (8)) and one where it does not (and $\lambda = 0$). For brevity, we write $\bar{\gamma}(D, N) \equiv \bar{\gamma}(D, N, 0)$.

Suppose the reserve constraint does not bind, $\lambda = 0$. Then combining (8) and (9) we obtain

$$R_{df} G(\bar{\gamma}(D, N)) + R_{\ell f} \left[1 - G(\bar{\gamma}(D, N)) \right] = \frac{R_d - R_r \chi}{1 - \chi}, \tag{10}$$

while combining (7) and (9) gives

$$A - c'(N)/\beta - \int (R_d - \chi R_r) \frac{\omega_1 \gamma}{R_d} dG(\gamma) \;=\; \left[\int_{\gamma < \bar{\gamma}(D,N)} R_{df} \Omega(\gamma) dG(\gamma) + \int_{\gamma \geq \bar{\gamma}(D,N)} R_{\ell f} \Omega(\gamma) dG(\gamma) \right] \quad , \tag{11}$$

where

$$\Omega(\gamma) \equiv 1 + \sum_s \omega_s \chi_s \gamma,$$

is the (expected) effective cost of funding one entrepreneurs when the shock is known to be γ. Indeed, an entrepreneur requires the one unit of initial investment which costs 1 in market 2. In addition, in market 1, a fraction $\gamma\omega_1$ will need to purchase one unit, which has a reserve cost of χ/R_d, and another fraction $\gamma\omega_2$ will need to purchase one unit at the reserve cost $1/R_e$. Finally, the deposit rate R_d is given by

$$(\beta R_d)^{-1} = \begin{cases} 1 & \text{if } Y^* \leq R_d D \\ \omega_1 u'(R_d D) + (1-\omega_1) & \text{otherwise} \end{cases} \tag{12}$$

Equations (10)-(12) form a system of three equations in three unknowns R_d, D, and N. If the solution (R_d, D, N) to this system satisfies the non-binding reserves constraint $(1-\chi)D \geq N$, then this is the solution for the equilibrium.

Suppose the reserve constraint binds $(\lambda > 0)$. Then

$$(1-\chi)D = N \tag{13}$$

and combining (7) and (8), we obtain

$$A - c'(N)/\beta - \int \omega_1 \gamma dG(\gamma) + \int \frac{\omega_1 \gamma}{R_d} \chi R_r dG(\gamma) \tag{14}$$

$$-\left[\int_{\gamma<\bar{\gamma}(D,N)} R_{df}\Omega(\gamma)dG(\gamma) + \int_{\gamma\geq\bar{\gamma}(D,N)} R_{\ell f}\Omega(\gamma)dG(\gamma)\right] = \frac{R_d - R_r\chi}{1-\chi} - R_m.$$

and the interbank market is given by

$$R_m = R_{df}G(\bar{\gamma}(D,N)) + R_{\ell f}\left[1 - G(\bar{\gamma}(D,N))\right]. \tag{15}$$

The deposit rate R_d still satisfies (12). The equilibrium is now given by R_d, R_m, D, and N that solve equations (12)-(15). This is an equilibrium whenever $\lambda > 0$, or $R_d - R_r\chi > (1-\chi) R_m$.

5 Simulations

In this section we parametrize the model and derive some simulations for this economy to answer two questions: First, what is the impact of increasing the share of CBDC use (ω_2) on investment and the interbank market rate? Second, what are the effects of different CBDC remuneration rates? We discuss the effects of caps on CBDC holdings and tiered remuneration in the following sections.

Parameter	Value	Description
β	0.96	discount factor
ω_2	0.4	share of tv-2 sellers (CBDC)
ω_1	$1-\omega_2$	share of tv-1 sellers (deposits)
χ	0.01	reserve requirement
R_r	1.01	interest rate on required reserves (tier 1)
R_e	1.00	interest rate on CBDC
R_{df}	1.01	deposit facility rate (tier 2)
R_{lf}	1.04	lending facility rate
ρ	0.3	CRRA parameter in $u'(X) = X^{-\rho}$
A	2.62	Output per unit of investment
$\bar{c}_L/\beta$	1.5	cost parameter in $c'(N) = \bar{c}_L N$
$\bar{c}_H/\beta$	2.15	cost parameter in $c'(N) = \bar{c}_H N^2$

Table 1: Parameter values

Parametrization

In all our simulations, we choose the following functional forms and parameter values (unless otherwise stated). The utility function in market 1 is CRRA, $u(x) = x^{(1-\rho)}/(1-\rho)$, while the banks' cost of investment is $c(N) = 0.5\bar{c}_L N^2$ where $\bar{c_L}$ is a constant. It will turn out that the curvature of this cost function is very important to determine the impact of CBDC on interbank markets. In some parametrization we also use $c(N) = \bar{c_H} N^3/3$. We use parameter values reported in Table 1. As will become clear below, we fix these parameter values so that we can analyze the effect of CBDC on the two equilibrium regimes of interest, with a binding and a lax constraint on required reserves. We assume that γ is uniformly distributed in [0,1] with $E(\gamma) = 0.5$.

CBDC and the demand for reserves

CBDC market share To understand the effects of introducing CBDC on the equilibrium, we first simulate the effect of an increase in the share of CBDC use by sellers, ω_2. This captures whether the central bank makes CBDC more or less attractive for users, with a higher ω_2 being associated with a more attractive CBDC, be it through the absence of fees, or non-pecuniary benefits such as ease of use or the preservation of privacy. Then we concentrate more precisely on the effect of remunerating CBDC: We fix ω_2 and increase R_e. In each simulation, we look at the

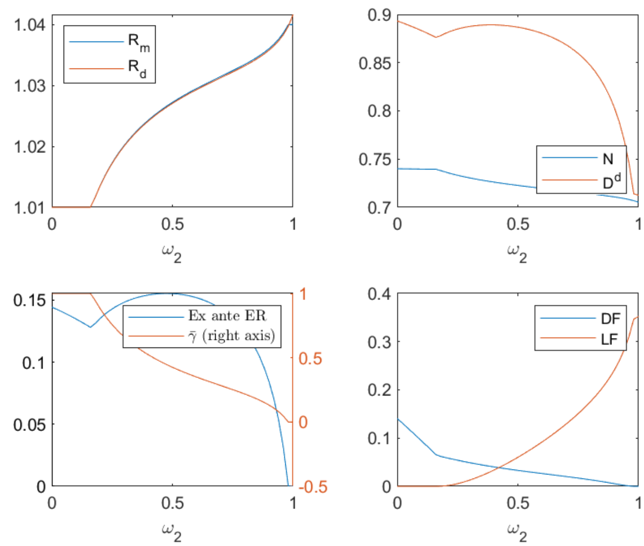

Figure 3: CBDC drains reserves.

level of bank intermediation N and D, and we analyze the effect on the deposit rate R_d and the interbank market rate R_m, as well as access to the central bank's deposit and lending facilities, shown by the acronyms DF and LF in Figure 3. To understand the effects we also report the aggregate amount of reserves in the economy prior to the refinancing shock, labelled as Ex ante ER, and $\bar{\gamma}$, the threshold above which a refinancing shock is large enough to cause a reserve shortfall for the banker.

Figure 3 shows three zones in each graph, each with different effects of CBDC.

Consider an increase in ω_2 from zero but below around 0.2 so that CBDC has a market share below 20% (of course this value depends on our parametrization) – this is zone 1. Figure 3 shows that there is no effect of increasing ω_2 on $\bar{\gamma}$ in that zone: All banks have enough reserves to satisfy even the highest refinancing shock. As a result, no bank needs to access the central bank's lending facility, while some banks access the deposit facility. In zone 1, both the interest rate on deposits and the interbank market rate equal the deposit facility rate R_{df}. Increasing ω_2 in zone 1 does not affect bank's investment N, but there is a decreasing demand for deposits from households since a fraction of sellers, albeit small, no longer accepts deposits but only CBDC. As a result, the level of reserves held by banks ahead of the interbank market, $ER = (1-\chi)D - N$ (ER in Figure 3), decreases.

Zone 2 is when ω_2 lies further above 0.2 but below a number close to 1. In that case, the relatively high usage of CBDC increases the need for reserves, which means that some banks may have insufficient reserves to handle a significant refinancing shock. As a result, $\bar{\gamma}$ falls below 1. As banks now envisage accessing the lending facility, the interbank market rate increases. In turn, this causes the value of reserves to increase, and banks attempt to obtain additional reserves by raising the deposit rate, R_d, above R_{df}. That increase in R_d initially leads to households demanding more deposits, rather than fewer. The effect of an increase in R_d dominates the effect of a higher ω_2 on the demand for deposits, as long as ω_2 remains relatively low. However, for higher values of ω_2, the direct effect of the CBDC market share eventually becomes dominant, leading to a reduction in the demand for deposits (D). However, reserves are now more expensive, which is reflected in a lower level of aggregate investment N. Accordingly, the level of reserves held by banks ahead of the refinancing shock first increases and then falls as ω_2 increases.

It is surprising that the interbank market rate increases although banks hold more excess reserves for some small to intermediate values of ω_2. To understand the mechanism, it is useful to look at the threshold $\bar{\gamma}$. In equilibrium, it is

$$\bar{\gamma}(D, N) = \frac{(1-\chi)D - N}{\sum_s \omega_s \chi_s N}.$$

So the interbank market rate will increase with ω_2 ($\bar{\gamma}$ will decline) whenever the change in reserve requirements (here $\sum_s \omega_s \chi_s N$) offsets the increase in excess reserves (captured by $(1-\chi)D - N$).

For even higher values of ω_2 closer to 1, the economy is in zone 3 where, given policy rates, the demand for CBDC is so high that banks will always access the lending facility. Then the reserve requirement binds ($\lambda > 0$). In that case, the deposit rate R_d increases above the interbank market rate which itself equals the lending facility rate $R_{\ell f}$. Banks hold no reserves when entering the interbank market.

From this narrative of the evolution of the economy as CBDC becomes more accepted by sellers, it is clear that banks will need more reserves (if only because they need 1 unit of reserves to purchase 1 unit of CBDC, while they would need only $\chi < 1$ reserves to issue 1 unit of deposits). As a result of this drain on reserves, the interbank market rate will tend to increase and banks will access more often the lending facility (zones 2-3). There is disintermediation in the sense of lower investment (in zone 2 and 3, N declines with ω_2). However, there is not necessarily disintermediation in the sense of a reduction of banks' liabilities (in zone 2, deposits D can increase with ω_2).

CBDC remuneration rate Above we have discussed how, everything else constant, an increase in CBDC usage induces an upward pressure on money market rates

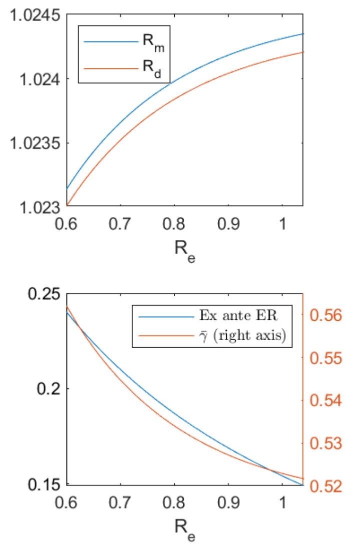

Figure 4: The demand for reserves and R_m are increasing in the CBDC rate.

by draining reserves. Is there any countervailing force to this, in particular, can the central bank reduce the attractiveness of CBDC by lowering its remuneration rate? We answer this question next by analyzing the effect of changing R_e given a fixed market share of CBDC, $\omega_2 = 0.4$, as shown in Figure 4. Notice that we have values of $R_e < 1$, which implies a rate penalizing CBDC usage (i.e. $i_e < 0$).[12]

As the CBDC rate R_e increases, CBDC gains purchasing power and becomes more attractive as a means of payment. Since the risk aversion parameter $\rho < 1$, households demand more of it. The increase in R_e also leads to a decline in $\Omega(\gamma)$ for all γ: This is the "funding effect"; a higher purchasing power reduces the effective cost of funding one entrepreneur. This funding effect plays to lower the demand for reserves. However, the funding effect gives rise to a second round "investment effect"; banks can afford to do more intermediation and N increases. This investment effect plays to increase the demand for reserves in the money market and, hence, to increase the money market rate R_m, as shown in Figure 4. Banks anticipating they will need additional reserves in market 1, boost their reserve holdings by raising the deposit rate R_d, causing a moderate increase in deposits. However, because investment (N) increases at a greater rate than deposits, the excess reserves $(1-\chi)D - N$ held by

[12]One may wonder if $R_e < 1$ would be possible in a world with cash. We think yes, as long as CBDC has additional benefits to cash, e.g. it can be used for online transactions.

banks prior to the refinancing shock decline.

Ambiguous effect of CBDC remuneration rate on R_m We may conclude that the effect of a larger market share of CBDC on the money market can be undone by decreasing its remuneration rate R_e, so as to make this payment instrument less attractive. However, the intuition that a lower R_e always reduces the pressure on the demand for reserves, and in turn the money market rate R_m, is deceptive. Indeed, as the previous paragraph explains, demand for reserves in the money market increases with R_e whenever the investment effect dominates the funding effect. This last condition may, however, not always be satisfied. For example, R_m could decrease with R_e if the cost of investment is steep, that is $c''(N)$ is large, as Figure 5 shows, for which we use the second cost function from Table 1. The intuition is simple: While banks find it cheaper to fund entrepreneurs when R_e is larger, they still incur the cost $c(N)$. If $c''(N)$ is too large, funding more entrepreneurs can be prohibitively costly. Then the need for more reserves due to the increase in intermediation N may not be enough to offset the stronger purchasing power of CBDC, thus reducing the need for reserves. In this case, R_m will drop. Since banks need less reserves, R_d is also decreasing in R_e, as is the demand for deposits D. As a consequence, the level of excess reserves held by banks ahead of the money market ($ER = (1 - \chi)D - N$) is lower. This may be the case, for example, in downturns when it is allegedly more difficult for banks to find projects with net positive present value.

For completeness, let us stress that the effects of ω_2 that we analyzed in the previous section are not substantially different when $c''(\cdot)$ is higher.

6 A cap on CBDC holdings

Quantitative limitations on the amount of CBDC holdings are a key element of central banks' discussions on CBDC. Limits are thought to deter the conversion of deposits into CBDC, thus reducing the risk of disintermediation. In a speech on the digital euro, ECB Member of the Executive Board Panetta (2022) states that "We intend to embed both types of tool – limits and tiered remuneration – in the design of a digital euro. Closer to the possible introduction of a digital euro, we will decide how to combine and calibrate them to preserve financial stability and our monetary policy stance and transmission." In this section, we investigate the role of such caps on CBDC holdings. We model the cap as a limit $\bar{E}$ on CBDC holdings: no one can hold more CBDC than $\bar{E}$. We first describe the bankers' problem in the presence of

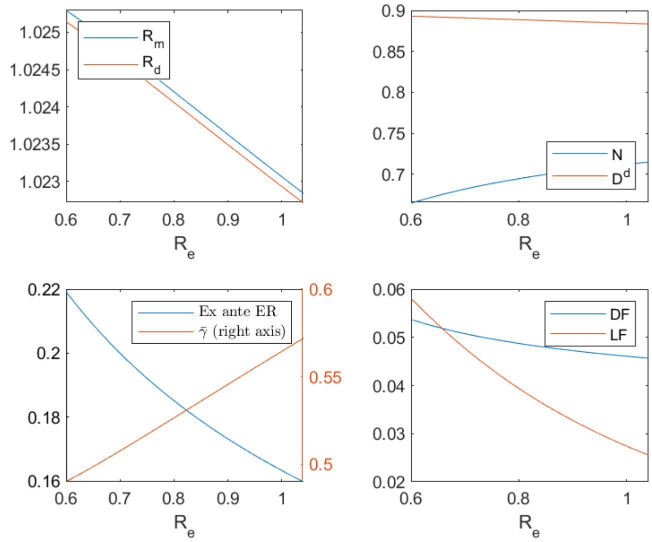

Figure 5: Large $c''(N)$: the demand for reserves and R_m are decreasing in the CBDC rate.

a CBDC cap and derive the new equilibrium. In simulations, we illustrate the effects of this quantitative limit on CBDC.

The banker's problem in the presence of a CBDC cap

We assume that entrepreneurs who need CBDC to purchase the investment good at the refinancing stage have to do so from one seller only. Since sellers can hold a maximum of $\bar{E}$ CBDC, entrepreneurs refinancing on tv-2 will have to liquidate some of their initial investment if the amount they need to refinance their entire initial investment exceeds the cap on CBDC, $\bar{E}$.[13]

Precisely, if $R_e\bar{E} < 1$, entrepreneurs can refinance at most $R_e\bar{E}$ units of their initial investment and have to scrap the remaining $1 - R_e\bar{E}$. We assume the initial investment has a scrapping value of zero. In that case, an entrepreneur can only produce $A(R_e\bar{E})$. All in all, the bank acquires an amount $\omega_2\gamma n \min\{1/R_e, \bar{E}\}$ of CBDC from the central bank such that the $\omega_2\gamma n$ entrepreneurs on tv-2 can purchase one unit of the investment good with CBDC if the cap is higher than $1/R_e$, or just

[13]It is equivalent to assume that entrepreneurs can purchase investment from the continuum of sellers, but they would be restricted to hold at most $\bar{E}$ of CBDC, thus reducing the amount of investment they can buy.

$R_e\bar{E}$ units of the investment good if the cap is below $1/R_e$. Therefore, the bank needs $\mathcal{R}(\gamma) = \frac{\omega_1 \gamma n_j}{R_d}\chi + \omega_2 \gamma n_j \min\{1/R_e, \bar{E}\}$ reserves at the refinancing stage. The banker has a reserve shortfall if

$$\mathcal{R}(\gamma) - (r_j - \chi d_j d_j + y_j) \equiv \sum_{s=1,2} \tilde{\chi}_s \omega_s \gamma n_j - (r_j - \chi d_j d_j + y_j) > 0$$

with

$$\tilde{\chi}_1 \equiv \chi/R_d \quad \text{and } \tilde{\chi}_2 \equiv \min\{1/R_e, \bar{E}\}.$$

In the Appendix, we describe how the equilibrium equations are affected by the cap on CBDC.

Simulations

In the following simulations, we compare two different caps on CBDC: a large one with $\bar{E}^{high} = 1.3$ and a small one with $\bar{E}^{low} = 0.95$. In the simulations in Figure 6 in which we increase the share of CBDC meetings, ω_2, the CBDC rate is kept constant at $R_e = 1$, implying that the small cap (dashed line) is always binding, since $\bar{E} < 1/R_e = 1$ while the large cap (solid line) is never binding.

When the cap binds, the entrepreneurs who refinance on tv-2 must liquidate some of the initial investment, thus affecting its effective productivity. The bank recognizes it and reduces investment. As a consequence, overall investment, N, is lower in the presence of a tight cap. The effect of the cap on investment is stronger as the share of CBDC meetings increases. Also, the binding cap reduces the demand for reserves, leading to lower money market and deposit rates and a lower demand for deposits.

Figure 7 shows the effects of the two caps when the CBDC rate is increased. When R_e is lower than around 0.77, both caps are binding (the large cap binds as long as $R_e < 1/\bar{E}^{high} \approx 0.77$), while only the small cap binds when $0.77 \leq R_e < 1.05 \approx 1/\bar{E}^{low}$. When the cap is binding, the bank needs less reserves compared to the case when it is not. Therefore, demand for reserves and, hence, the money market rate is lower. The bank also lowers the deposit rate since it needs less reserves. As R_e increases, the funding costs decrease and less CBDC is needed for reinvestment. Therefore, the cap becomes "less binding."

So for high levels of R_e, the high cap (solid line) does not bind. Then the money market rate increases in R_e but with a smaller slope because the effect of the surplus of reserves that was due to the binding CBDC cap has vanished. The demand for reserves is only driven by the funding and investment effects that we described above

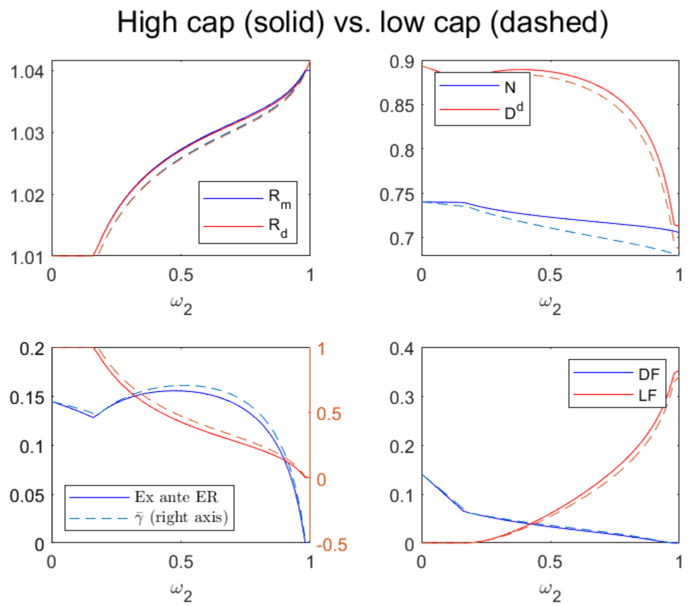

Figure 6: The effects of CBDC caps when the share of CBDC meetings increases.

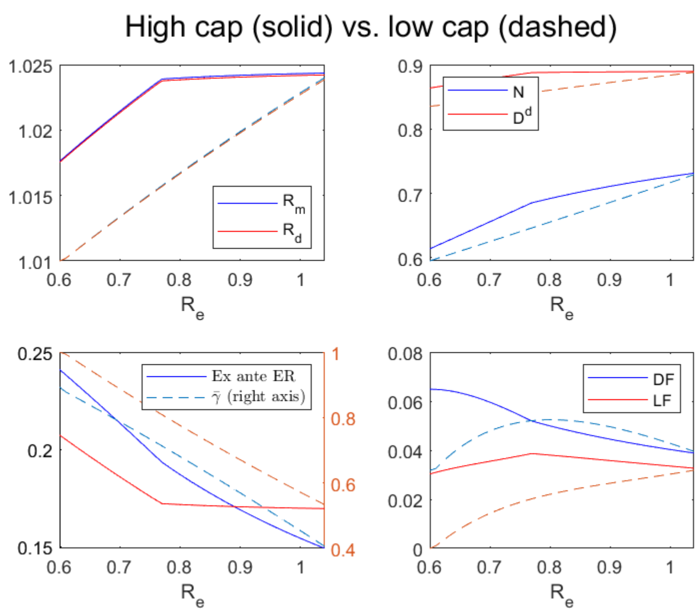

Figure 7: The effects of CBDC caps when the CBDC rate increases.

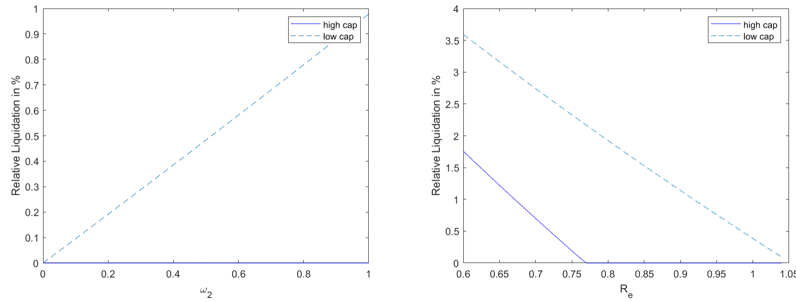

Figure 8: The effects of CBDC caps on the liquidation amount relative to the effective production when the share of CBDC meetings (left) and the CBDC rate (right) increase.

(with the latter dominating). As R_e becomes even larger, also the low cap no longer binds and both the solid and dotted lines become the same.

Figure 8 shows the average amount of liquidation, as measured by $\omega_2 N(1 - R_e\bar{E}) \int \gamma dG(\gamma)$, as a percentage of the effective production, $\mathcal{A}(\bar{E}, R_e)N$, for both the high non-binding cap (solid line) and the low binding cap (dashed line). When ω_2 increases, there are two counteracting effects on liquidation: there are more CBDC meetings, but the bank reduces its investment. However the left panel of Figure 8 shows that the first (direct) effect is stronger when the cap binds and liquidation increases. For the large cap however, there is no liquidation. The right panel in Figure 8 illustrates the relative average amount of liquidation for both the high cap (solid line) and the low cap (dashed line) as a function of the CBDC rate, R_e. When $i_e < 0$, the gross rate R_e is less than 1, and CBDC does not have a large purchasing power. Then both caps bind, and some projects have to be liquidated. However, the purchasing power of CBDC improves when R_e increase. As a consequence, the number of liquidated projects decline until they reach zero when the respective cap is not binding anymore.

To conclude this section on caps, it is important to note that the central bank can compensate the decrease in investment due to a cap on CBDC by increasing the CBDC rate. Then one unit of CBDC can buy more, which relaxes the constraint that the cap places on transactions with CBDC.

7 Two-tier remuneration system for CBDC

Another central element of the CBDC design often discussed in policy circles is the possibility of tiered remuneration (see, e.g., Bindseil (2020)). Specifically, central banks could set two different interest rates: the central bank would remunerate the first $x of CBDC holdings at a higher rate and anything above $x at a lower rate. Again, this design feature is thought to dis-incentivize a large shift from bank deposits into CBDC holdings.

The banker's problem in the presence of a tiered remuneration

Suppose CBDC holdings ε are remunerated at the following contingent rate,

$$R_e(\varepsilon) = \begin{cases} R_e^1 & \text{if } \varepsilon \leq \bar{\varepsilon} \\ R_e^2 & \text{if } \varepsilon > \bar{\varepsilon} \end{cases},$$

where $\bar{\varepsilon}$ is the tiering threshold.

Tiering has no bite in our original setup because firms can purchase a small amount of the investment goods from a very large number of sellers, to guarantee that each of them hold a lower amount of CBDC than the tiering threshold. So for tiering to have a bite, we make a slight modification to our model and we assume that the bank is investing in k firms with still a net discounted payoff of $Ak - c(k)/\beta$. However, it is the bank that has to refinance a fraction γ of its balance sheet, and it can only do it from one seller.[14] All in all, given γ, the bank has to purchase $\omega_2\gamma k$ with CBDC from the same seller.

At the refinancing stage, the seller charges a price $p(k)$ to sell k units of capital, such that

$$k = R_e^1 \min\left[p(k)k; \bar{\varepsilon}\right] + R_e^2 \max\left[p(k)k - \bar{\varepsilon}; 0\right].$$

Therefore

$$p(k) = \begin{cases} \frac{1}{R_e^1} & \text{if } k \leq R_e^1\bar{\varepsilon} \\ \frac{1}{R_e^2} + \left(R_e^2 - R_e^1\right)\frac{\bar{\varepsilon}}{R_e^2 k} & \text{if } k > R_e^1\bar{\varepsilon} \end{cases}.$$

Hence, on the γk units of capital needed, the bank pays $p(\gamma k)$. So the bank needs

[14] Then the bank cannot make sure to always be below the remuneration threshold by purchasing a tiny bit from a very large number of sellers.

$\omega_2 \gamma k p(\gamma k)$ in CBDC which gives

$$\omega_2 \gamma k p(\gamma k) = \begin{cases} \omega_2 \frac{\gamma k}{R_e^1} & \text{if } \gamma \leq \frac{R_e^1 \bar{\varepsilon}}{k} \\ \omega_2 \left[\frac{\gamma k}{R_e^2} + \left(R_e^2 - R_e^1\right) \frac{\bar{\varepsilon}}{R_e^2} \right] & \text{if } \gamma > \frac{R_e^1 \bar{\varepsilon}}{k} \end{cases}.$$

The bank is short in reserves whenever

$$\mathcal{R}(\gamma, k) \equiv \chi \omega_1 \frac{\gamma k}{R_d} + \omega_2 \gamma k p(\gamma k) - (r - \chi d + y) > 0$$

Notice that at this stage, k is fixed. Therefore, we can define two cases depending on γ:

$$\mathcal{R}(\gamma, k) = \begin{cases} \chi \omega_1 \frac{\gamma k}{R_d} + \omega_2 \frac{\gamma k}{R_e^1} - (r - \chi d + y) & \text{if } \gamma \leq \frac{R_e^1 \bar{\varepsilon}}{k} \equiv \hat{\gamma}, \\ \chi \omega_1 \frac{\gamma k}{R_d} + \omega_2 \frac{\gamma k}{R_e^2} + \omega_2 \left(R_e^2 - R_e^1\right) \frac{\bar{\varepsilon}}{R_e^2} - (r - \chi d + y) & \text{otherwise} \end{cases}$$

Then for $\gamma \leq \hat{\gamma}$, $\mathcal{R}(\gamma, k) \geq 0$ iff

$$\gamma \geq \frac{(r - \chi d + y)}{k(\omega_1 \chi \frac{1}{R_d} + \omega_2 \frac{1}{R_e^1})} \equiv \bar{\gamma}_1,$$

while for $\gamma > \hat{\gamma}$, $\mathcal{R}(\gamma, k) \geq 0$ iff

$$\gamma \geq \frac{(r - \chi d + y) + \omega_2 \left(R_e^1 - R_e^2\right) \frac{\bar{\varepsilon}}{R_e^2}}{k \left(\omega_1 \chi \frac{1}{R_d} + \omega_2 \frac{1}{R_e^2} \right)} \equiv \bar{\gamma}_2.$$

Given these thresholds it is straightforward to write down the value function of the bank and to compute the first order conditions with respect to y, k, and excess reserves.

Simulations

In the following simulations, we set $R_e^1 = 1$ and $R_e^2 = 0.7$ and the tiering threshold $\bar{\varepsilon} = 0.4$. In Figure 9, we show two different thresholds of γ: $\hat{\gamma}$ is the threshold above which the reinvestment shock is so large that the second remuneration rate applies. Since (aggregate) investment K is decreasing in ω_2, this threshold is increasing in ω_2. As before, $\bar{\gamma}$ is the threshold above which a bank receives such a large refinancing shock that it is short in reserves. Depending on whether the second remuneration rate applies, this threshold is either $\bar{\gamma}_1$ or $\bar{\gamma}_2$. That is, if $\gamma \leq \hat{\gamma}, \bar{\gamma} = \bar{\gamma}_1$ and if

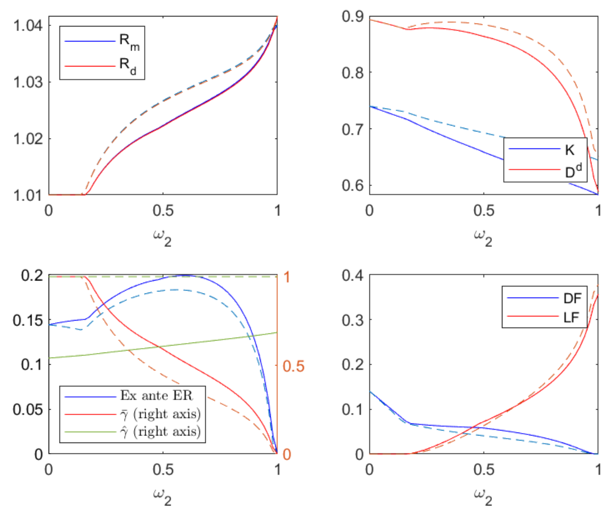

Figure 9: Increasing the share of CBDC meetings in the presence of a two-tiered CBDC remuneration

$\gamma > \hat{\gamma}, \bar{\gamma} = \bar{\gamma}_2$. For comparison, we also add for each variable a dashed line to the plots corresponding to the case with no two-tiered remuneration (which is equivalent to$\bar{\varepsilon} = \infty$) and $R_e = 0.85$ (the average of the two-tiered rates).

As Figure 9 shows, the main effects of an increase in the share of CBDC meetings, ω_2, are the same as before. However, now for banks who are hit by a large refinancing shock and, therefore, need a large amount of CBDC, the second and lower remuneration rate applies. This leads to CBDC loosing purchasing power in relative terms for a high refinancing shock. As a result, the expected refinancing cost of banks is higher. Therefore, banks choose to reduce their overall investment relative to the case without a tiered remuneration: Figure 9 shows that the solid line corresponding to aggregate investment, K with tiered remunation is below the dashed line showing K without the tiered remuneration.. The larger ω_2 becomes, the stronger is the effect of the second (lower) rate and the lower is investment K.

To simulate the effects of an increase in the remuneration of CBDC, we set $R_e^1 = R_e$ and $R_e^2 = R_e - 0.3$. As Figure 10 shows, an increase in both remuneration rates of CBDC has the same effects as before. However, the average effective remuneration rate of CBDC is lower now, leading to larger refinancing costs for banks as CBDC is worth less, and, again, lower investment. This decrease in investment leads to a smaller demand for reserves and, hence, to a lower money market rate.

In sum, we find that a tiered remuneration of CBDC with a lower CBDC rate

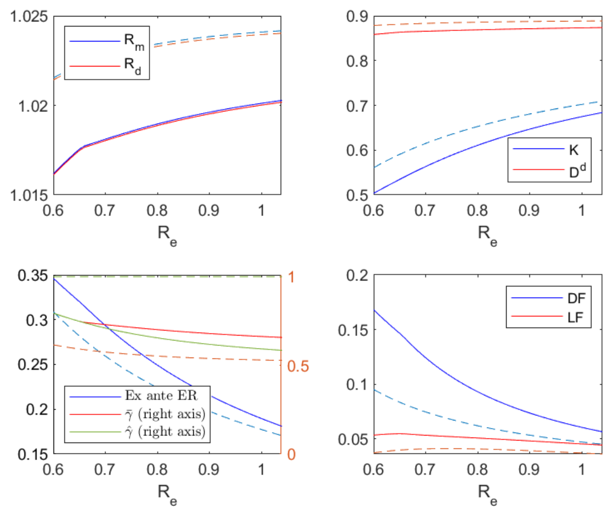

Figure 10: Increasing the remuneration rates of CBDC in the presence of a two-tiered CBDC remuneration

that is applied after a certain tiering threshold, is equivalent to a decrease in the CBDC rate when there is no two-tiered system in place.

8 Conclusion and future research

We developed a model of the interbank market featuring uncertainty in the form of a Poole (1968) shock, in which CBDC is introduced in a framework similar to Chiu et al. (2022). In the model, there are two types of payments: bank deposits that only need partial reserve backing, and CDBC which is equivalent to a 100% reserve requirement. We find that, as the market share of CBDC increases, the demand for reserves increases, leading eventually to a higher interbank market rate. While this result is intuitive, the effects from an increase in the CBDC rate are ambiguous. Since a higher remuneration leads to CBDC increasing in value, the costs to fund the same investment level decrease and less reserves are needed. However, now the banks can fund more entrepreneurs, increasing their investment level and therefore, the demand for reserves. Depending on the model parameters, either the investment or the funding effect dominates, pushing the demand for reserves and, hence, the interbank rate up or down.

Furthermore, we show that quantitative limitations on the amount of CBDC that can be held decrease the reserve demand and the interbank rate and reduces the investment level. Another policy design feature that is actively discussed at central banks, e.g. in Bindseil (2020), is a tiered remuneration of CBDC. We introduce such a tiering system in our model and find that introducing a second, lower CBDC rate for large amounts of CBDC holdings is equivalent to a decrease in the CBDC rate when there is no two-tiered system in place.

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9 Appendix

It will be useful to define the (expected) effective productivity of entrepreneurs as

$$\mathcal{A}(\bar{E}, R_e) = A \int \left[(1-\gamma) + \gamma(\omega_1 + \omega_2 \left(R_e \min\{1/R_e, \bar{E}\}\right)\right] dG(\gamma)$$

Now, the equations characterizing the equilibrium when the reserve constraint does not bind, hence, $(1-\chi)D \geq N$ with $\lambda = 0$, read as:

$$R_{df}G(\bar{\gamma}(D,N)) + R_{\ell f}\left[1 - G(\bar{\gamma}(D,N))\right] = \frac{R_d - R_r\chi}{1-\chi},$$

$$\begin{aligned}\mathcal{A}(\bar{E}, R_e) - c'(N)/\beta - \int \omega_1\gamma dG(\gamma) + \int \frac{\omega_1\gamma}{R_d}\chi R_r dG(\gamma) &= \\ \left[\int_{\gamma<\bar{\gamma}(D,N)} R_{df}\Omega(\gamma)dG(\gamma) + \int_{\gamma\geq\bar{\gamma}(D,N)} R_{\ell f}\Omega(\gamma)dG(\gamma)\right] &,\end{aligned}$$

where

$$\Omega(\gamma) \equiv 1 + \sum_s \omega_s\tilde{\chi}_s\gamma.$$

Finally, the deposit rate R_d is given by

$$(\beta R_d)^{-1} = \begin{cases} 1 & \text{if } Y^* \leq R_d D \\ \omega_1 u'\left(R_d D\right) + (1-\omega_1) & \text{otherwise.}\end{cases}$$

When the reserve constraint binds and $(\lambda > 0)$, these equations become:

$$(1-\chi)D = N$$

$$\begin{aligned}&\mathcal{A}(\bar{E}, R_e) - c'(N)/\beta - \int \omega_1\gamma dG(\gamma) + \int \frac{\omega_1\gamma}{R_d}\chi R_r dG(\gamma) \\ &-\left[\int_{\gamma<\bar{\gamma}(D,N)} R_{df}\Omega(\gamma)dG(\gamma) + \int_{\gamma\geq\bar{\gamma}(D,N)} R_{\ell f}\Omega(\gamma)dG(\gamma)\right] = \frac{R_d - R_r\chi}{1-\chi} - R_m.\end{aligned}$$

$$R_m = R_{df}G(\bar{\gamma}(D,N)) + R_{\ell f}\left[1 - G(\bar{\gamma}(D,N))\right].$$

where, again,

$$\Omega(\gamma) \equiv 1 + \sum_s \omega_s\tilde{\chi}_s\gamma.$$

$$(\beta R_d)^{-1} = \begin{cases} 1 & \text{if } Y^* \leq R_d D \\ \omega_1 u'\left(R_d D\right) + (1-\omega_1) & \text{otherwise.}\end{cases}$$