Managing the transition to central bank digital currency *

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Abstract

We develop a two-country DSGE model with financial frictions to study the transition from a steady-state without CBDC to one in which the home country issues a CBDC. The CBDC provides households with a liquid, convenient and storage-cost-free means of payments which reduces the market power of banks on deposits. In the steady-states CBDC unambiguously improves welfare without disintermediating the banking sector. But macroeconomic volatility in the transition period to the new steady-state increases for plausible values of the latter. Demand for CBDC and money overshoot, thereby crowding out bank deposits and leading to initial declines in investment, consumption and output. We use non-linear solution methods with occasionally binding constraints to explore how alternative policies reduce volatility in the transition, contrasting the effects of restrictions on non-residents, binding caps, tiered remuneration and central bank asset purchases. Binding caps reduce disintermediation and output losses in the transition most effectively, with an optimal level of around 40% of steady-state CBDC demand.

Keywords: Central bank digital currency, open-economy DSGE models, steady-state transition, occasionally binding constraints

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Non-technical summary

A large academic literature has developed and examined various potential effects of central bank digital currencies (CBDCs) on banks, the financial sector more broadly and the rest of the economy, as well as on international spillovers. These papers have studied how shocks propagate differently through the economy once CBDCs are available, that is around the new equilibrium with CBDC well-established as a monetary instrument.

In this paper, we look at the issue from a different angle and study the macroeconomic effects of CBDC in the transition to the new equilibrium—that is from the moment of the initial launch up to the longer run when it is firmly established as a payment instrument. We pay particular attention to policies that can help balance trade-offs in the transition phase between risks to macroeconomic and financial stability arising from excess demand for CBDC against welfare losses occurring when CBDC demand is overly constrained and the resulting menu of monetary instruments available to households excessively reduced.

We develop a two-country model with financial frictions to study the transition from a steady-state without CBDC to one in which the home country issues a CBDC. Households need liquidity to finance transactions and for this reason they hold money in the form of either cash, bank deposits or—if available—CBDC units. Deposits are remunerated, while cash and CBDC (in the baseline configuration) are not. Cash is subject to storage costs, unlike deposits and the CBDC. CBDC provides variety to the menu of monetary instruments available, which households value: their marginal utility decreases in the amount of each type of instrument held. We also assume that banks have (monopolistic) market power in the deposit market. As monopolists, banks set the deposit rate as a mark-down on the interest rate on loans to firms, which is accepted by households because deposits provide liquidity services. To contain deposit outflows when the CBDC is introduced and the menu of monetary instruments available to households expands, banks increase the deposit rate. The availability of CBDC thus reduces the market power of banks and can lead to either an increase or a decrease in deposits.

Comparing the equilibria with and without the CDBC, we find that deposit rates and deposits increase when the CBDC is available. Output increases marginally and welfare improves, both in the home and in the foreign economy, although effects are more limited in the foreign economy. Next, we solve the model non-linearly to study the transition between the steady states without and with a CBDC. For low steady-state CBDC demand (5% of steady-state output in our simulations), introducing a CBDC has no material macroeconomic impact. For plausibly higher steady-state demand (30% of steady-state output in our simulations) we find that the transition to the new steady-state is characterized by significant volatility in CBDC, cash and deposits, leading to volatility in loan rates, investment and consumption. Demand for CBDC and cash overshoot their new steady-state values in the short run, thereby crowding out bank deposits and leading to a fall in investment and consumption. Output declines initially and—after the initial volatility has subsided—increases slowly towards the new steady-state.

We investigate how alternative policies reduce volatility during the transition. We consider different instruments: first, the presence of soft and hard holding limits; second, a two-tiered CBDC remuneration scheme that penalizes "excessive" holdings of CBDC by applying a negative interest rate to CBDC holdings above a certain limit; and third, restrictions on non-residents' CBDC holdings that either preclude non-residents to hold CBDC or result in higher cross-border transaction costs in CBDC for non-residents. In addition, we investigate whether active central-bank balance-sheet policies, where the central bank purchases private-sector assets to balance CBDC issuance, are effective in smoothing the transition. We find that binding caps are most effective in reducing disintermediation and output losses in the transition and in minimizing international spillovers. Their optimal level—i.e. minimizing welfare losses—is around 40% of steady-state CBDC demand.

1 Introduction

Interest in central bank digital currency (CBDC) is now quasi universal. More than 90% of the central banks participating in a recent BIS survey reported that they were engaged in CBDC work.[1] Major central banks are looking seriously into the issue, including the People's Bank of China, which is running pilots of its e-yuan on over 200 million test users, the Bank of England, which stressed in the summer of 2023 that a digital pound would be "likely needed in the future" and the European Central Bank, which launched in October 2023 a two-year preparation phase before the possible launch of a digital euro.

A large academic literature has developed and examined various potential effects of CBDCs on banks, the financial sector more broadly and the rest of the economy, as well as on international spillovers, such as e.g. Agur et al. (2022), Andolfatto (2021), Barrdear and Kumhof (2022), Brunnermeier and Niepelt (2019), Burlon et al. (2022), Chiu et al. (2023), Ferrari Minesso et al. (2022), Fernández-Villaverde et al. (2021), Fernández-Villaverde et al. (2021) Keister and Sanches (2023), Li (2023) and Niepelt (2023) among many others. Many papers have focused on the impact of CBDCs for the economy in the steady-state—when it is well-established as a monetary instrument.[2]

In this paper, we look at the issue from a different angle and study the macroeconomic effects of CBDC in the transition to the steady-state—from its initial launch up to the longer run when it is firmly established. We pay particular attention to policies that can help balance trade-offs in the transition phase between risks to macroeconomic and financial stability arising from excess demand for CBDC against welfare losses occurring when CBDC demand is overly constrained and the resulting menu of monetary instruments available to households excessively reduced.[3]

Our question is motivated by the novelty of CBDCs as an additional, albeit not yet existing, means of payment. Uncertainty about the potential effects at its launch

[1] See Kosse and Mattei (2023). Efforts related to retail CBDC—a digital version of banknotes—are more advanced than those for wholesale CBDC—the extension of settlement in central bank money to financial institutions beyond banks. So far only small emerging economies have launched CBDCs, such as Nigeria (e-Naira), The Bahamas (Sand dollar) or Jamaica (JAM-DEX).

[2] And where standard perturbation methods can be used to solve models.

[3] Simulating the transition between two steady-states with occasionally binding constraints, as we do and explain below, is also notoriously more cumbersome than studying perturbations around one steady-state, as in standard models, because it combines two complications: first, one needs a global solution method, as the model cannot be approximated around one fixed point (the unique steady-state); second, the constraint might bind or not.

and during the transition towards a new equilibrium is large because the extent of CBDC adoption is uncertain. The range of potential CBDC demand estimates available is wide.[4] While take-up at the lower end of the estimated range would probably not create major challenges to banks, take-up at the upper range would potentially disrupt the financial sector, in particular if it were to occur rapidly (Bank of Canada et al., 2021; Bank of England, 2023). Substitution of bank deposits for CBDC holdings might lead to an increase in banks' funding costs[5], with potential adverse effects on credit, investment and ultimately the economy at large.[6]

For this reason, different mechanisms to limit CBDC demand have been proposed to contain imminent risks to financial stability resulting from the introduction of CBDCs, such as Bindseil (2020), who discusses holding caps and tiered remuneration schemes to control the quantity of CBDC. Importantly, such considerations are reflected in the proposal for a regulation on the establishment of a digital euro, which foresees that "the European Central Bank shall develop instruments to limit the use of the digital euro as a store of value and shall decide on their parameters and use" (Art. 16).[7] Bank of England (2023) explicitly considers limits on holdings of a digital pound during a transition period following its introduction to constrain outflows from bank deposits and allow UK authorities to learn more about its impact (p. 40).[8]

[4] For instance, Adalid et al. (2022) estimate that potential take-up of a digital euro could lie between EUR 500 billion and EUR 7 trillion depending on assumptions made. Li (2023) estimates that aggregate CBDC holdings could reach between 4 % and 52 % of total liquid assets held by households in Canada. Lambert et al. (2023) find that individual holdings might range between 1% and 35% of total individual liquid assets in the euro area.

[5] For instance, Whited et al. (2022) shows that 80 cents of bank deposits would be replaced by a one-dollar introduction of CBDC, leading to a 20 cent drop in bank lending and a shift to wholesale funding for banks. Fernández-Villaverde et al. (2021) investigate potential competition between CBDC and the traditional maturity-transforming role of commercial banks.

[6] Relatedly, several studies analyze welfare implications that arise when a CBDC is present, pointing to different rules on how CBDC demand could be managed to balance trade-offs. Keister and Sanches (2023) point to different trade-offs between financial inclusion and the facilitation of illicit activities, and between promoting efficient exchange and efficient investment. Andolfatto (2021) shows that, by improving competition in the banking sector, the presence of a CBDC can actually raise deposits and increase bank lending. Similarly, Chiu et al. (2023) show that if banks have market power in the deposit market, a CBDC can enhance competition, raising the deposit rate and expanding intermediation. Garratt and Zhu (2021) develop a model with a CBDC that offers a convenience value to users and thereby levels the playing field in an heterogeneous banking sector by shifting deposits and lending from large to small banks. Burlon et al. (2022) find that, depending on the precise design of rules, the presence of a CBDC can induce significant welfare gains.

[7] See the full text of the proposal, which is available at `https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:52023PC0369`. While the regulation does not further specify the nature of these instruments, it also states that "within the framework of the Regulation, the digital euro shall not bear interest" (Art. 16 (8)).

[8] Bank of England (2023) envisages an individual holding limit of between GBP 10,000 and GBP

How to optimally manage CBDCs in the transition to the steady-state hence remains an open question. In a typical macroeconomic setup, any restrictions that keep CBDC holdings below the optimal steady-state value reduces welfare.[9] But there might be a case to impose limits during the transition from a steady-state without CBDC to one with CBDC, especially if the transition in question involves significant overshooting of CBDC demand and increases macroeconomic volatility. In such instances, holding limits or introducing a penalizing remuneration on CBDC could help smoothing the transition to the new equilibrium. International linkages are an additional complication: Although central banks are developing retail CBDCs for domestic use, residents in e.g. unstable economies may decide to hold foreign CBDCs as a store of value, even if they cannot use them for transactions in their own country—with potential cross-border spillover and spillback effects.[10]

We study the impact of alternative policies in the transition to the new steady-state in a unified framework. We develop a two-country model with financial frictions to study the transition from a steady-state without CBDC to one in which the home country issues a CBDC. Households face a cash-in-advance constraint that forces them to hold money in the form of either cash, bank deposits or—if available—CBDC units, to purchase consumption goods. Deposits are remunerated, while cash is not. Cash is subject to storage costs, unlike deposits and the CBDC. CBDC is also not remunerated in the baseline configuration—an assumption that we relax when we introduce tiered remuneration—unlike deposits. However, CBDC provides variety to the menu of monetary instruments available, which households value: Their marginal utility decreases in the amount of each type of instrument held. Importantly, households may have different preferences across cash, deposits and CBDC in terms of liquidity services provided, which are captured by differences in loadings on each instrument in their cash-in-advance constraint. Such

20,000, at least during the introduction period of the digital pound, and no remuneration (p. 79-80).

[9] For instance, Assenmacher et al. (2021) assess the welfare effects of a cap, collateral constraints and interest rate on CBDC and find that tools to limit CBDC demand unambiguously reduce welfare. Ahnert et al. (2023) show that holding limits can improve social welfare but only for high levels of CBDC remuneration.

[10] The literature on CBDCs in open economies is limited. George et al. (2020) find that, with imperfect substitutability of CBDC and bank deposits, the interest rate on CBDC can be used as an additional instrument to stabilize the economy, thereby increasing welfare. Ferrari Minesso et al. (2022) compare the international transmission of monetary policy and technology shocks in the presence of a CBDC to a configuration without CBDC and find that international spillovers are amplified. Kumhof et al. (2023) find that, in a two-country model, CBDC policies can significantly reduce exchange-rate volatility and the volatility of cross-border banking balances.

heterogeneity in preferences may reflect, for instance, differences in attitude vis-à-vis anonymity, digitalization, payment habits as well as other tastes. Following Andolfatto (2021) and Niepelt (2023), we assume that banks have (monopolistic) market power in the deposit market but act as price-takers in the lending market where monitoring costs give rise to a financial accelerator mechanism as in Bernanke et al. (1999). As monopolists, banks set the deposit rate as a mark-down on the interest rate on final loans, which is accepted by households because deposits provide liquidity services. To contain deposit outflows when the CBDC is introduced and the menu of monetary instruments available to households expands, banks increase the deposit rate. The availability of CBDC thus reduces the market power of banks and can lead to either an increase or a decrease in deposits.

Using calibration values from Ferrari Minesso et al. (2022), we first compare the steady-state with CBDC to the steady-state without CBDC and find that deposit rates and deposits increase. Output increases marginally and welfare improves in the domestic economy. In the foreign country, instead, output remains stable and welfare contracts because the economy reacts more to international shocks, as in Ferrari Minesso et al. (2022).[11] Next, we solve the model non-linearly to study the transition between the steady-state without and the one with a CBDC. The effects depend importantly on unobservable steady-state demand for CBDC. For low steady-state demand (5% of steady-state output in our simulations), introducing a CBDC has no material macroeconomic impact. For plausibly higher steady-state demand (30% of steady-state output in our simulations)[12] we find that the transition to the new steady-state is characterized by significant volatility in CBDC, cash and deposits, leading to volatility in loan rates, investment and consumption. Demand for CBDC and cash overshoot their new steady-state values in the short run, thereby crowding out bank deposits and leading to a fall in investment and consumption. Output declines initially and—after the initial volatility has subsided—increases slowly towards the new steady state.

Finally, we investigate how alternative policies—implemented as occasionally binding

[11]Note that we assume that issuance of CBDC does not increase the central bank's balance sheet. The effects on output and welfare thus only reflect the efficiency gains resulting from the availability of another means of payment, not from expansion in central bank assets as in e.g. Barrdear and Kumhof (2022)

[12]To put that number in perspective, currency in circulation amounted to around 45% of euro area output in 2023.

constraints—reduce volatility during the transition. We consider alternative instruments: first, the presence of soft and hard holding limits occasionally binding in the transition to the new steady-state; second, a two-tiered CBDC remuneration scheme that penalizes "excessive" holdings of CBDC by applying a negative interest rate to CBDC holdings above a certain limit; and third, restrictions on non-residents' CBDC holdings that either preclude non-residents to hold CBDC altogether or result in higher cross-border transaction costs in CBDC for non-residents. In addition, we investigate whether active central bank balance-sheet policies, where the central bank purchases private-sector assets to balance CBDC issuance, are effective in smoothing the transition. We also determine the optimal level of the holding limit, i.e. the level that minimizes welfare losses during the transition and allows to reach the new steady-state as fast as possible. Binding caps are most effective in reducing disintermediation and output losses in the transition as well as international spillovers. Their optimal level—i.e. minimizing welfare losses—is around 40% of steady-state CBDC demand.

The remainder of the paper is structured as follows: Section 2 presents the main features of our model. Section 3 discusses the simulations, while section Section 4 offers some conclusions.

2 The model

The model extends Ferrari Minesso et al. (2022) to include financial frictions, occasionally binding constraints and uses non-linear solution methods to study transition dynamics from a steady-state without to one with a CBDC. Figure 1 gives an overview of the model setup from the perspective of the home economy and depicts how the different agents in the model interact with each other.

There are two symmetric economies (home and foreign), which trade goods and financial assets (bonds). Bond markets are incomplete, therefore uncovered interest parity (UIP) does not hold. Consumers supply labor to firms, save and consume final aggregate goods. They also need liquid assets to purchase final goods. Households can invest in three financial assets: bonds, deposits and money, which in our setup means cash. Money can be used for payment, is not remunerated and subject to a linearly increasing storage

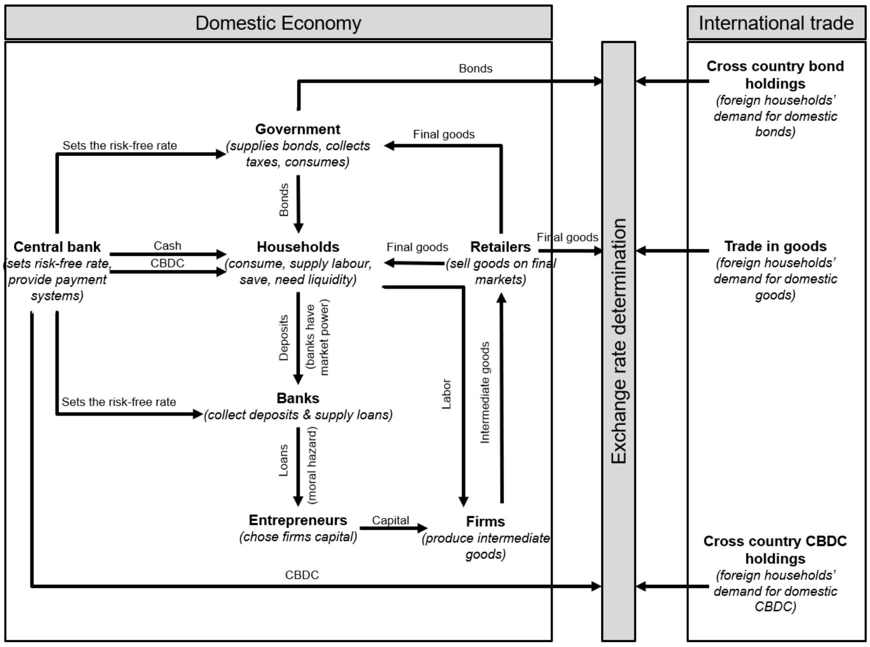

Figure 1: Model overview

Notes: The chart shows the home economy and the international trade block. The foreign economy is symmetric to the home economy with the only difference that the foreign central bank does not issue a CBDC.

cost.[13] Bonds, which are also traded internationally, are remunerated but cannot be used for payment. Deposits are remunerated *and* can be used for payment—though the degree of liquidity services they provide relative to cash may not be identical (see below). Since deposits provide liquidity services to households, banks can extract rents from issuing deposits, which hence are remunerated below the risk-free rate, as in Andolfatto (2021). Because of the computational complexity of solving the model nonlinearly, we rely on a constant elasticity of substitution (CES) aggregator to define the demand for payment instruments and maintain tractability. CES constitutes a general specification for preferences that can capture different assumptions for payment markets. For example, Ferrari Minesso et al. (2022) and Assenmacher et al. (2023) show how liquidity demand can be micro-funded through preferences for anonymity or decentralized markets.

The financial sector is populated by finitely-lived banks that combine net worth and deposits to finance loans to firms. We assume that there is a financial friction similar to the "financial accelerator" mechanism of Bernanke et al. (1999) and Christensen and Dib (2008). Specifically, we assume that banks cannot observe the outcome of firms' investment projects at no cost. Because of this friction, highly-leveraged entrepreneurs

[13]Unlike the CBDC that will be introduced below, money is a physical, not digital, payment instrument. The linear cost reflects, e.g. the costs of storing banknotes in a vault.

have an incentive to misreport performance of their firms and default on their debts. As a result, banks charge higher interest rates to more leveraged firms, which makes credit spreads over the risk-free rate counter-cyclical. As mentioned above, banks also have market power in setting the deposit rate, which stands below the interest rate on loans since deposits provide liquidity services to households.

The production sector of the economy is populated by three different types of firms: capital good producers, intermediate good producers and retailers. Capital good producers are modeled as a continuum of identical firms which use undepreciated capital and a fraction of final goods to produce new capital goods. Entrepreneurs combine own net worth and bank loans to purchase new capital goods that are used with labor to produce final undifferentiated goods. Entrepreneurs accumulate profits but exit with exogenous probability $1-\nu$ in each period. Final goods are bundled by retailers and sold to domestic and foreign consumers. We apply the Calvo setup and assume that final goods prices are not perfectly flexible. The model is closed with a public sector that decides on public expenditures and a central bank that sets the nominal interest rate with a Taylor rule.

Importantly, we assume that—in addition to cash—the public sector in the home country can issue a central bank digital currency (CBDC). The CBDC is a liability of the central bank that is directly accessible to households and can be used for payment. In the baseline configuration it is not remunerated.[14] By introducing another payment instrument with the CBDC, households' liquidity constraint is relaxed and banks can extract less rents from deposits. Moreover, the CBDC can be traded across countries subject to certain limits to be discussed below. It is a digital payment instrument—a digital version of a banknote—and is not subject to storage costs, unlike cash. Following Brunnermeier and Niepelt (2019), we assume that issuance of CBDC is "monetary policy neutral"—i.e. managed such that credit allocation in the economy is not altered.[15] In this way, aggregate output and welfare effects resulting from CBDC issuance are not affected by the way how the central bank expands its balance sheet but remain driven by economic fundamentals, such as the marginal productivity of factors, aggregate demand and the availability of capital. Adalid et al. (2022) show how this assumption is consistent with

[14]Below we explore tiered remuneration for the CBDC, as suggested by Bindseil et al. (2021), as one of the policy options to smooth the transition.

[15]This can be achieved if any potential lengthening of the central bank's balance sheet through the issuance of a CBDC is funded by the central bank acquiring claims vis-à-vis the banking sector, thereby automatically providing substitute funding for banks.

the current composition of eurosystem liabilities.

In what follows, we present the problem from the perspective of the home economy. The foreign economy's problem is symmetric except for CBDC supply and demand. Foreign variables are denoted by an asterisk.

2.1 Households

The intra-period utility of the representative household is:

$$U_t = \exp\left(e_t^C\right) \ln(C_t - hC_{t-1}) - \frac{\chi}{1+\varphi} l_t^{1+\varphi} \tag{2.1}$$

where C_t denotes consumption, l_t hours worked; χ is a scaling parameter; h governs habit formation[16] and φ is the inverse of the Frisch elasticity of labor supply; e_t^C is a consumption preference shock, which follows an AR(1) process. Households optimize utility subject to a budget constraint and a cash-in-advance constraint. The budget constraint is:

$$\begin{aligned} P_t C_t + B_t^H + NER_t B_t^F + D_t + M_t + DC_t \leq W_t l_t + R_{t-1} B_{t-1}^H + \\ + R_{t-1}^* NER_t B_{t-1}^F - \frac{\phi^B}{2}\left(\frac{NER_t B_t^F}{P_t}\right)^2 P_t + D_{t-1} R_{t-1}^D + \xi^\$ M_{t-1} + R_{t-1}^{DC} DC_{t-1} + \Pi_t \end{aligned} \tag{2.2}$$

where P_t is the price level. Funds are used for consumption (C_t)[17], purchases of risk-free domestic bonds (B_t^H) and foreign bonds (B_t^F), adjused for the nominal exchange rate (NER_t, defined as units of domestic currency per unit of foreign currency)[18] and invested into bank deposits (D_t), cash (M_t) and CBDC (DC_t) (when the latter is available).

Sources of funds are labor income $(W_t l_t)$, interest earned on domestic and foreign bonds (R_t and R_t^*, the latter adjusted for the exchange rate), on deposits $(D_{t-1} R_t^D)$ and on CBDC holdings (if the CBDC is remunerated). As stressed above, cash is subject to linearly increasing storage costs $\xi^\$ \in [0, 1]$), resulting from, e.g. the need to keep large

[16]Habit formation is key to generate in-model trade-offs between assets classes that match empirical data; see Jermann (1998).

[17]Aggregate consumption goods are defined as $C_t = \left[\omega^{1-\rho}\left(C_{H,t}\right)^\rho + (1-\omega)^{1-\rho}\left(C_{F,t}\right)^\rho\right]^{\frac{1}{\rho}}$ with ω being the degree of home bias and ρ the elasticity of substitution between home $(C_{H,t})$ and foreign goods $(C_{F,t})$. Aggregate investment goods are defined accordingly: $I_t = \left[\omega^{1-\rho}\left(I_{H,t}\right)^\rho + (1-\omega)^{1-\rho}\left(I_{F,t}\right)^\rho\right]^{\frac{1}{\rho}}$. See the online appendix.

[18]In other words, a fall in NER_t is an appreciation of the domestic currency.

amounts of cash in a vault.[19] The interest rate on CBDC is distinct from the risk-free policy rate (which is determined by a Taylor rule, as we explain below). Finally, Π_t denotes profits of firms net of lump sum taxes.[20] Financial frictions on transactions in foreign bonds ($\frac{\phi^B}{2}(\frac{NER_t B_t^F}{P_t})^2 P_t$) prevent uncovered interest parity to hold fully, in line with standard empirical evidence.

Households need liquidity to purchase final goods. To maintain tractability, we define households' demand for payment instruments through a CES aggregator:

$$\mathcal{L}_t = \chi_L \left[\mu_M M^{1-\eta_L} + \mu_D D^{1-\eta_L} + \mu_{DC} DC^{1-\eta_L}\right]^{\frac{1}{1-\eta_L}} \tag{2.3}$$

total liquidity $\mathcal{L}_t$ aggregates cash, deposits and—if available—CBDC holdings. In the foreign economy, CBDC needs to be converted into foreign currency at the prevailing exchange rate. Moreover, μ_M, μ_D and μ_{DC} are scaling parameters capturing the velocity of circulation of cash, deposits and CBDC respectively,[21] whereas η_L defines the elasticity of substitution between different payment instruments. μ_{DC} and μ_M are calibrated to have cash and CBDC account for about 30% of steady-state output.[22] χ_L pins down the level of liquidity in the steady state. $\mathcal{L}_t$ is a concave function to capture the fact that each payment instrument relaxes the cash-in-advance constraints with diminishing returns to scale. In turn, this captures a preference of households for variety in payment instruments.[23]

Optimality conditions are:

$$\lambda_t + \gamma_t = \frac{\exp\left(e_t^C\right)}{C_t - hC_{t-1}} - h\beta E_t\left[\frac{\exp\left(e_{t+1}^C\right)}{C_{t+1} - hC_t}\right] \tag{2.4}$$

$$\chi l_t^{\phi} = \lambda_t W_t \tag{2.5}$$

[19]In our baseline calibration, we assume that storage costs are zero for reasons of simplicity, i.e. $\xi^{\$} = 1$.

[20]Specifically, firms' profits are $\left[\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\nu} di - MC_t\right] Y_{H,t} + \left[\int_0^1 \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\nu} NER_t di - MC_t\right] Y_{F,t}$.

[21]Put differently, μ_{DC} represents the liquidity services provided by the CBDC relative to cash (μ_M) and deposits (μ_D). Moreover, households may have different preferences across instruments in terms of liquidity services provided, which can be accounted for by different loadings on each instrument in the cash-in-advance constraint.

[22]As noted above, currency in circulation amounts to around 45% of quarterly GDP in the euro area.

[23]Preferences can be micro-founded assuming heterogeneity within households as in Ferrari Minesso et al. (2022) or features of payment instruments as in Agur et al. (2022).

$$E_t\left(\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{R_t}{\pi_{t+1}}\right)=1 \tag{2.6}$$

$$E_t\left(\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{NER_{t+1}}{NER_t}\frac{R_t^*}{\pi_{t+1}}\right)=\left(1+\phi^B NER_t B_t^F\right) \tag{2.7}$$

$$\gamma_t\mu_D\chi_L C_t^{\eta_L}D_t^{-\eta_L}=\lambda_t-\beta E_t\left(\lambda_{t+1}\frac{R_t^D}{\pi_{t+1}}\right) \tag{2.8}$$

$$\gamma_t\mu_M\chi_L C_t^{\eta_L}M_t^{-\eta_L}=\lambda_t-\beta E_t\left(\lambda_{t+1}\frac{\xi}{\pi_{t+1}}\right) \tag{2.9}$$

$$\gamma_t\mu_{DC}\chi_L C_t^{\eta_L}DC_t^{-\eta_L}=\lambda_t-\beta E_t\left(\lambda_{t+1}\frac{R_t^{DC}}{\pi_{t+1}}\right) \tag{2.10}$$

where $\{\lambda_t\}_{t=0}^{\infty}$ and $\{\gamma_t\}_{t=0}^{\infty}$ are the sequences of Lagrange multipliers associated with the budget constraint and the cash-in-advance constraint, respectively. Equation (2.8) to Equation (2.10) define demand for monetary instruments in the model. The deposit rate is $\lambda_t-\gamma_t\mu_D\chi_L\left(\frac{C_t}{D_t}\right)^{\eta_L}\frac{E_t(\pi_{t+1})}{E_t(\lambda_{t+1})}$, which implies that if deposit holdings are above steady-state, the remuneration of deposits needs to increase, as they lose value as payment instrument. Combining Equation (2.6) with Equation (2.8) leads to:

$$\beta E_t\left(\frac{\lambda_{t+1}}{\pi_{t+1}}\right)\left(R_t-R_t^D\right)=\gamma_t\mu_D\chi_L C_t^{\eta_L}D_t^{-\eta_L} \tag{2.11}$$

the higher liquidity services provided by deposits are (the right-hand side of Equation (2.11)), the larger the spread between the risk-free rate and the deposit rate is. In other words, because households benefit from holding deposits for liquidity services, banks can extract a rent proportional to the liquidity services provided. The problem for the foreign economy is symmetric, with the only difference that we allow for a cross-country transaction cost for CBDC ($\phi^{*,DC}$). The foreign demand for CBDC is:

$$\gamma_t^*\mu_{DC}^*\chi_L^* C_t^{*\eta_L^*}\frac{DC_t^*}{NER_t}^{-\eta_L^*}=\lambda_t^*-\beta^*E_t\left(\lambda_{t+1}^*\frac{R_t^{DC}}{\pi_{t+1}^*}\frac{NER_t}{NER_{t+1}}\right)-\lambda_t^*\phi^{*,DC}\frac{DC_t^*}{NER_t} \tag{2.12}$$

where $\lambda_t^*\phi^{*,DC}\frac{DC_t^*}{NER_t}$ is the marginal cross-country transaction cost for CBDC faced by foreigners. A derivation of the full problem for households can be found in Appendix A.

2.2 Entrepreneurs and production

As in Bernanke et al. (1999) we assume that entrepreneurs manage capital producing firms, are risk neutral and finitely-lived.[24] The existence of an incentive-compatibility constraint gives rise to a credit friction: The lower a firm's net worth is, the more severe agency problems become. Banks therefore charge higher rates to more leveraged firms as they need to be monitored more intensively. Each entrepreneur i uses net worth (N) and bank loans (L) to purchase new capital goods (K) at price Q. Capital is produced by specialized agents, capital producers, who sell new investment goods to entrepreneurs. The law of motion of capital is:

$$K_{t+1} = (1-\delta)K_t + I_t \tag{2.13}$$

in aggregate, net borrowings are:

$$L_t = Q_{t+1}K_{t+1} - N_t \tag{2.14}$$

entrepreneurs' risk-adjusted returns on capital must equal expected financing costs ($E_t F_{t+1}$), hence:

$$E_t F_{t+1} = E_t \left[\frac{r^k_{t+1} + (1-\delta)Q_{t+1}}{Q_t} \right] \tag{2.15}$$

where r^k are returns on capital, Q is the cost of capital and δ the capital depreciation rate. Bernanke et al. (1999) show that the optimal debt contract implies an external finance premium, $\Psi(\bullet)$, which depends on the entrepreneur's leverage ratio. In this setup, the external financing cost is equal to the prime (real) lending rate plus the external finance premium. The demand for capital is:

$$E_t F_{t+1} = E_t \left[\frac{R_t}{\pi_{t+1}} \Psi(\bullet) \right] \tag{2.16}$$

where $\Psi(\bullet)$ is a function of leverage with tightness parameter ψ, i.e. $\Psi(\bullet) = \Psi(\frac{Q_t K_t}{N_t}; \psi_t)$ with $\Psi'(\bullet) < 0$ and $\Psi(1) = 1$. ψ_t defines the steady-state lending spread and captures

[24] There is an exogenous probability, ν, that an entrepreneur survives until the next period, which ensures that entrepreneur's retained earnings remain insufficient to finance the acquisition of new capital.

aggregate risk shocks as in Christiano et al. (2014). The external finance premium depends on the leverage ratio, i.e. how large an entrepreneur's stake in the project is. The more firms are leveraged, the higher are the incentives for entrepreneurs to misreport revenues and declare bankruptcy—and the riskier is the loan.[25] For this reason, under the Bernanke et al. (1999) financial friction, banks charge higher rates to more leveraged firms. The external finance premium is counter-cyclical: In good times, firms have higher profits and entrepreneurs accumulate wealth; as a result $\Psi(\bullet)$ decreases. Conversely, when a negative shock hits the economy, firm losses erode entrepreneurs' net worth, the cost of credit increases, which reduces investment and extends the duration of the downturn. We define the external finance premium as:

$$\Psi(\bullet) = \left(\frac{Q_t K_{t+1}}{N_t}\right)^{\psi_t} \tag{2.17}$$

aggregate entrepreneurial net worth evolves as:

$$N_{t+1} = \nu V_t + (1 - \nu_t) g \tag{2.18}$$

where ν is the survival rate of entrepreneurs and g a lump-sum transfer to new entrepreneurs.[26] V is the end-of-period net worth which equals profits minus costs, i.e.: $F_t Q_{t-1} K_t - E_{t-1} F_{t-1} L_{t-1}$.

Intermediate goods firms, which are perfectly competitive and indexed by i, choose capital and labor optimally to produce undifferentiated intermediate goods under a Cobb-Douglas technology that are sold domestically $(Y(i)_H)$ and exported $(X(i)_F)$.[27]

Finally, retailers aggregate domestic and foreign goods produced by competitive firms, differentiate them at negligible costs and sell them on final goods markets with some degree of market power. For this reason, final prices are above the marginal cost of production. We follow Calvo (1983) and assume that retailers can update prices with probability ξ. The full problem for the production sector and all first-order conditions are reported in Appendix A.

[25] When loan riskiness increases, agency costs to monitor firms rise and lenders expect higher losses. Because of that, higher external finance premia are paid by successful (non-defaulting) entrepreneurs to offset these higher losses.

[26] To keep the population of entrepreneurs constant, in each period a fraction $(1 - \nu_t)$ of entrepreneurs become workers and an equal number of workers become entrepreneurs.

[27] The production function is: $Y(i)_{H,t} + X(i)_{F,t} = \exp(A_t) K(i)_t^{\alpha} l(i)_t^{1-\alpha}$ with $\alpha \in (0, 1)$.

2.3 Banks

Banks, indexed by i, intermediate funds between households and firms. Specifically, they collect deposits (D) from households, combine them with bank capital (N^B) and supply loans to entrepreneurs. Banks are run by finitely-lived and risk-neutral bankers. When a banker exits, a new banker takes her place and receives an endowment of capital proportional to total funds intermediated, as in Gertler and Karadi (2011). The budget constraint of the representative bank i is:[28]

$$L_t = N_t^B + D_t \tag{2.19}$$

bankers maximize profits under a Dixit-Stiglitz demand for loans as in Andrés and Arce (2012) and Gerali et al. (2010): $D(i)_t = \left(\frac{R(i)_t^D}{R_t^D}\right)^{\theta_D} D_t$. The optimal deposit rate is endogenously determined as a mark-down on the lending rate:

$$F_t = R_t^D \frac{\theta_{t,D} - 1}{\theta_{t,D}} \tag{2.20}$$

where $\mu_{t,B} = \frac{\theta_{t,D}-1}{\theta_{t,D}}$ defines the (time-varying) mark-down of the deposit rate (R^D) over the loan rate (F). Bankers are able to remunerate deposits below returns on loans because households derive liquidity services from holding deposits. The higher the value of the services provided, the larger is the interest rate spread.[29] The law of motion of aggregate bank net worth, assuming symmetry across banks, is:

$$N_t^B = \nu_B \left(L_{t-1}F_{t-1} - D_{t-1}R_{t-1}^D\right) + \omega_B L_{t-1} \tag{2.21}$$

where ν_B is the survival probability of bankers and ω_B the transfer to new bankers. ω_B also pins down the steady-state level of banks' net worth.

2.4 CBDC issuance

We assume that the government in the home economy issues a CBDC, that is a liability of the central bank directly accessible to households. In our baseline configuration, the CBDC is not remunerated and can be acquired by foreign residents subject to cross-

[28] We suppress here index i to simplify notation.

[29] The steady-state mark-down is a function of the demand for all payment instruments in the model.

country transactions costs. The CBDC is a digital version of a banknote and is not subject to storage costs, unlike cash. The CBDC is also monetary-policy neutral as in Niepelt (2023), which concretely means that issuance of a CBDC leads to a swap between central bank liabilities, for example between excess reserves and CBDC, as discussed e.g. in Adalid et al. (2022). Therefore, issuing CBDC does not affect the overall size of the central bank's balance sheet in the baseline configuration and its issuance does not result in additional money creation. In turn, output and inflation effects are muted in the steady-state—a key requirement for the "equivalence" result of Brunnermeier and Niepelt (2019).[30]

In our framework the CBDC has two effects in the steady-state. First, it expands the supply of payment instruments available to consumers, thereby relaxing their cash-in-advance constraint— with potential improvements in welfare. Second, because consumers can choose from a more diverse menu of payment instruments, banks lose part of their market power—which reduces the rent they can extract from issuing deposits. However, the macroeconomic impact of this channel is a priori ambiguous. Deposits could decrease in equilibrium, therefore leading to bank disintermediation, because alternative payment instruments are available, as discussed in Fernández-Villaverde et al. (2021). If, however, banks decide endogenously to pay higher interest rates on deposits to keep or attract savers, deposit supply might remain stable or even expand, as suggested in Andolfatto (2021).

Armed with the model, we can consider the impact of alternative policies to mitigate volatility *along the transition path* to the new steady-state from an economy without CBDC to one with CBDC. In particular, we consider four alternative mitigating policies: imposing quantity limits, tiered remuneration, expansion of the central bank's balance sheet and limiting access of foreigners to CBDC.

Quantity limits. Excess CBDC demand might lead to bank disintermediation and credit contraction. For this reason, it has been argued that quantity limits might be put

[30] If instead the CBDC is assumed to be monetary policy non-neutral and CBDC issuance leads to an expansion of the central bank's balance sheet, as we discuss below—e.g. when there are no excess reserves to swap with newly created CBDC units—issuing a CBDC could have sizable effects on output. Then, similarly to quantitative easing policies, the central bank would acquire assets to balance newly-created CBDC units on the liability side of its balance sheet, which reduces interest rates and boosts credit, with potentially large quantitative effects on output, as we show below; see e.g. Barrdear and Kumhof (2022) and Burlon et al. (2022).

in place as safeguards during the transition period or in the new steady-state, e.g. by Bank of England (2023); Bindseil et al. (2021). We model quantity limits as an occasionally binding constraint, according to which CBDC holdings of households are defined as:

$$DC_t = \begin{cases} \text{Equation (2.10)} & \text{if } DC_t < \bar{DC} \\ \bar{DC} & \text{if } DC_t \geq \bar{DC} \end{cases} \tag{2.22}$$

$$DC_t^* = \begin{cases} \text{Equation (2.12)} & \text{if } DC_t^* < \bar{DC}^* \\ \bar{DC}^* & \text{if } DC_t^* \geq \bar{DC}^* \end{cases} \tag{2.23}$$

where $\bar{DC}$ and $\bar{DC}^*$ are the quantity limits. Note that quantity limits can be set at different levels for domestic and foreign households.

Tiered remuneration. Two-tiered remuneration has been suggested as another means to reduce excess CBDC demand e.g. by Bindseil (2020). We can think about this as CBDC holdings being not remunerated up to a certain threshold $\bar{DC}$, in line with our baseline assumption of no remuneration. Any amount of CBDC held above $\bar{DC}$ faces a negative interest rate (or a negative spread on the CBDC's remuneration rate, if the latter is positive), so as to discourage large CBDC holdings. We implement two-tiered remuneration in the model with an occasionally binding constraint according to which CBDC remuneration R^{DC} is defined as:

$$R_t^{DC} = \begin{cases} 1 & \text{if } DC_t < \bar{DC} \\ 1\frac{\bar{DC}}{DC_t} + R_{-}^{DC}\frac{DC_t - \bar{DC}}{DC_t} & \text{if } DC_t \geq \bar{DC} \end{cases} \tag{2.24}$$

since there is no consensus on how negative the remuneration rate should be to be effective, we consider alternative calibrations for R_{-}^{DC}. We set the thresholds ($\bar{DC}$, $\bar{DC}^*$) to 50% of steady state CBDC demand in each country and the penalty rate at elevated values i.e. $R_{-}^{DC} = 0.97$ or 300 basis points below parity and $R_{-}^{DC} = 0.95$ or 500 basis points below parity.

Central bank balance sheet expansion. In the baseline scenario we assume that CBDC issuance is monetary-policy neutral in the steady-state, i.e. excess reserves are substituted with CBDC on the central bank's balance sheet. During the transition period, however, the central bank could consider balancing newly created CBDC units with purchases of bank loans to reduce bank disintermediation arising from excess CBDC demand, as discussed e.g. in Brunnermeier and Niepelt (2019). In the model, we assume that these purchases, denoted AP, are proportional to excess CBDC demand with $\chi_{AP} \in (0,1)$:

$$AP_t = \begin{cases} 0 & \text{if } DC_t < DC_{ss} \\ DC_t - \chi_{AP} DC_{ss} & \text{if } DC_t \geq DC_{ss} \end{cases} \tag{2.25}$$

moreover, we assume that the central bank transfers revenues from such asset purchases to the government, which uses them to reduce taxes.

Limited access of foreigners to CBDC. Finally, yet another mitigating policy during the transition period could restrict access of foreigners to CBDC either fully or partially. We model full exclusion by setting $DC^* = 0$ and partial exclusion by increasing the value of parameter $\phi^{*,DC}$ relative to the new equilibrium.

2.5 Solution method

We solve the model using global methods to account for the full set of nonlinearities arising from the transition between the two steady states and the occasionally binding constraint. Specifically, under rational expectations, the nonlinear system of equilibrium equations describing the model, in a generic period τ, can be written as:

$$E_\tau \left[f\left(x_{\tau+1}, x_\tau, x_{\tau-1}, \eta_\tau \right) \right] = 0 \tag{2.26}$$

where x is the vector of N endogenous variables and η_τ a vector of structural shocks. Note that the information set of agents at time τ includes the sequence of shocks $\{\eta_t\}_\tau^T$; i.e. only shocks in period τ are not expected. The model can be solved nonlinearly by solving the stacked system of equations described by Equation (2.26) for all periods from 0 to T; the system can be written in compact notation as

$$\mathcal{F}\left(X \right) = 0 \tag{2.27}$$

where $X = (x'_0, x'_1, ..., x'_T)$ and has $N \times T$ variables. This class of problems is typically solved numerically with a Newton-type algorithm.[31]

We use the nonlinear solution method to compute the transition path to the new steady-state with CBDC. Technically, this is a "two-boundary value problem", i.e. a problem in which initial and terminal conditions—namely the old and new steady states—are known. Note that we assume absence of additional shocks during the transition, i.e. $\eta_t = 0$ for all $t > \tau$.

3 Simulations

In this section we present steady-state effects and transition dynamics from the model without vs. with CDBC. We consider changes in the new steady-state with CBDC and the transition path towards the new equilibrium separately. To account for nonlinearities in the transition between the two steady-states—i.e. without CBDC and with CBDC—as well as for the presence of mitigating policies—modeled mainly with occasionally binding constraints, as discussed above—we use a global solution method to compute the transition path. Studying the transition is crucial because e.g. having a CDBC might be efficient in the long run (i.e. in the new steady-state equilibrium), while excess demand for CBDC in the transition period might disintermediate the banking system, thereby leading to declines in credit, output and welfare in the short run—a challenging trade-off for policy-makers. To manage these effects, alternative mitigating policies aimed at limiting excess CBDC demand can be put in place, the effects of which can be examined with our model.

The model is calibrated following Ferrari Minesso et al. (2022), Christiano et al. (2014) and Gertler and Karadi (2011) (see Appendix B for details on the calibration). The discount factor is chosen in such a way that a period corresponds to a quarter. Parameters for the foreign country are calibrated on data for the United States whereas those for the home country are based on data for Germany, the euro area's largest economy

[31]The solution algorithm we implement works as in Adjemian et al. (2022):

1. Start with an initial guess X^j and verify if Equation (2.27) is satisfied;
2. If not, try a new updated solution (X^{j+1}) that is found according to $\mathcal{F}(X^j) + \mathcal{F}'(X^j)(X^{j+1} - X^j) = 0$ where $\mathcal{F}'(X^j)$ is the Jacobian matrix of $\mathcal{F}(\bullet)$ evaluated at X^j;
3. Iterate steps 1)-2) until convergence, i.e. $||\mathcal{F}(X^j)|| < \epsilon$.

(see Eichenbaum et al. (2021)).

3.1 Steady-state effects

Consider first how issuing a CBDC –without supply constraints– changes the new steady-state compared to the steady-state without CBDC. We assume that the CDBC is not remunerated, is monetary-policy neutral and accessible to foreign households subject to a small cross-border transaction cost.

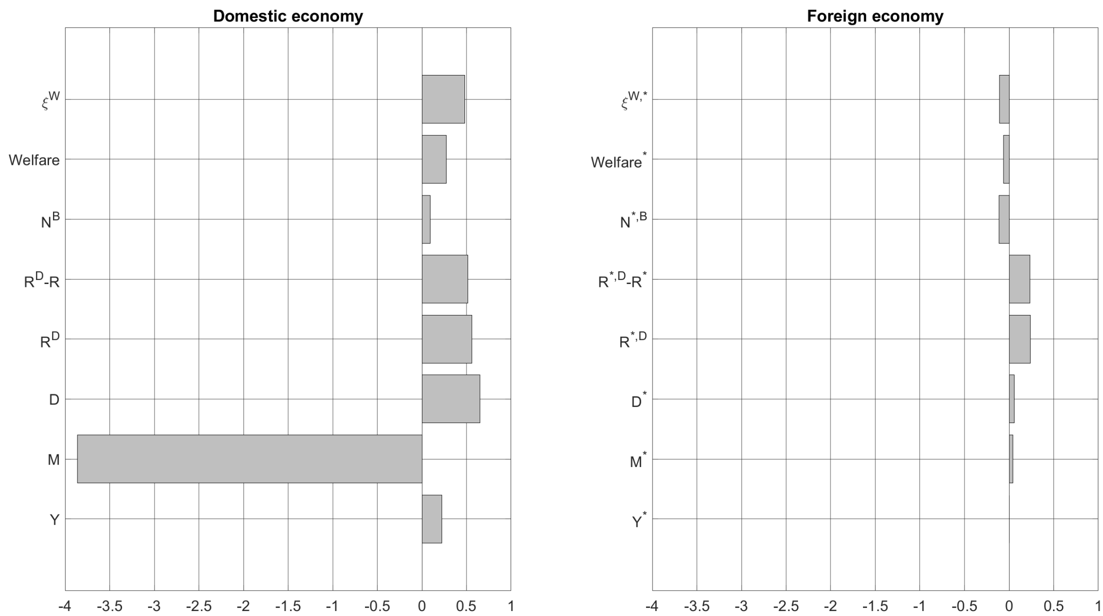

Figure 2: Percentage changes between the stochastic steady-states without and with CDBC.
Notes: The bars (except ξ^W) show percentage changes between the stochastic steady-states of the model without CBDC (i.e. $DC = 0$, $DC^* = 0$) and with CBDC. We consider the stochastic steady-state to factor in the impact of uncertainty and thereby solve the model at the second order with pruning. ξ is a measure of the welfare-consumption equivalent, i.e. the change in consumption needed to make consumers in the baseline model indifferent to the introduction of the CBDC. ξ is defined as $\exp\left[(1-\beta)\left(\mathcal{W}^{CBDC} - \mathcal{W}^{NoCBDC}\right)\right] - 1$; welfare is defined in recursive form: $\mathcal{W}_t = U_t + \beta E_t\left(\mathcal{W}_{t+1}\right)$. The CBDC is issued in the home country and, in this exercise, there are no restrictions in place to limit demand for CBDC

Figure 2 reports percentage changes in the stochastic steady-state once the CBDC is issued.[32] In the home economy (left panel), output remains broadly stable as, in the configuration considered, the CDBC does not improve the productivity of capital and labor in the production sector, which ultimately determines steady-state output. Welfare increases slightly, by about 0.3% of steady-state consumption, as captured by

[32] We consider the stochastic steady-state to factor in the impact of uncertainty, which can change after the CBDC is issued. For example, the CBDC could be used as a store of value and act as a shock absorber, thereby helping to smooth business cycle effects and generating welfare gains.

the consumption-equivalent metric ξ. The financial impact of CBDC is more apparent. Demand for cash falls by about 4% because CBDC expands variety in the menu of payment instruments, which appeals to households. Moreover, banks lose part of their monopoly power over deposit issuance, which is contested by the CBDC. As a result, banks need to increase the interest rate paid on deposits to keep or attract customers, which rises by about 0.5 percentage points. Interestingly, the endogenous increase in the remuneration of deposits increases total deposits by more than 0.5%. In net terms, this is profitable for banks because losses at the intensive margin (i.e. higher deposit rates) are compensated by gains at the extensive margin (more deposits allowing to finance more loans, in turn leading to higher profits). For this reason, output is marginally higher, as more investment opportunities are financed.

In the foreign economy the introduction of a CBDC has no or just marginal effects, with the exception of an increase in deposit rates. The reason is that the CBDC—although it does not bear interest in the baseline configuration—increases the exposure of foreign households to exchange rate valuation effects. Hence, the interest rate paid on bank deposits needs to increase endogenously to make households indifferent between holding CBDC or bank deposits.

3.2 Transition without mitigating policies

3.2.1 Baseline

We first consider the transition without mitigating policies. Figure 3 reports the transition path between the steady-state without CBDC to the steady-state with CBDC in the home (black solid line) and foreign (gray solid line) economies. Variables are reported in percentage deviations from the new steady-state with CBDC. As a result, if the new steady-state is above the steady-state without CDBC, the starting point of the simulation will be negative. Upon issuance, the demand for CBDC in the home economy overshoots the new steady-state by almost 2%. Home households substitute deposits with CBDC, thereby reducing credit available to firms, which triggers a fall in investment and capital in the home economy. As investment falls, home output contracts by about 0.6% relative to the new steady-state, lowering home consumption, prices and welfare. The home central bank reacts by reducing its policy rate, which contributes to a depreciation of the home currency. The larger part of output losses during the transition is absorbed relatively quickly, while it takes more time for CBDC demand to adjust to the new steady-state level; therefore consumption, capital and bank deposits take more time to converge to their new equilibrium.

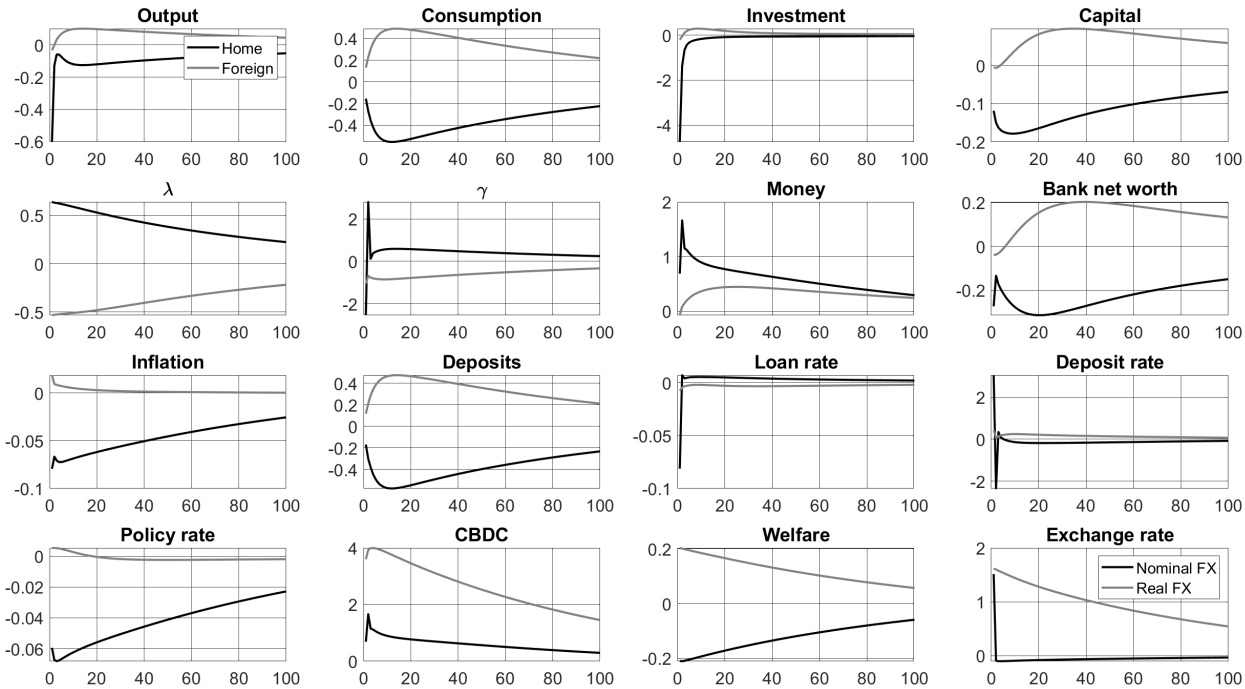

Figure 3: Transition to the new steady-state with CBDC without mitigating policies.
Notes: Variables are reported in percentage deviations from the steady-state with CBDC. The black line shows the transition in the home economy and the gray line in the foreign economy. The model is solved with global methods as in Equation (2.27). The CBDC is issued in the home country and, in this exercise, there are no restrictions in place to limit demand for CBDC.

Interestingly, transition dynamics are different in the foreign economy. Households do not substitute bank deposits, with CBDC (see Figure C.1). Hence credit supply does not decline but, in fact, increases because deposit rates are higher and also because CBDC units held by foreign households are denominated in a currency that appreciates over time (after an initial depreciation; see Figure 3), which makes foreign households richer. Because credit expands, demand and output in the foreign economy increase in tandem. Exports of the foreign economy, in contrast, contract because the foreign currency appreciates—an effect which, however, fades out over time. All in all, during the transition, the foreign economy overheats somewhat relative to the new equilibrium.[33]

[33]The transition is marginally smoothed in case the CBDC is announced in advance. Figure C.5 in the Appendix shows transition dynamics in case the issuance of CBDC is announced 3 years in advance. Agents anticipate the CBDC issuance, so output losses are reduced, but only by a small extent.

3.2.2 Lower steady-state demand for CBDC

Steady-state demand for CDBC is an important determinant of transition dynamics. If stead-state demand is low, introducing a CDBC will not alter materially demand for deposits in the transition period. Therefore, macroeconomic effects should be limited. To examine this conjecture, we calibrate μ_D and μ_D^* to generate much lower demand for CBDC in the new steady-state than in the baseline—at around 5% of GDP. Unconstrained transition dynamics are reported in Figure 4. Unsurprisingly, if stead-state demand for CDBC is low, outflows from deposits are limited, at 0.1% of steady-state level instead of 0.5% in the baseline simulation. Lower steady-state demand for CBDC also results in lower excess demand for CBDC in initial stages of transition—about 0.25% of the steady-state level instead of 2% in the baseline. All in all, the limited impact of CBDC on household choices of monetary instruments implies that credit supply is not materially hit during the transition, therefore investment remains more stable and output contracts negligibly (by about 0.1% of steady-state level or, 6 times less than in the baseline).

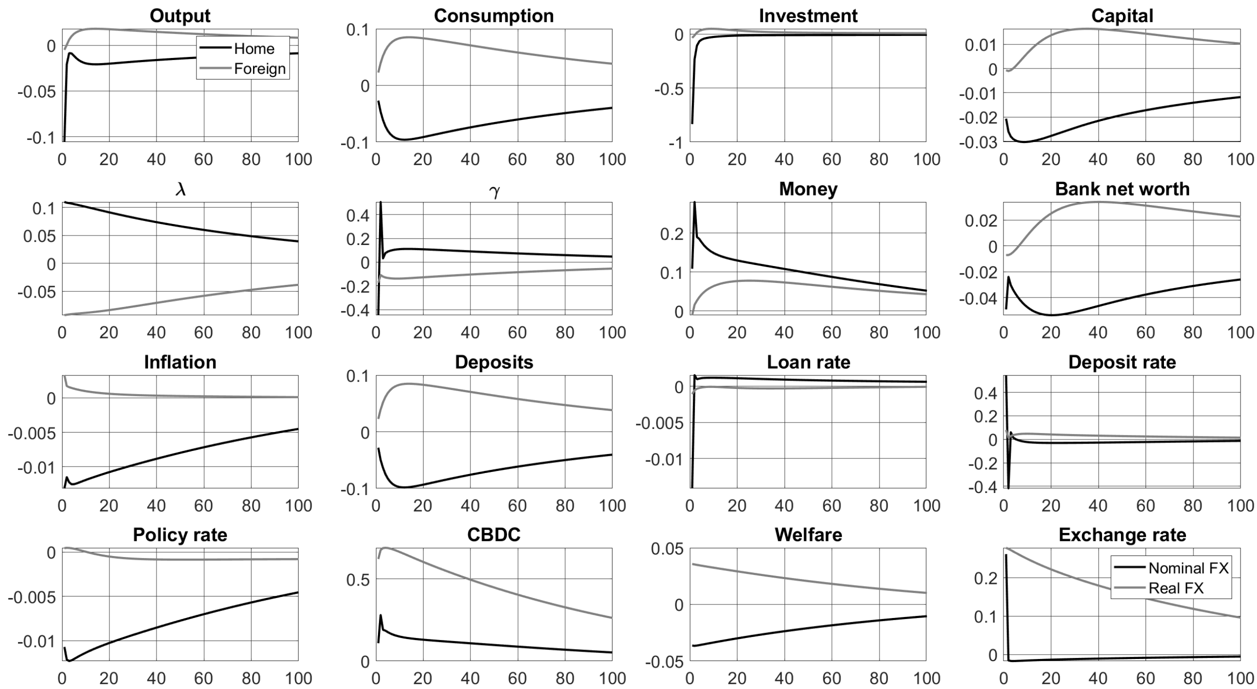

Figure 4: Transition to the new steady-state with CBDC without mitigating policies when CBDC demand is 5% of GDP in the new steady-state.

Notes: Variables are reported in percentage deviations from the steady-state with CBDC. The black line shows the transition in the home economy and the gray line in the foreign economy. The model is solved with global methods as in Equation (2.27). The CBDC is issued in the home country and, in this exercise, there are no restrictions in place to limit demand for CBDC.

3.2.3 Higher storage costs for money

Steady-state demand for cash might also alter trade-offs between alternative payment instruments. Figure 5 reports transition dynamics when we assume a 10% holding cost for money in both countries. Under the new calibration, in the steady-state, money demand drops by about 35%, to about 20% of GDP. In turn, demand for deposits and CBDC increases, by 1.5 and 1.2 percentage points respectively.

Transition dynamics are largely unaffected, however. Output losses and excessive demand for CDBC remain in line with the baseline model. Rebalancing outside money is stronger as holding money is costlier under this calibration. Households hence choose to liquidate monetary holdings to acquire other types of assets in the transition. However, because money accounts for a much smaller share in the mix of households' liquid assets, such stronger rebalancing does not change much the output effects of CBDC introduction relative to the baseline.

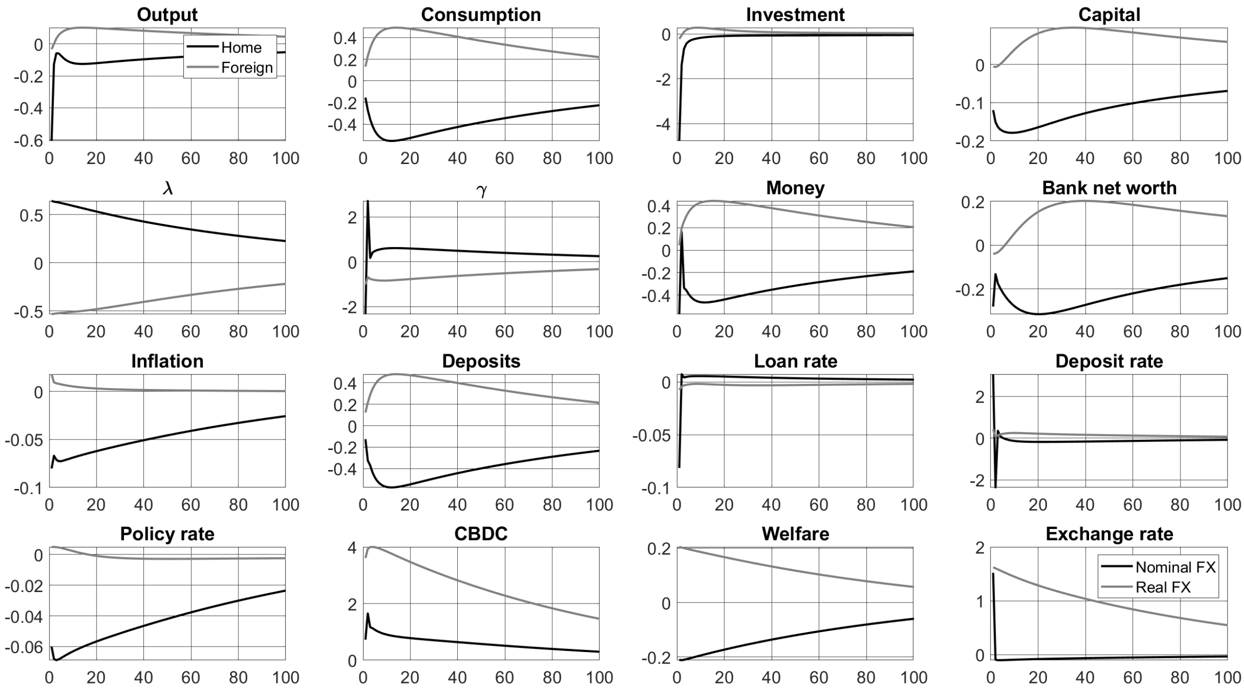

Figure 5: Transition to the new steady-state with CBDC without mitigating policies when Money has a 10% holding cost.

Notes: Variables are reported in percentage deviations from the steady-state with CBDC. The black line shows the transition in the home economy and the gray line in the foreign economy. The model is solved with global methods as in Equation (2.27). The CBDC is issued in the home country and, in this exercise, there are no restrictions in place to limit demand for CBDC.

3.3 Transition with holding limits

We now come back to the baseline calibration and consider the transition with holding limits. The home central bank limits CBDC issuance according to Equation (2.22). We consider two different calibration for these limits. First, a soft limit set at the steady-state level of CBDC demand, which is intended to prevent overshooting during the transition to the new steady-state. Second, a tighter limit at 50% of the steady-state level of CBDC demand, which curbs demand even more. That tighter constraint is maintained at 50% of steady-state demand until the economy is close to the new steady-state, up to period 100, and thereafter gradually relaxed; this is in the spirit of recent policy proposals (e.g. Panetta (2023)).

The transition with a soft limit is reported in Figure 6. The black solid lines shows the transition in the home economy when the limit—modeled as an occasionally binding constraint—is present while the gray dots show the unconstrained transition. Essentially, the transition to the new steady-state unfolds similarly whether the soft limit is present or not; changes, if any, are marginal.

A tighter constraint prevents the materialization of output losses during the transition, as Figure 7 shows. Initial output losses are reduced to almost zero, while deposits even increase as demand for bank deposits is no longer crowded out by excess demand for CBDC. Investments remain stable as loan supply is almost unchanged, because higher deposit from households compensate for lower bank net-worth during the transition. Overall, a hard constraint limits excess demand for CBDC preventing a disorderly withdrawal of bank funding and stabilizing credit supply.

Soft holding limits have also marginal effects on the transition in the foreign economy (see Figure C.6.) Harder limits, instead, reduces positive spillovers to foreign output in the transition to an extent such that they turn marginally negative (see Figure C.7). Insofar as foreign households are constrained in the amount of CDBC they can hold, demand for deposits remains above steady-state, which leads to higher deposit rates.[34] Bank net worth contracts in tandem with credit to the real economy shortly after CBDC introduction. The credit contraction is however short-lived, investments turn positive subsequently, despite small output losses of 0.05% relative to steady-state on impact.

[34] As implied by equation Equation (2.8) a higher deposits reduce their marginal value as means of payment, hence banks need to remunerate them more to make them appealing.

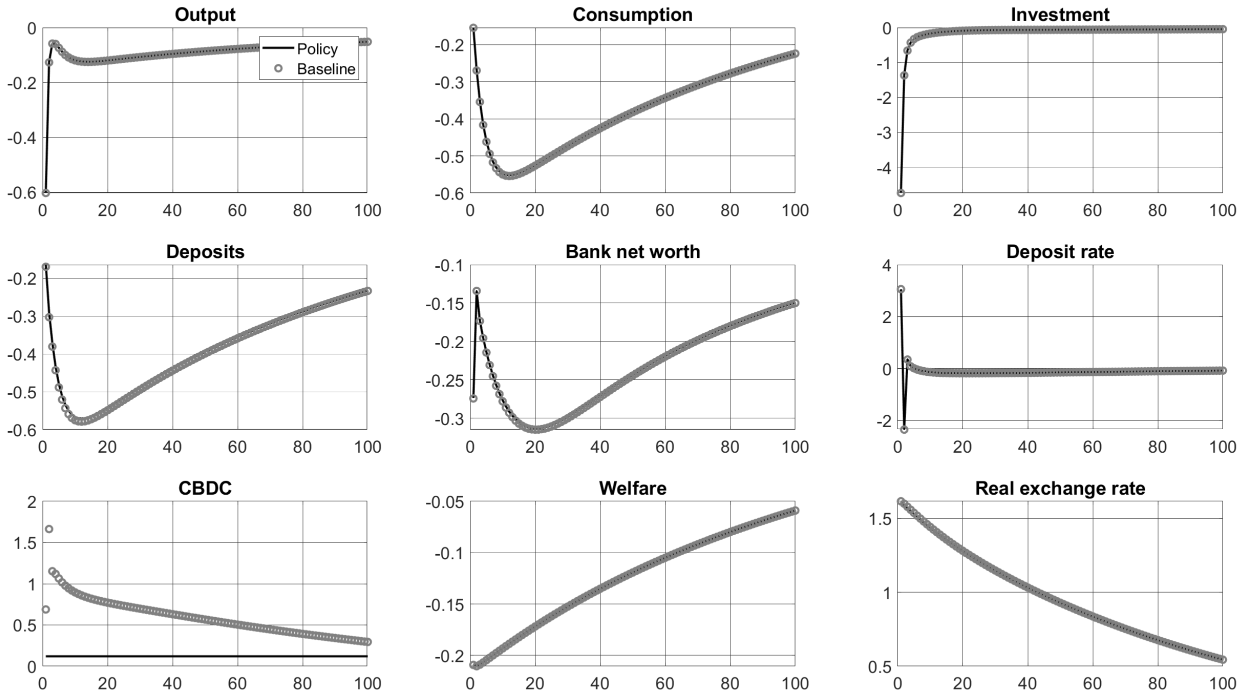

Figure 6: Transition to new steady-state with CBDC and soft holding limit calibrated to the steady-state level of CBDC demand.

Notes: Variables are reported in percentage deviations from the steady-state with CBDC. The black line shows the transition in the home economy under the occasionally binding constraint and the gray dots the unconstrained transition path. The model is solved with global methods as in Equation (2.27). The CBDC is issued in the home economy and, during the transition period, supply of CBDC is defined as in Equation (2.22) with the limits $\bar{DC}$ and $\bar{DC}^*$ set to steady-state demand.

Because demand for CBDC is constrained and output contracts, the foreign currency appreciates by less, by about two-thirds less compared to the baseline, which also limits output losses via the trade channel.

3.4 Transition with tiered remuneration

A two-tiered remuneration scheme, as described in Equation (2.24), is also effective in smoothing the transition—provided that the penalty interest rate is extremely high; see Figure 8. Assuming that CBDC holdings above 50% steady-state demand bear a negative interest rate of 300 basis point (while holdings below 50% of steady-state demand are not remunerated) leads to a marked fall in excess CBDC demand in the transition. Hence home households substitute deposits with CBDC less, which reduces bank disintermediation and the negative knock-on effects on home investment, consumption and output. A higher penalty rate (500 basis points) makes those effects stronger—and even expansionary. Households dislike then the CBDC so much that they move into deposits,

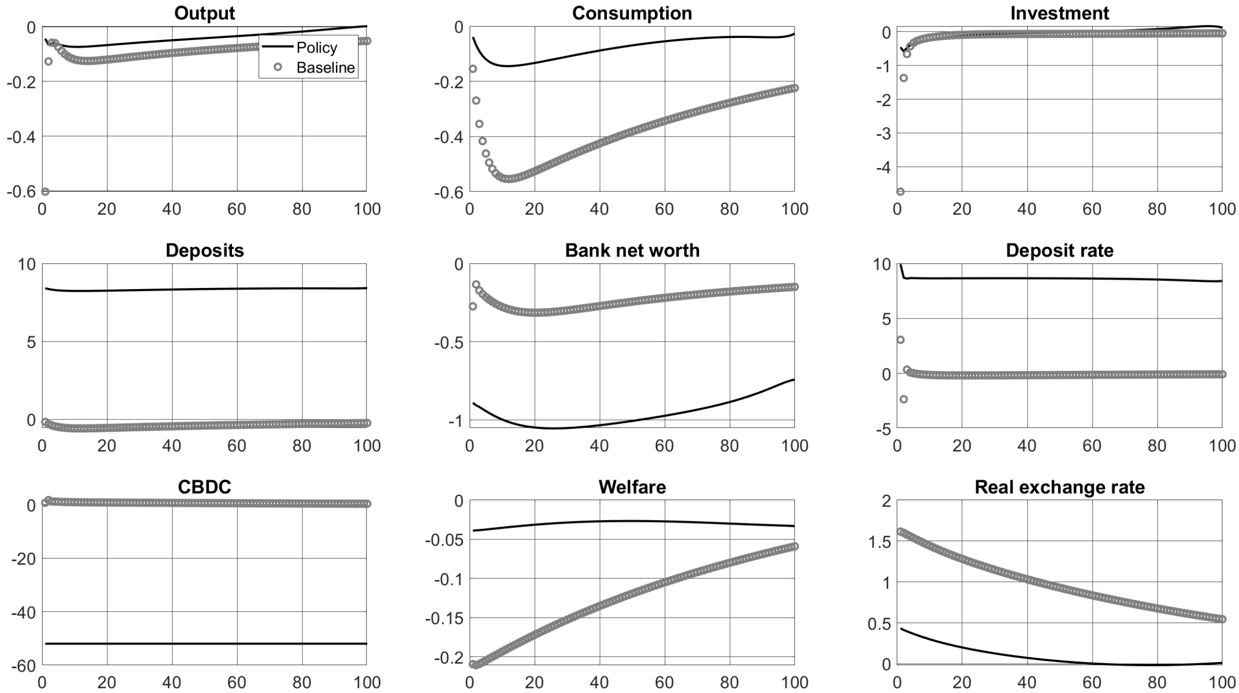

Figure 7: Transition to new equilibrium with CBDC and holding limit calibrated to 50% of steady-state demand for CBDC.

Notes: Variables are reported in percentage deviations from the steady-state with CBDC. The black line shows the transition in the home economy under the occasionally binding constraint and the gray dots the unconstrained transition path. The model is solved with global methods as in Equation (2.27). The CBDC is issued in the home economy and, during the transition period, supply of CBDC is defined as in Equation (2.22) where the limits $\bar{DC}$ and $\bar{DC}^*$ are set to 50% of steady-state demand. The limit is gradually lifted back to the level of steady-state CBDC demand after period 100.

which boosts investment and output.

A 300 basis point penalty rate on excessive CBDC holdings reduces CBDC demand in the foreign economy in the transition (see Figure C.8). That leads to slightly lower investment and output as banks can maintain stronger market power and keep deposit rates lower than in the baseline scenario. Overall effects are limited, however. A higher (500 basis points) penalty rate, instead, reduces foreign demand for the CDBC more markedly. Households incur higher losses on their CBDC holdings, leading to lower savings and consumption. The foreign economy experiences a mild recession, with output contracting in the transition, while the foreign currency depreciates.

3.5 Transition with central bank balance sheet expansion

In Figure 9 we show transition dynamics if the central bank purchases bank loans to balance CBDC demand, as in Equation (2.25). This policy reduces significantly out-

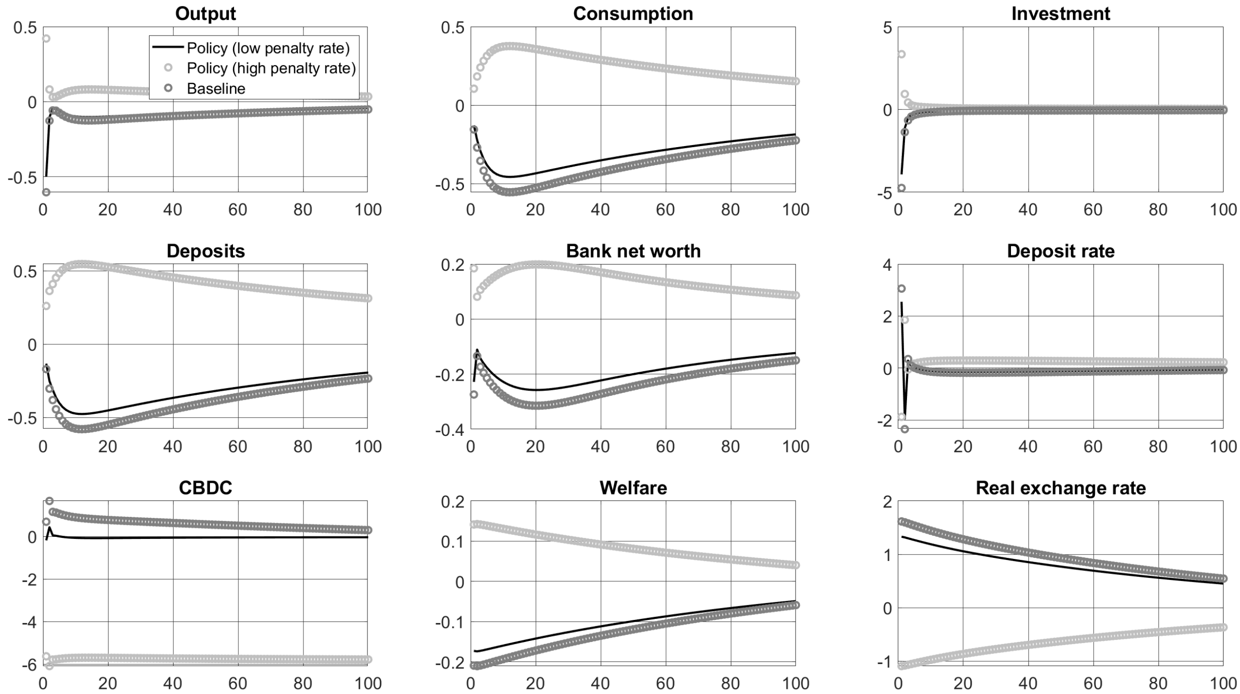

Figure 8: Transition to new equilibrium with CBDC and tiered remuneration.
Notes: Variables are reported in percentage deviations from the steady-state with CBDC. The black line shows the transition in the home economy under the occasionally binding constraint with low penalty rate (300 bps), the light gray dots the occasionally binding constraint with high penalty rate (500 bps) and the dark gray dots the unconstrained transition path. The model is solved with global methods as in Equation (2.27). The home economy issues a CBDC with a tiered remuneration scheme as in Equation (2.24) in the transition period.

put losses relative to the baseline transition without policy, notably through a smaller contraction—about two thirds—of private investment. Purchases by the central bank substitute loans from the banking sector sustaining credit supply to the private sector as in Brunnermeier and Niepelt (2019). Because investment remains more stable, output contracts by less and consumption remains higher (i.e. decreases by two-thirds less relative to the transition without the mitigating policy). The economy contracts by less, therefore the policy rate remains higher than in the baseline scenario; see Figure C.2. Notice however that this policy is less effective in preventing bank disintermediation—a fall in deposits—than hard quantity limits. That reflects the fact that purchases by the central bank substitute bank credit supply to firms; however, households continue to liquidate deposits because they buy almost as much CBDC as in the baseline simulation from the beginning of transition.

Figure C.3 and Figure C.4 in the Appendix show that these effects become somewhat stronger if the central bank purchases private-sector assets more aggressively (for an amount equal to CBDC demand 50% above steady-state).

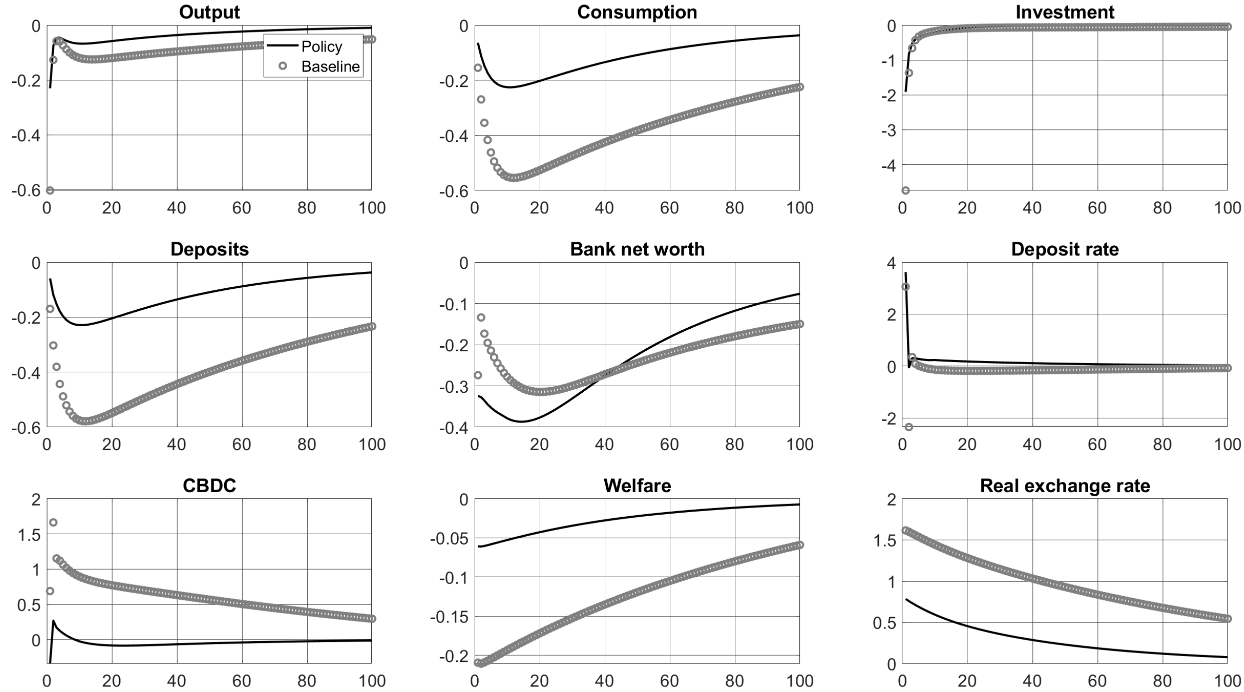

Figure 9: Transition to new equilibrium with CBDC currencies with central bank balance sheet expansion.

Notes: Variables are reported in percentage deviations from the steady-state with CBDC. The black line shows the transition in the home economy under the occasionally binding constraint and the gray dots the unconstrained transition path. The model is solved with global methods as in Equation (2.27). The CBDC is issued in the home economy and, during the transition, the central bank purchases private-sector assets for the amount of CBDC demand above equilibrium.

Although the home central bank purchases domestic bank loans only, this mitigating policy reduces spillovers to the foreign economy in the transition, too. Output losses are reduced, while the home currency depreciates less (see Figure C.9). In turn, the CBDC becomes less attractive to foreign households because expected exchange rate valuation gains are reduced. As a result, foreign demand for the CBDC halves, which stabilizes investment, consumption and output to some extent.

3.6 Transition with restricted access of foreigners to CBDC

Restricting foreigners' access is ineffective in smoothing the transition path in the home economy, no matter whether foreigners are completely excluded from accessing CDBC, as in Figure 10, or partially through higher cross-border transaction costs on CBDC, as in Figure 11.

The reason is that, in this model, restricting foreigners' access to CBDC does not affect the key mechanism that leads to output losses in the home economy during the transition—i.e. the substitution of CBDC with home deposits over and beyond endogenous increases in interest rates paid on deposits to keep or attract savers—which occurs in the home economy and is mostly unaffected by developments in the foreign economy.

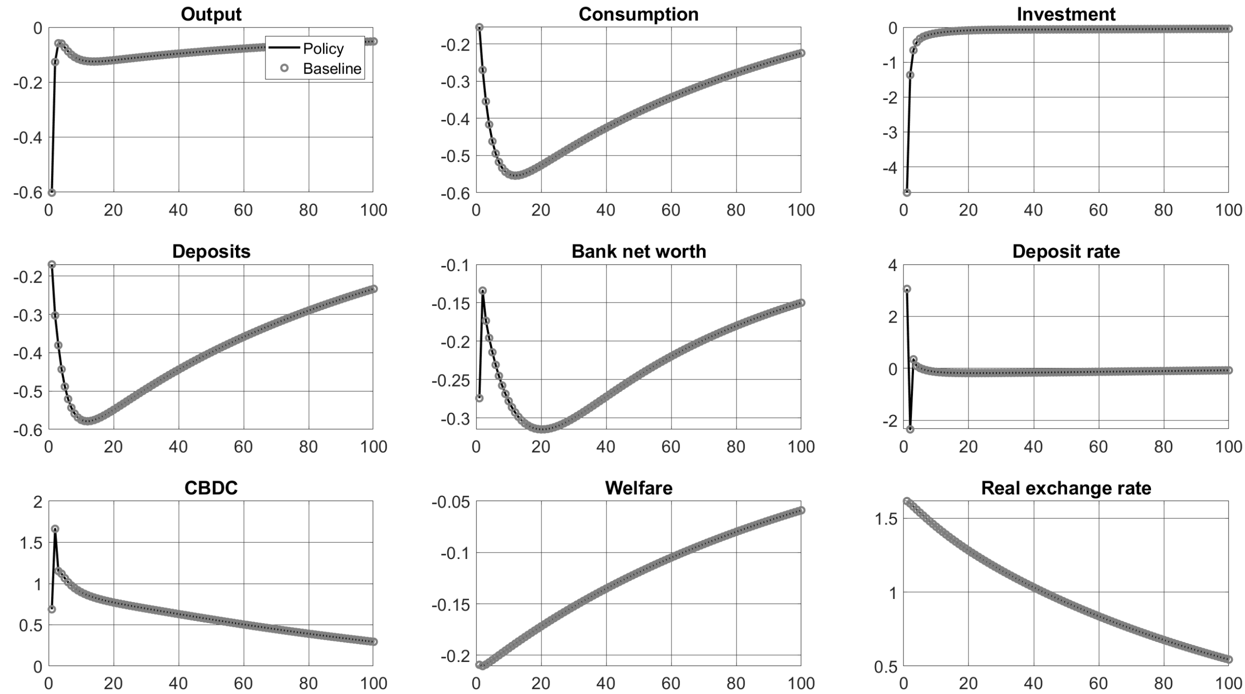

Figure 10: Transition to the new equilibrium with CBDC with no access of foreigners.
Notes: Variables are reported in percentage deviations from the steady-state with CBDC. The black line shows the transition in the home economy if foreigners have no access to the CBDC and the gray dots the unconstrained transition path. The model is solved with global methods as in Equation (2.27). The CBDC is available in the home country only.

If only domestic households can purchase the CDBC, there is no foreign demand by construction. However, CBDC issuance still generates foreign spillovers through other channels, as Figure C.10 shows. Without mitigating policies, in fact, output contracts in the domestic economy during the transition, forcing the domestic central bank to cut the policy rate. The exchange rate depreciates and foreign households rebalance their portfolios, selling bonds issued by the domestic economy and purchasing other assets, namely foreign deposits and bonds. That increases credit supply and boosts output in the foreign country. Higher cross-border CBDC holding costs are not very effective in limiting such spillovers during the transition, however. Although foreign demand for CBDC is lower and the foreign deposit rate increases by less than in the baseline simulation, the aggregate impact on credit and output is negligible.

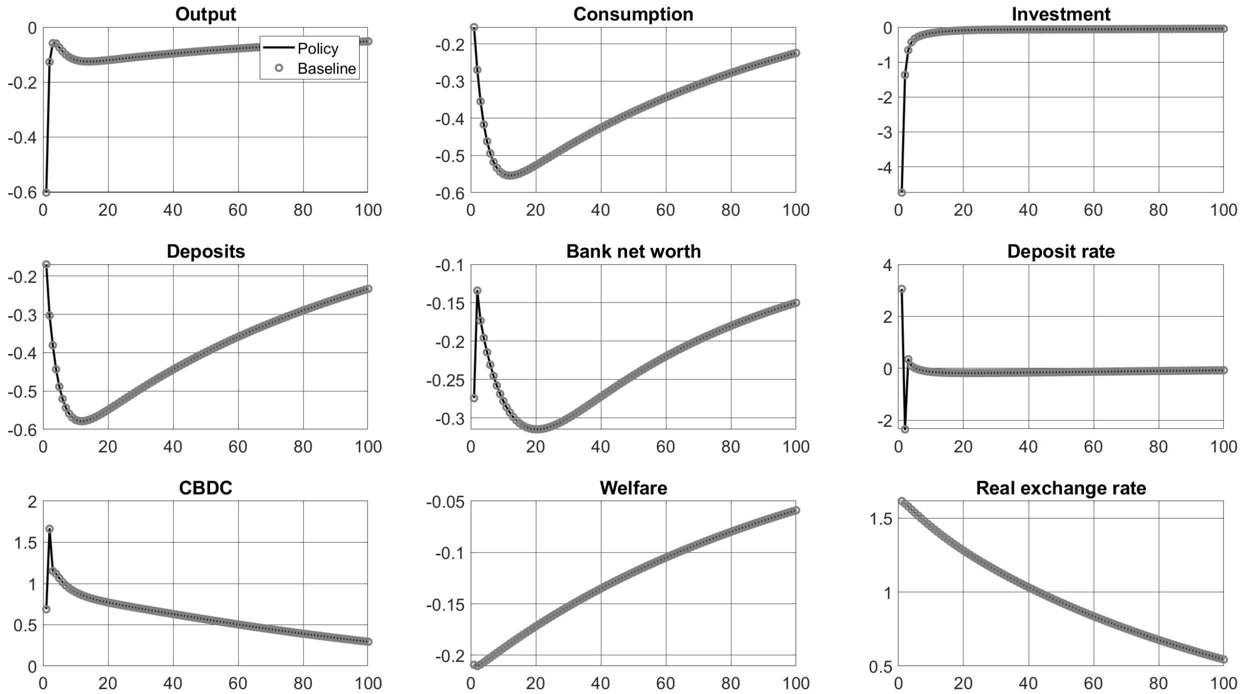

Figure 11: Transition to the new equilibrium with CBDC with partial access by foreigners (higher cross-border transaction costs).

Notes: Variables are reported in percentage changes deviations from the steady-state with CBDC. The black line shows the transition in the home economy when cross-border transaction costs ($\phi^{*,DC}$) are 50 times higher than in the baseline calibration and the gray dots shows the unconstrained transition path. The model is solved with global methods as in Equation (2.27).

3.7 Optimal level of holding limits

The results above suggest that holding limits are well-suited to manage the effects of CBDC during the transition to the new steady-state. These limits trade offs risks of banking disintermediation if demand for CBDC is too strong against welfare losses in terms of reduction in payment options for households if demand for CBDC is too constrained. What is the optimal level for such holding limits? In this section we determine the level of holding limits that maximizes welfare during the transition relative to the equilibrium without CBDC. In the following simulation, welfare ($\mathcal{W}$) is defined as:

$$\mathcal{W}_c^{CBDC} = \sum_{t=0}^{\tau} \beta^t U_{c,t}^{CBDC} + \frac{\beta^{\tau+1}}{1-\beta} U_{c,ss}^{CBDC} \tag{3.1}$$

where τ is the length of the simulation used (200), c is a country index and the subscript "ss" indicates the steady-state value of utility, i.e. after period τ we assume welfare remains at steady-state with no additional shocks. Welfare in the steady-state without CBDCs is: $\mathcal{W}_c^{No\ CBDC} = \frac{1}{1-\beta} U_{c,ss}^{No\ CBDC}$. There are two components to welfare during the transition. The first element in Equation (3.1) captures welfare gains or losses during the transition. The second element defines (permanent) welfare in the new steady-state, conditional on the absence of other shocks.

Figure 12 reports the percent change in welfare relative to the steady-state without CBDC conditional on specific levels of holding limits during the transition (expressed in percentage of steady-state CBDC demand). Interestingly, limit levels above 70% of steady-state CBDC demand generate small net welfare losses, as reductions in welfare costs from heightened volatility in macro variables in the transition period (the first element in Equation (3.1)) remain smaller than the welfare benefits coming from availability of the CBDC at this level (the second element in Equation (3.1)). Net welfare gains become positive for limits below 60% of steady-state CBDC demand a reach a maximum at around 40%, about 2000 euros per capita in our calibration.

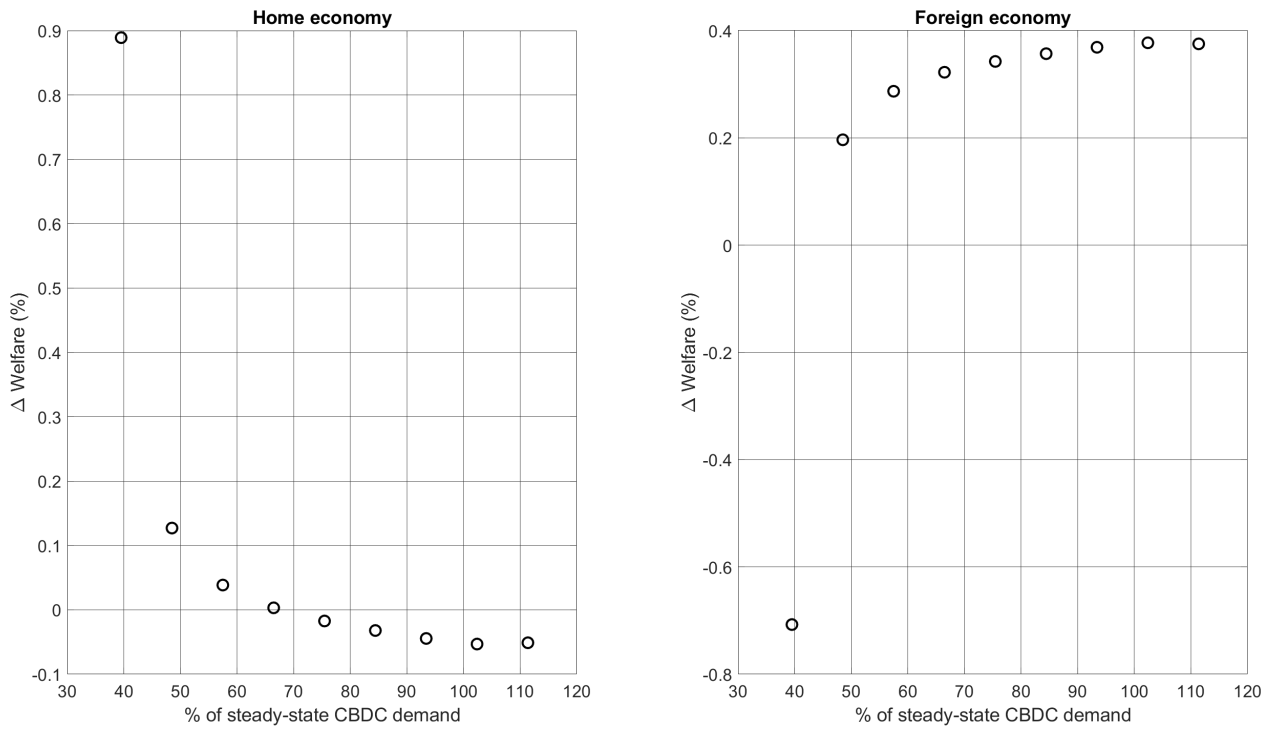

Figure 12: Welfare gains (or losses) for alternative levels of CBDC holding limit.

Notes: The figure shows the change (in percentage points) in welfare (W) relative to the steady-state without a CBDC for alternative levels of the CBDC holding limit during the transition, expressed in percent of steady-state demand.

4 Conclusion

In his paper, we developed a two-country DSGE model with financial frictions to study the transition from a steady-state without CBDC to one in which the home country issues a CBDC. We found that CBDC unambiguously improves welfare without disintermediating the banking sector. Deposits increase in the new steady-state as banks endogenously raise deposit rates. The effects in the transition depend importantly on unobservable steady-state demand for CBDC. For low steady-state demand, introducing a CBDC has no material macroeconomic impacts during the transition. But for higher, plausible values of steady-state demand for CBDC, our simulations point to increased macroeconomic volatility in the transition period. Policies can mitigate these effects. Binding caps reduce disintermediation and output losses in the transition most effectively, with an optimal level of around 40% of steady-state CBDC demand.

These findings have implications for future research. In particular: how long is the transition to a stable equilibrium with CBDC, where its benefits for the economy fully materialize? what determines the length of the transition? how can it be made shorter? These are all questions that could be usefully explored in future research to inform central banks as they are studying whether to issue CBDCs or not—and the broader public at large.

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Appendix

A Derivations

A.1 Households

Households in the domestic economy maximize the discounted sum of period-utility under the budget constraint and the cash-in-advance constraint (CIA). Utility is defined as:

$$U_t = \exp\left(e_t^C\right)\ln(C_t - hC_{t-1}) - \frac{\chi}{1+\varphi} l_t^{1+\varphi} \tag{A.1}$$

where C_t is aggregate consumption, l_t labor supply, h defines habit formation, χ is the weight of labor disutility and φ is the inverse of Frisch elasticity of labor supply. The budget constraint is:

$$P_tC_t + B_t^H + NER_tB_t^F + D_t + M_t + DC_t \leq W_tl_t + R_{t-1}B_{t-1}^H + \\ + R_{t-1}^*NER_tB_{t-1}^F - \frac{\phi^B}{2}\left(\frac{NER_tB_t^F}{P_t}\right)^2 P_t + D_{t-1}R_{t-1}^D + \xi^{\$}M_{t-1} + R_{t-1}^{DC}DC_{t-1} + \Pi_t \tag{A.2}$$

with P_t the aggregate price level, B_t^H and B_t^F domestic and foreign bond holdings, D_t deposits, M_t cash holdings, DC_t CBDC holdings, W_tl_t the wage bill, R_t and R_t^* the domestic and foreign risk-free rate respectively, ϕ^B a cross-country bond holding cost, R_t^{DC} the remuneration on CBDC holdings and Π_t profits net of tax. The cash-in-advance constraint is:

$$\frac{C_t}{P_t} = \mathcal{L}_t = \chi_L\left[\mu_M M^{1-\eta_L} + \mu_D D^{1-\eta_L} + \mu_{DC} DC^{1-\eta_L}\right]^{\frac{1}{1-\eta_L}} \tag{A.3}$$

where χ_L, μ_M, μ_D, μ_{DC} scaling parameters and η_L the elasticity of substitution between different liquidity instruments. First order conditions are:

$$\lambda_t + \gamma_t = \frac{\exp\left(e_t^C\right)}{C_t - hC_{t-1}} - h\beta E_t\left[\frac{\exp\left(e_{t+1}^C\right)}{C_{t+1} - hC_t}\right] \tag{A.4}$$

$$\chi l_t^{\phi} = \lambda_t W_t \tag{A.5}$$

$$E_t\left(\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{R_t}{\pi_{t+1}}\right)=1 \tag{A.6}$$

$$E_t\left(\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{NER_{t+1}}{NER_t}\frac{R^*_t}{\pi_{t+1}}\right)=\left(1+\phi^B NER_t B^F_t\right) \tag{A.7}$$

$$\gamma_t\mu_D\chi_L C_t^{\eta_L}D_t^{-\eta_L}=\lambda_t-\beta E_t\left(\lambda_{t+1}\frac{R^D_t}{\pi_{t+1}}\right) \tag{A.8}$$

$$\gamma_t\mu_M\chi_L C_t^{\eta_L}M_t^{-\eta_L}=\lambda_t-\beta E_t\left(\lambda_{t+1}\frac{\xi}{\pi_{t+1}}\right) \tag{A.9}$$

$$\gamma_t\mu_{DC}\chi_L C_t^{\eta_L}DC_t^{-\eta_L}=\lambda_t-\beta E_t\left(\lambda_{t+1}\frac{R^{DC}_t}{\pi_{t+1}}\right) \tag{A.10}$$

where $\{\lambda_t\}_{t=0}^{\infty}$ and $\{\gamma_t\}_{t=0}^{\infty}$ are the sequences of Lagrangian multipliers associated to the budget constraint and the cash-in-advance constraint respectively. $\pi_t=\frac{P_t}{P_{t-1}}$ is the headline inflation rate.

The households' problem is symmetric in the foreign economy, with the only difference concerning CBDC issuance and demand. Utility is defined as:

$$U^*_t=\exp\left(e^{*,C}_t\right)\ln(C^*_t-h^*C^*_{t-1})-\frac{\chi^*}{1+\varphi^*}l_t^{*1+\varphi^*} \tag{A.11}$$

the budget constraint is:

$$\begin{aligned}P^*_tC^*_t+B^{*,H}_t+\frac{B^{*,F}_t}{NER_t}+D^*_t+M^*_t+DC^*_t\leq W^*_tl^*_t+R^*_{t-1}B^{*,H}_{t-1}+\\+R_{t-1}\frac{B^{*,F}_{t-1}}{NER_t}-\frac{\phi^{*,B}}{2}\left(\frac{B^{*,F}_t}{P^*_tNER_t}\right)^2P^*_t+D^*_{t-1}R^{*,D}_{t-1}+\\+\xi^{*,\$}M^*_{t-1}+R^{DC}_{t-1}\frac{DC^*_{t-1}}{NER_t}-\frac{\phi^{*,DC}}{2}\left(\frac{DC^*_t}{P^*_tNER_t}\right)^2P^*_t+\Pi^*_t\end{aligned} \tag{A.12}$$

where $\phi^{*,DC}$ are CBDC cross-country holding consts. The cash-in-advance constraint is:

$$\frac{C^*_t}{P^*_t}=\mathcal{L}^*_t\chi^*_L\left[\mu^*_M M^{*1-\eta^*_L}+\mu^*_D D^{*1-\eta^*_L}+\mu^*_{DC}\frac{DC^*}{NER_t}^{1-\eta^*_L}\right]^{\frac{1}{1-\eta^*_L}} \tag{A.13}$$

first-order conditions are:

$$\lambda^*_t+\gamma^*_t=\frac{\exp\left(e^{*,C}_t\right)}{C^*_t-h^*C^*_{t-1}}-h^*\beta^*E_t\left[\frac{\exp\left(e^{*,C}_{t+1}\right)}{C^*_{t+1}-h^*C^*_t}\right] \tag{A.14}$$

$$\chi^*L^{*\phi^*}=\lambda^*_tW^*_t \tag{A.15}$$

$$E_t\left(\beta^*\frac{\lambda^*_{t+1}}{\lambda^*_t}\frac{R^*_t}{\pi^*_{t+1}}\right)=1 \tag{A.16}$$

$$E_t\left(\beta^*\frac{\lambda^*_{t+1}}{\lambda^*_t}\frac{NER_t}{NER_{t+1}}\frac{R_t}{\pi^*_{t+1}}\right)=\left(1+\phi^{*,B}\frac{B^{*,F}_t}{NER_t}\right) \tag{A.17}$$

$$\gamma^*_t\mu^*_D\chi^*_L C_t^{*\eta^*_L}D_t^{*-\eta^*_L}=\lambda^*_t-\beta^*E_t\left(\lambda^*_{t+1}\frac{R^{*,D}_t}{\pi^*_{t+1}}\right) \tag{A.18}$$

$$\gamma^*_t\mu^*_M\chi^*_L C_t^{*\eta^*_L}M_t^{*-\eta^*_L}=\lambda^*_t-\beta^*E_t\left(\lambda^*_{t+1}\frac{\xi^*}{\pi^*_{t+1}}\right) \tag{A.19}$$

$$\gamma^*_t\mu^*_{DC}\chi^*_L C_t^{*\eta^*_L}\frac{DC^*_t}{NER_t}^{-\eta^*_L}=\lambda^*_t-\beta^*E_t\left(\lambda^*_{t+1}\frac{R^{DC}_t}{\pi^*_{t+1}}\frac{NER_t}{NER_{t+1}}\right)-\lambda^*_t\phi^{*,DC}\frac{DC^*_t}{NER_t} \tag{A.20}$$

where $\{\lambda^*_t\}^\infty_{t=0}$ and $\{\gamma^*_t\}^\infty_{t=0}$ the sequences of Lagrangian multipliers associated to the budget constraint and the cash-in-advance constraint, respectively. The consumption shock processes are:

$$\begin{aligned} e^C_t &= \rho_C e^C_{t-1}+\varepsilon^C_t \\ e^{*,C}_t &= \rho^*_C e^{*,C}_{t-1}+\varepsilon^{*,C}_t \end{aligned} \tag{A.21}$$

where ε^C_t and $\varepsilon^{*,C}_t$ are IID shocks.

A.2 Entrepreneurs

Entrepreneurs manage firms, are risk neutral and finitely-lived, ν being the survival probability between two periods. Entrepreneurs use net worth (N_t) and bank's loans (L_t) to acquire new capital (K_t) at the price Q_t. Net borrowings are:

$$L_t=Q_{t+1}K_{t+1}-N_t \tag{A.22}$$

because of risk neutrality, returns on capital must equal the expected financing cost (E_tF_{t+1}), hence:

$$E_tF_{t+1}=E_t\left[\frac{r^k_{t+1}+(1-\delta)Q_{t+1}}{Q_t}\right] \tag{A.23}$$

where r^k_t are returns on capital, Q_t the cost of capital and δ the capital's depreciation rate. As in Bernanke et al. (1999) we assume that the external financing cost is equal to the prime (real) lending rate plus the external finance premium. Demand for capital is given by:

$$E_t F_{t+1} = E_t \left[\frac{R_t}{\pi_{t+1}} \left(\frac{Q_t K_{t+1}}{N_t} \right)^{\psi_t} \right] \tag{A.24}$$

ψ_t defines the steady-state lending spread and captures aggregate risk shocks as in Christiano et al. (2014). Aggregate entrepreneurial net worth evolves as:

$$N_{t+1} = \nu V_t + (1 - \nu_t) g \tag{A.25}$$

g is a lump-sum transfer to new entrepreneurs. V_t is end-of-period net worth which equals profits minus costs, that is: $F_t Q_{t-1} K_t - E_{t-1} F_{t-1} L_{t-1}$. The problem is symmetric for the foreign economy. Loans demand is:

$$L_t^* = Q_{t+1}^* K_{t+1}^* - N_t^* \tag{A.26}$$

the expected financing cost is defined as:

$$E_t F_{t+1}^* = E_t \left[\frac{r_{t+1}^{*,k} + (1 - \delta^*) Q_{t+1}^*}{Q_t^*} \right] \tag{A.27}$$

demand for capital is:

$$E_t F_{t+1}^* = E_t \left[\frac{R_t^*}{\pi_{t+1}^*} \left(\frac{Q_t^* K_{t+1}^*}{N_t^*} \right)^{\psi_t^*} \right] \tag{A.28}$$

the law of motion of entrepreneurial net worth is:

$$N_{t+1}^* = \nu^* V_t^* + (1 - \nu_t^*) g^* \tag{A.29}$$

with $V_t^* = F_t^* Q_{t-1}^* K_t^* - E_{t-1} F_{t-1}^* L_{t-1}^*$. Aggregate riskiness, ψ and ψ^* is:

$$\begin{aligned} \psi_t &= \bar{\psi} \exp(\Psi_t) \\ \Psi_t &= \rho_\psi \Psi_{t-1} + \varepsilon_t^\psi \\ \psi_t^* &= \bar{\psi}^* \exp(\Psi_t^*) \\ \Psi_t^* &= \rho_\psi^* \Psi_{t-1}^* + \varepsilon_t^{*,\psi} \end{aligned} \tag{A.30}$$

where ε_t^ψ and $\varepsilon_t^{*,\psi}$ are IID shocks and $\bar{\psi}$, $\bar{\psi}^*$ define steady-state capital demand.

A.3 Capital producers

Capital producers produce new investment goods with a linear production technology subject to quadratic costs. Profits are:

$$\Pi_t^K = Q_t I_t - I_t - \frac{\chi_I}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t \tag{A.31}$$

where I_t are new investment goods and χ_I a scaling parameter. Profit maximization implies:

$$Q_t = 1 - \chi_I\left(\frac{I_t}{K_t} - \delta\right) \tag{A.32}$$

the law of motion of capital is;

$$K_{t+1} = (1-\delta)K_t + I_t \tag{A.33}$$

in the foreign economy the problem is symmetric. Profits are:

$$\Pi_t^{*,K} = Q_t^* I_t^* - I_t^* - \frac{\chi_I^*}{2}\left(\frac{I_t^*}{K_t^*} - \delta^*\right)^2 K_t^* \tag{A.34}$$

with the optimality condition and the law of motion of capital being:

$$Q_t^* = 1 - \chi_I^*\left(\frac{I_t^*}{K_t^*} - \delta^*\right) \tag{A.35}$$

$$K_{t+1}^* = (1-\delta^*)K_t^* + I_t^* \tag{A.36}$$

A.4 Banks

Banks are run by finitely-lived and risk-neutral bankers. In each period there is a probability ν_B that a bankers exits and is replaced with a new banker with initial endowment proportional to total intermediated assets as in Gertler and Karadi (2011). The balance sheet of representative bank i is:

$$L(i)_t = N(i)_t^B + D(i)_t \tag{A.37}$$

where N^B is bank net worth. Profits are:

$$\Pi(i)_t^B = L(i)_t F_t - R(i)_t^D D(i)_t \tag{A.38}$$

under the deposit demand $D(i)_t = \left(\frac{R(i)_t^D}{R_t^D}\right)^{\theta_{t,D}} D_t$. Assuming symmetry across banks, profit maximization implies that

$$F(i)_t = R(i)_t^D \frac{\theta(i)_{t,D} - 1}{\theta(i)_{t,D}} \tag{A.39}$$

where $\frac{\theta(i)_{t,D}-1}{\theta(i)_{t,D}}$ is the (time-varying) mark-down of the deposit rate relative to the loan rate. This depends on the existence of a payment use for deposits, which allows banks to extract a rent. We define the mark-down in the steady-state of the model. The law of motion of (aggregated) bankers net worth is:

$$N(i)_t^B = \nu_B \left(L(i)_{t-1} F(i)_{t-1} - D(i)_{t-1} R(i)_{t-1}^D\right) + \omega_B L(i)_{t-1} \tag{A.40}$$

where ω_B defines steady-state bank net worth. In the foreign country, the balance sheet is:

$$L(i)_t^* = N(i)_t^{*,B} + D(i)_t^* \tag{A.41}$$

with equilibrium rates being:

$$F(i)_t^* = R(i)_t^{*,D} \frac{\theta(i)_{t,D}^* - 1}{\theta(i)_{t,D}^*} \tag{A.42}$$

and net worth:

$$N(i)_t^{*,B} = \nu_B^* \left(L(i)_{t-1}^* F(i)_{t-1}^* - D(i)_{t-1}^* R(i)_{t-1}^{*,D}\right) + \omega_B^* L(i)_{t-1}^* \tag{A.43}$$

assuming symmetry across banks allows aggregation.

A.5 Goods aggregation

Domestic goods are used for consumption (C), investment (I) and government spending (G):

$$C_{H,t} + G_{H,t} + I_{H,t} = Y_{H,t} \tag{A.44}$$

where index H denotes consumption of goods from the home country. Similarly, in the foreign economy, goods produced in the home country are used in consumption, investment and purchased by the foreign government:

$$C^*_{F,t} + G^*_{F,t} + I^*_{F,t} = X_{F,t} \tag{A.45}$$

where index F denotes consumption in the foreign economy. Symmetrically, the foreign economy demands foreign domestic goods and exports to the domestic economy:

$$C^*_{H,t} + G^*_{H,t} + I^*_{H,t} = Y^*_{H,t} \tag{A.46}$$

$$C_{F,t} + G_{F,t} + I_{F,t} = X^*_{F,t} \tag{A.47}$$

final domestically purchased and exported goods are aggregated across firms i through a CES aggregator function. The aggregation technology is:

$$\begin{aligned}
Y_{H,t} &= \left[\int_0^1 Y(i)_{H,t}^{\frac{\nu}{\nu-1}} di\right]^{\frac{\nu}{\nu-1}} & X_{F,t} &= \left[\int_0^1 X(i)_{H,t}^{\frac{\nu^*}{\nu^*-1}} di\right]^{\frac{\nu^*}{\nu^*-1}} \\
Y^*_{H,t} &= \left[\int_0^1 \left(Y(i)^*_{H,t}\right)^{\frac{\nu^*}{\nu^*-1}} di\right]^{\frac{\nu^*}{\nu^*-1}} & X^*_{F,t} &= \left[\int_0^1 \left(X(i)^*_{H,t}\right)^{\frac{\nu}{\nu-1}} di\right]^{\frac{\nu}{\nu-1}}
\end{aligned} \tag{A.48}$$

where ν is the elasticity of substitution across varieties i. In both countries, total profits are the sum of profits from domestically-produced and exported goods:

$$\begin{aligned}
\Pi_t^{AG} &= P_{H,t}\left[\int_0^1 Y(i)_{H,t}^{\frac{\nu}{\nu-1}} di\right]^{\frac{\nu}{\nu-1}} - \int_0^1 P(i)_{H,t} Y(i)_{H,t} di + \\
&+ P_{F,t}\left[\int_0^1 (X(i)^*)_{F,t}^{\frac{\nu}{\nu-1}} di\right]^{\frac{\nu}{\nu-1}} - \int_0^1 P(i)_{F,t} X(i)^*_{F,t} di \\
\Pi_t^{*,AG} &= P^*_{H,t}\left[\int_0^1 \left(Y(i)^*_{H,t}\right)^{\frac{\nu^*}{\nu^*-1}} di\right]^{\frac{\nu^*}{\nu^*-1}} - \int_0^1 P(i)^*_{H,t} Y(i)^*_{H,t} di + \\
&+ P^*_{F,t}\left[\int_0^1 X(i)_{F,t}^{\frac{\nu^*}{\nu^*-1}} di\right]^{\frac{\nu^*}{\nu^*-1}} - \int_0^1 P(i)^*_{F,t} X(i)_{H,t} di
\end{aligned} \tag{A.49}$$

profit maximization implies:

$$
\begin{aligned}
Y(i)_{H,t} &= \left(\frac{P(i)_{H,t}}{P_{H,t}}\right)^{-\nu} Y_{H,t} \quad X(i)^*_{F,t} = \left(\frac{P(i)_{F,t}}{P_{F,t}}\right)^{-\nu} X^*_{F,t} \\
Y(i)^*_{H,t} &= \left(\frac{P(i)^*_{H,t}}{P^*_{H,t}}\right)^{-\nu^*} Y^*_{H,t} \quad X(i)_{F,t} = \left(\frac{P(i)^*_{F,t}}{P^*_{F,t}}\right)^{-\nu^*} X_{H,t}
\end{aligned}
\tag{A.50}
$$

according to Equation (A.50), demand for of each variety i depends on the price of variety i relative to the aggregate price of the same variety and on total demand. Substituting demand into the profit functions defines price aggregates:

$$
\begin{aligned}
P_{H,t} &= \left(\int_0^1 P(i)_{H,t}^{1-\nu} di\right)^{\frac{1}{1-\nu}} \qquad P_{F,t} = \left(\int_0^1 P(i)_{F,t}^{1-\nu} di\right)^{\frac{1}{1-\nu}} \\
P^*_{H,t} &= \left(\int_0^1 P(i)^{*\,1-\nu^*}_{H,t} di\right)^{\frac{1}{1-\nu^*}} \quad P^*_{F,t} = \left(\int_0^1 P(i)^{*\,1-\nu^*}_{F,t} di\right)^{\frac{1}{1-\nu^*}}
\end{aligned}
\tag{A.51}
$$

retailers produce total consumption, investment and government consumption goods, aggregating domestically-produced and imported goods:

$$
\begin{aligned}
C_t &= \left[\omega^{1-\rho} (C_{H,t})^\rho + (1-\omega)^{1-\rho} (C_{F,t})^\rho\right]^{\frac{1}{\rho}} \\
I_t &= \left[\omega^{1-\rho} (I_{H,t})^\rho + (1-\omega)^{1-\rho} (I_{F,t})^\rho\right]^{\frac{1}{\rho}} \\
G_t &= \left[\omega^{1-\rho} (G_{H,t})^\rho + (1-\omega)^{1-\rho} (G_{F,t})^\rho\right]^{\frac{1}{\rho}}
\end{aligned}
\tag{A.52}
$$

ω is the degree of home bias and ρ the elasticity of substitution between domestic and imported goods. C_t, I_t, G_t are final bundles of private consumption, investments and government consumption respectively. Retailers' profits are:

$$
\begin{aligned}
P_t C_t - P_{H,t} C_{H,t} - P_{F,t} C_{F,t} \\
P_t I_t - P_{H,t} I_{H,t} - P_{F,t} I_{F,t} \\
P_t G_t - P_{H,t} G_{H,t} - P_{F,t} G_{F,t}
\end{aligned}
$$

where $P_{H,t}$ is the price of goods produced in the home country and $P_{F,t}$ the price of goods produced in the foreign country. Both prices are expressed in home currency units.

First-order conditions define demand for aggregate domestic and imported goods:

$$\begin{aligned} C_{H,t} &= \left(\frac{P_{H,t}}{P_t}\right)^{\frac{1}{\rho-1}} \omega C_t \\ I_{H,t} &= \left(\frac{P_{H,t}}{P_t}\right)^{\frac{1}{\rho-1}} \omega I_t \\ G_{H,t} &= \left(\frac{P_{H,t}}{P_t}\right)^{\frac{1}{\rho-1}} \omega G_t \end{aligned} \tag{A.53}$$

demand for domestically-produced goods depends on their price relative to the aggregate price level, home bias and total consumption. Similarly, demand functions for imported goods are:

$$\begin{aligned} C_{F,t} &= \left(\frac{P_{F,t}}{P_t}\right)^{\frac{1}{\rho-1}} (1-\omega) C_t \\ I_{H,t} &= \left(\frac{P_{H,t}}{P_t}\right)^{\frac{1}{\rho-1}} (1-\omega) I_t \\ G_{H,t} &= \left(\frac{P_{H,t}}{P_t}\right)^{\frac{1}{\rho-1}} (1-\omega) G_t \end{aligned} \tag{A.54}$$

combining Equation (A.44) and Equation (A.53) defines total demand of domestic goods in the domestic economy as:

$$Y_{H,t} = \omega \left(\frac{P_{H,t}}{P_t}\right)^{\frac{1}{\rho-1}} (C_t + I_t + G_t) \tag{A.55}$$

symmetrically, Equation (A.47) and Equation (A.54) give demand for imported goods:

$$X^*_{F,t} = (1-\omega) \left(\frac{P_{F,t}}{P_t}\right)^{\frac{1}{\rho-1}} (C_t + I_t + G_t) \tag{A.56}$$

The foreign economy's problems are fully symmetric. Final consumption bundles are:

$$\begin{aligned} C^*_t &= \left[(\omega^*)^{1-\rho^*} \left(C^*_{H,t}\right)^{\rho^*} + (1-\omega^*)^{1-\rho^*} \left(C^*_{F,t}\right)^{\rho^*}\right]^{\frac{1}{\rho^*}} \\ I^*_t &= \left[(\omega^*)^{1-\rho^*} \left(I^*_{H,t}\right)^{\rho^*} + (1-\omega^*)^{1-\rho^*} \left(I^*_{F,t}\right)^{\rho^*}\right]^{\frac{1}{\rho^*}} \\ G^*_t &= \left[(\omega^*)^{1-\rho^*} \left(G^*_{H,t}\right)^{\rho^*} + (1-\omega^*)^{1-\rho^*} \left(G^*_{F,t}\right)^{\rho^*}\right]^{\frac{1}{\rho^*}} \end{aligned} \tag{A.57}$$

profits are:

$$P_t^* C_t^* - P_{H,t}^* C_{H,t}^* - P_{F,t}^* C_{F,t}^*$$
$$P_t^* I_t^* - P_{H,t}^* I_{H,t}^* - P_{F,t}^* I_{F,t}^*$$
$$P_t^* G_t^* - P_{H,t}^* G_{H,t}^* - P_{F,t}^* G_{F,t}^*$$

demand for goods produced domestically in the foreign economy is:

$$\begin{aligned} C_{H,t}^* &= \left(\frac{P_{H,t}^*}{P_t^*}\right)^{\frac{1}{\rho^*-1}} \omega^* C_t^* \\ I_{H,t}^* &= \left(\frac{P_{H,t}^*}{P_t^*}\right)^{\frac{1}{\rho^*-1}} \omega^* I_t^* \\ G_{H,t}^* &= \left(\frac{P_{H,t}^*}{P_t^*}\right)^{\frac{1}{\rho^*-1}} \omega^* G_t^* \end{aligned} \tag{A.58}$$

demand for imported goods is:

$$\begin{aligned} C_{F,t}^* &= \left(\frac{P_{F,t}^*}{P_t^*}\right)^{\frac{1}{\rho^*-1}} (1-\omega^*) C_t^* \\ I_{H,t}^* &= \left(\frac{P_{H,t}^*}{P_t^*}\right)^{\frac{1}{\rho^*-1}} (1-\omega^*) I_t^* \\ G_{H,t}^* &= \left(\frac{P_{H,t}^*}{P_t^*}\right)^{\frac{1}{\rho^*-1}} (1-\omega^*) G_t^* \end{aligned} \tag{A.59}$$

combining Equation (A.46) and Equation (A.58) gives total demand for domestically-produced goods in the foreign economy:

$$Y_{H,t}^* = \omega^* \left(\frac{P_{H,t}^*}{P_t^*}\right)^{\frac{1}{\rho^*-1}} (C_t^* + I_t^* + G_t^*) \tag{A.60}$$

combining Equation (A.47) and Equation (A.59) defines total demand for imported goods in the foreign economy:

$$X_{F,t} = (1-\omega^*) \left(\frac{P_{F,t}^*}{P_t^*}\right)^{\frac{1}{\rho^*-1}} (C_t^* + I_t^* + G_t^*) \tag{A.61}$$

aggregate price indices are:

$$P_t = \left[\omega\left(P_{H,t}\right)^{\frac{\rho}{\rho-1}} + (1-\omega)\left(P_{F,t}\right)^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}$$
$$P_t^* = \left[\omega^*\left(P_{H,t}^*\right)^{\frac{\rho^*}{\rho^*-1}} + (1-\omega^*)\left(P_{F,t}^*\right)^{\frac{\rho^*}{\rho^*-1}}\right]^{\frac{\rho^*-1}{\rho^*}} \tag{A.62}$$

A.6 Intermediate goods production

There is a continuum of perfectly competitive firms (indexed by i) that produce undifferentiated intermediate goods. The production function is:

$$Y(i)_{H,t} + X(i)_{F,t} = A_t K(i)_t^{\alpha} l(i)_t^{1-\alpha} \tag{A.63}$$

where $Y(i)_H$ and $X(i)_F$ are goods produced for the domestic market and for export respectively and A_t total factor productivity. Total costs are $TC(i)_t = r(i)_t^k K(i)_t + W(i)_t l(i)_t$, where $r(i)_t^k$ are returns on capital and $W(i)_t$ the real wage.

Cost minimization implies that:

$$r_t^k = \int_0^1 \alpha MC(i)_t A_t K(i)_t^{\alpha-1} L(i)_t^{\alpha} di$$
$$W_t = \int_0^1 (1-\alpha) MC(i)_t A_t K(i)_t^{\alpha} l(i)_t^{-\alpha} di \tag{A.64}$$

$\{MC(i)_t\}_{t=0}^{\infty}$ is the sequence of Lagrangian multipliers associated with the problem which defines the real marginal cost of production. The production function in the foreign economy is:

$$Y(i)_{H,t}^* + X(i)_{F,t}^* = A_t^* \left(K(i)_t^*\right)^{\alpha^*} \left(l(i)_t^*\right)^{1-\alpha^*} \tag{A.65}$$

Total costs are $TC(i)_t^* = r(i)_t^{*,k} K(i)_t^* + W(i)_t^* l(i)_t^*$.

First-order conditions are:

$$r_t^{*,k} = \int_0^1 \alpha^* MC(i)_t^* A_t^* \left(K(i)_t^*\right)^{\alpha^*-1} \left(L(i)_t^*\right)^{\alpha^*} di$$
$$W_t^* = \int_0^1 (1-\alpha^*) MC(i)_t^* A_t^* \left(K(i)_t^*\right)^{\alpha^*} \left(l(i)_t^*\right)^{-\alpha^*} di \tag{A.66}$$

$\{MC(i)_t^*\}_{t=0}^{\infty}$ is the sequence of Lagrangian multipliers associated with the problem which defines the real marginal cost of production. Total factor productivity (TFP) is defined as:

$$\begin{aligned}A_t &= \exp(a_t)\\ a_t &= \rho_A a_{t-1} + \varepsilon_t^A\\ A_t^* &= \exp(a_t^*)\\ a_t^* &= \rho_{*,A} a_{t-1}^* + \varepsilon_t^{*,A}\end{aligned} \tag{A.67}$$

where ε_t^A and $\varepsilon_t^{*,A}$ are IID shocks.

A.7 Price setting

Prices are set by monopolists, indexed by i, who purchase undifferentiated final goods from retailers and sell them with some market power on final markets. They maximize profits under the Calvo formalism, that is monopolists can update prices in each period only with probability ξ, and subject to the demand functions and a steady-state tax τ. Profits are the sum of domestic sales and exported goods to the foreign economy. $\hat{P}(i)_H$ and $\hat{P}(i)_F^*$ are the optimal prices on the domestic and exported goods, respectively, if a monopolist is able to update the price.
Total profits are:

$$\begin{aligned}E_t \sum_{j=0}^{\infty} (\beta\xi)^j \lambda_{t+j} \Bigg[&\left(\frac{\hat{P}(i)_{H,t}}{P_{t+j}} - MC_{t+j} \right) \left(\frac{\hat{P}(i)_{H,t}}{P_{H,t+j}} \right)^{-\nu} Y_{H,t+j} + \\ &+ \left(\frac{NER_{t+j}}{P_{t+j}} \hat{P}(i)_{F,t}^* - MC_{t+j} \right) \left(\frac{\hat{P}(i)_{F,t}^*}{P_{F,t+j}^*} \right)^{-\nu^*} X_{F,t+j} \Bigg]\end{aligned} \tag{A.68}$$

the optimal new domestic price $\hat{P}(i)_{H,t}$ is:

$$E_t \sum_{j=0}^{\infty} (\beta\xi)^j \lambda_{t+j} \left[\frac{\hat{P}(i)_{H,t}}{P_t} \frac{P_t}{P_{t+j}} \left(\frac{P_{H,t}}{P_{H,t+j}} \right)^{-\nu} Y_{H,t+j} - \frac{1}{1+\tau} \frac{\nu}{\nu-1} MC_{t+j} \left(\frac{P_{H,t}}{P_{H,t+j}} \right)^{-\nu} Y_{H,t+j} \right] = 0 \tag{A.69}$$

because all monopolists are equal, it is possible to drop index i and the previous equation can be written recursively:

$$
\begin{aligned}
F_{H,t}p\hat{_{H,t}} &= K_{H,t} \\
F_{H,t} &= \lambda_t Y_{H,t} + \beta\xi E_t\left(\pi_{t+1}^{-1}\pi_{H,t+1}^{\nu}F_{H,t+1}\right) \\
K_{H,t} &= \lambda_t \frac{1}{1+\tau}\frac{\nu}{\nu-1}MC_t Y_{H,t} + \beta\xi E_t\left(\pi_{H,t+1}^{\nu}K_{H,t+1}\right)
\end{aligned}
\tag{A.70}
$$

where $p\hat{_{H,t}} = \frac{P\hat{_{H,t}}}{P_t}$, $\pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}}$ and $\pi_t = \frac{P_t}{P_{t-1}}$. The price index is:

$$
P_{H,t} = \left[(1-\xi)P\hat{_{H,t}}^{1-\nu} + \xi P_{H,t-1}^{1-\nu}\right]^{\frac{1}{1-\nu}}
\tag{A.71}
$$

similarly, the optimal new price for exported goods, $\hat{P}(i)_{F,t}^{*}$, is:

$$
\begin{aligned}
E_t\sum_{j=0}^{\infty}(\beta\xi)^j\lambda_{t+j}\Bigg[&\frac{NER_{t+j}}{NER_t}\frac{NER_tP_t^*}{P_t}\frac{\hat{P}(i)_{F,t}^*}{P_t^*}\frac{P_t}{P_{t+j}}\left(\frac{P_{F,t}^*}{P_{F,t+1}^*}\right)^{-\nu^*}X_{F,t+j} + \\
&- \frac{1}{1+\tau}\frac{\nu^*}{\nu^*-1}MC_{t+j}\left(\frac{P_{F,t}^*}{P_{t+j}^*}\right)^{-\nu^*}X_{F,t+j}\Bigg] = 0
\end{aligned}
\tag{A.72}
$$

as monopolists are all equal, index i can be dropped and the first order condition can be written recursively as:

$$
\begin{aligned}
F_{F,t}RER_t p\hat{_{F,t}^*} &= K_{F,t} \\
F_{F,t} &= \lambda_t X_{F,t} + \beta\xi E_t\left[\frac{NER_{t+1}}{NER_t}\pi_{t+1}^{-1}\left(\pi_{F,t+1}^*\right)^{\nu^*}F_{F,t+1}\right] \\
K_{F,t} &= \lambda_t\frac{1}{1+\tau}\frac{\nu}{\nu-1}MC_tX_{F,t} + \beta\xi E_t\left[\pi_{F,t}^*K_{H,t+1}\right]
\end{aligned}
\tag{A.73}
$$

where $p\hat{_{F,t}^*} = \frac{P\hat{_{F,t}^*}}{P_t^*}$ and $\pi_{F,t}^* = \frac{P_{F,t}^*}{P_{F,t-1}^*}$. The price index is:

$$
P_{F,t}^* = \left[(1-\xi)P\hat{_{F,t}^*}^{1-\nu^*} + \xi P_{F,t-1}^{*,1-\nu^*}\right]^{\frac{1}{1-\nu^*}}
\tag{A.74}
$$

in the foreign economy monopolists face the same problem. Their profit function is:

$$E_t \sum_{j=0}^{\infty} (\beta^* \xi^*)^j \lambda^*_{t+j} \left[\left(\frac{\hat{P}(i)^*_{H,t}}{P^*_{t+j}} - MC^*_{t+j} \right) \left(\frac{\hat{P}(i)^*_{H,t}}{P^*_{H,t+j}} \right)^{-\nu^*} Y^*_{H,t+j} + \right.$$
$$\left. + \left(\frac{1}{NER_{t+j} P^*_{t+j}} \hat{P}(i)_{F,t} - MC^*_{t+j} \right) \left(\frac{\hat{P}(i)_{F,t}}{P_{F,t+j}} \right)^{-\nu} X^*_{F,t+j} \right] \tag{A.75}$$

the first-order condition relative to $\hat{P}(i)^*_{H,t}$ is:

$$E_t \sum_{j=0}^{\infty} (\beta^* \xi^*)^j \lambda^*_{t+j} \left[\frac{\hat{P}(i)^*_{H,t}}{P^*_t} \frac{P^*_t}{P^*_{t+j}} \left(\frac{P^*_{H,t}}{P^*_{H,t+j}} \right)^{-\nu^*} Y^*_{H,t+j} - \frac{1}{1+\tau^*} \frac{\nu^*}{\nu^*-1} MC^*_{t+j} \left(\frac{P^*_{H,t}}{P^*_{H,t+j}} \right)^{-\nu^*} Y^*_{H,t+j} \right] = 0 \tag{A.76}$$

the previous equation can be written in recursive form as:

$$\begin{aligned} F^*_{H,t} \hat{p^*_{H,t}} &= K^*_{H,t} \\ F^*_{H,t} &= \lambda^*_t Y^*_{H,t} + \beta^* \xi^* E_t \left(\left(\pi^*_{t+1} \right)^* \pi^{*,\nu}_{H,t+1} F^*_{H,t+1} \right) \\ K^*_{H,t} &= \lambda^*_t \frac{1}{1+\tau^*} \frac{\nu^*}{\nu^*-1} MC^*_t Y^*_{H,t} + \beta^* \xi^* E_t \left(\pi^{*,\nu}_{H,t+1} K^*_{H,t+1} \right) \end{aligned} \tag{A.77}$$

where $\hat{p^*_{H,t}} = \frac{\hat{P^*_{H,t}}}{P^*_t}$, $\pi^*_{H,t} = \frac{P^*_{H,t}}{P^*_{H,t-1}}$ and $\pi^*_t = \frac{P^*_t}{P^*_{t-1}}$.

The price index is:

$$P^*_{H,t} = \left[(1-\xi^*) \hat{P^*_{H,t}}^{1-\nu^*} + \xi^* P^{*,1-\nu^*}_{H,t-1} \right]^{\frac{1}{1-\nu^*}} \tag{A.78}$$

the first-order condition for $\hat{P}(i)_{F,t}$ is:

$$E_t \sum_{j=0}^{\infty} (\beta^* \xi^*)^j \lambda^*_{t+j} \left[\frac{NER_t}{NER_{t+j}} \frac{P_t}{NER_t P^*_t} \frac{\hat{P}(i)_{F,t}}{P_t} \frac{P^*_t}{P^*_{t+j}} \left(\frac{P_{F,t}}{P_{F,t+1}} \right)^{-\nu} X^*_{F,t+j} + \right.$$
$$\left. - \frac{1}{1+\tau^*} \frac{\nu}{\nu-1} MC^*_{t+j} \left(\frac{P_{F,t}}{P_{t+j}} \right)^{-\nu} X^*_{F,t+j} \right] = 0 \tag{A.79}$$

which has a recursive form representation:

$$
\begin{aligned}
F^*_{F,t}\frac{\hat{p_{F,t}}}{RER_t} &= K^*_{F,t} \\
F^*_{F,t} &= \lambda^*_t X^*_{F,t} + \beta^*\xi^* E_t\left[\frac{NER_t}{NER_{t+1}}\left(\pi^*_{t+1}\right)^{-1}\left(\pi_{F,t+1}\right)^{\nu} F^*_{F,t+1}\right] \\
K^*_{F,t} &= \lambda^*_t \frac{1}{1+\tau^*}\frac{\nu^*}{\nu^*-1} MC^*_t X_{F,t} + \beta^*\xi^* E_t\left[\pi_{F,t} K^*_{H,t+1}\right]
\end{aligned}
\tag{A.80}
$$

where $\hat{p_{F,t}} = \frac{\hat{P_{F,t}}}{P_t}$ and $\pi_{F,t} = \frac{P_{F,t}}{P_{F,t-1}}$.

The price index is:

$$
P_{F,t} = \left[(1-\xi^*)\hat{P_{F,t}}^{1-\nu} + \xi^* P^{1-\nu}_{F,t-1}\right]^{\frac{1}{1-\nu}} \tag{A.81}
$$

A.8 Market clearing and government

Total production in the domestic economy is:

$$
Y^{tot}_t = \int_0^1 Y(i)_{H,t} di + \int_0^1 X(i)_{F,t} di = A_t \int_0^1 K(i)^{\alpha}_t L(i)^{1-\alpha}_t di \tag{A.82}
$$

which can be written recursively as:

$$
\begin{aligned}
Y^{tot}_t &= d_{H,t} Y_{H,t} + d_{F,t} X_{F,t} \\
d_{H,t} &= (1-\xi)\left(\frac{P_{H,t}}{P_t}\right)^{\nu} \hat{p_{H,t}}^{-\nu} + \xi \pi^{\nu}_{H,t} d_{H,t-1} \\
d_{F,t} &= (1-\xi)\left(\frac{P^*_{F,t}}{P^*_t}\right)^{\nu^*} (\hat{p^*_{F,t}})^{-\nu^*} + \xi (\pi^*_{F,t})^{\nu^*} d_{F,t-1}
\end{aligned}
\tag{A.83}
$$

symmetrically, in the foreign economy aggregate production is:

$$
Y^{*,tot}_t = \int_0^1 Y(i)^*_{H,t} di + \int_0^1 X(i)^*_{F,t} di = A^*_t \int_0^1 \left(K(i)^*_t\right)^{\alpha^*} \left(L(i)^*_t\right)^{1-\alpha^*} di \tag{A.84}
$$

which can also be written recursively as:

$$
\begin{aligned}
Y^{*,tot}_t &= d^*_{H,t} Y^*_{H,t} + d^*_{F,t} X^*_{F,t} \\
d^*_{H,t} &= (1-\xi^*)\left(\frac{P^*_{H,t}}{P^*_t}\right)^{\nu^*} (\hat{p^*_{H,t}})^{-\nu^*} + \xi^* (\pi^*_{H,t})^{\nu^*} d^*_{H,t-1} \\
d^*_{F,t} &= (1-\xi^*)\left(\frac{P_{F,t}}{P_t}\right)^{\nu} (\hat{p_{F,t}})^{-\nu} + \xi (\pi_{F,t})^{\nu} d^*_{F,t-1}
\end{aligned}
\tag{A.85}
$$

government spending in each country is exogenous:

$$\frac{G_t}{G_{ss}} = \left(\frac{G_{t-1}}{G_{ss}}\right)^{\rho_G} \varepsilon_t^G \frac{G_t^*}{G_{ss}^*} = \left(\frac{G_{t-1}^*}{G_{ss}^*}\right)^{\rho_G^*} \varepsilon_t^{*,G} \tag{A.86}$$

where G_{ss} is the steady-state level of government consumption and ε^G and $\varepsilon^{*,G}$ IID shocks. There is zero net supply of bonds:

$$\begin{aligned} B_t^H + B_t^{*,F} &= 0 \\ B_t^{*,H} + B_t^F &= 0 \end{aligned} \tag{A.87}$$

monetary policy in the Home and the Foreign countries follows a Taylor rule:

$$\begin{aligned} \ln R_t &= (1-\varrho)\ln R_{t-1} + \varrho\left[\ln R_{ss} + \theta_\pi \ln \pi_t + \theta_y\left(\ln Y_t - \ln Y_{ss}\right)\right] + \mathcal{E}_t \\ \ln R_t^* &= (1-\varrho^*)\ln R_{t-1}^* + \varrho^*\left[\ln R_{ss}^* + \theta_\pi^* \ln \pi_t^* + \theta_y^*\left(\ln Y_t^* - \ln Y_{ss}^*\right)\right] + \mathcal{E}_t^* \end{aligned} \tag{A.88}$$

where Y_{ss} is the steady-state level of output.

Monetary policy innovations $\mathcal{E}_t$ are:

$$\begin{aligned} \mathcal{E}_t &= \rho_R \mathcal{E}_{t-1} + \varepsilon_t^R \\ \mathcal{E}_t^* &= \rho_R^* \mathcal{E}_{t-1}^* + \varepsilon_t^{*,R} \end{aligned} \tag{A.89}$$

with ε^R and $\varepsilon^{*,R}$ IID shocks.

The real exchange rate is defined as:

$$RER_t = NER_t \frac{P_t^*}{P_t} \tag{A.90}$$

welfare $(\mathcal{W}_t)$ is defined recursively:

$$\begin{aligned} \mathcal{W}_t &= U_t + \beta E_t\left(\mathcal{W}_{t+1}\right) \\ \mathcal{W}_t^* &= U_t^* + \beta^* E_t\left(\mathcal{W}_{t+1}^*\right) \end{aligned} \tag{A.91}$$

B Tables

Table B.1: Calibration

Parameter	Description	Value	Parameter	Description	Value
		Structural parameters			
h	Habit persistence	0.75	ξ	Calvo parameter	0.76
h^*	Habit persistence	0.75	ξ^*	Calvo parameter	0.33
β	Discount factor	0.9926	ν	Elasticity of demand	6
β^*	Discount factor	0.9926	ν^*	Elasticity of demand	6
φ	Inverse of Frisch's labor elasticity	1	ϱ	Interest rate smoothing	0.75
φ^*	Inverse of Frisch's labor elasticity	1	ϱ^*	Interest rate smoothing	0.75
χ	Weight of labor in utility	0.969072	θ_y	Sensitivity to output	0.26
χ^*	Weight of labor in utility	0.969056	θ_y^*	Sensitivity to output	0.04
σ	Elasticity of consumption	1	θ_π	Sensitivity to inflation	1.49
σ^*	Elasticity of consumption	1	θ_π^*	Sensitivity to inflation	1.68
ϕ^B	Cross-country bond holding cost	0.001	G_{ss}	Share of government spending in steady-state consumption	0.2
$\phi^{*,B}$	Cross-country bond holding cost	0.001	G_{ss}	Share of government spending in steady-state consumption	0.2
ω	Home bias	0.9	$\bar{\psi}$	External finance premium parameter	0.005
ω^*	Home bias	0.9	$\bar{\psi}^*$	External finance premium parameter	0.005
α	Technology parameter	0.25	$\xi^{\$}$	Storage cost of cash	1
α^*	Technology parameter	0.25	$\xi^{*,\$}$	Storage cost of cash	1
ρ	Elasticity of substitution across goods	0.333333	ν	Survival probability of firms	0.9
ρ^*	Elasticity of substitution across goods	0.333333	ν^*	Survival probability of firms	0.9
δ	Depreciation rate of capital	0.025	ω_B	Share of assets transferred to new banks	0.0001
δ^*	Depreciation rate of capital	0.025	ω_B^*	Share of assets transferred to new banks	0.0001
		Liquidity parameters			
χ_I	Elasticity of investment	0.2	μ_M	Weight of cash	0.03
χ_I^*	Elasticity of investment	0.2	μ_M^*	Weight of cash	0.03
η_L	Elasticity of substitution across liquid assets	6	μ_D	Weight of deposits	1
η_L^*	Elasticity of substitution across liquid assets	6	μ_D^*	Weight of deposits	1
χ_L	Scaling parameter for liquidity	2.5	μ_{DC}	Weight of CBDC	0.02
χ_L^*	Scaling parameter for liquidity	2.5	μ_{DC}^*	Weight of CBDC	0.02
$\phi^{*,DC}$	Cross-country CBDC holding cost	0.001			
		Shock processes			
σ_R	Volatility of monetary policy shocks	0.01	ρ_A	Autoregressive component of TFP shocks	0.96
σ_R^*	Volatility of monetary policy shocks	0.01	ρ_A^*	Autoregressive component of TFP shocks	0.95
σ_A	Volatility of TFP shocks	0.01	ρ_G	Autoregressive component of government spending shocks	0.86
σ_A^*	Volatility of TFP shocks	0.01	ρ_G^*	Autoregressive component of government spending shocks	0.95
σ_G	Volatility of government spending shocks	0.01	ρ_C	Autoregressive component of consumption preference shocks	0.81
σ_G^*	Volatility of government spending shocks	0.01	ρ_C^*	Autoregressive component of consumption preference shocks	0.8
σ_C	Volatility of consumption preference shocks	0.01	ρ_ψ	Autoregressive component of financial shocks	0.92
σ_C^*	Volatility of consumption preference shocks	0.01	ρ_ψ^*	Autoregressive component of financial shocks	0.97
σ_ψ	Volatility of financial shocks	0.01			
σ_ψ^*	Volatility of financial shocks	0.01			

Notes: A * denotes parameters for the foreign economy.

C Figures

C.1 Domestic economy

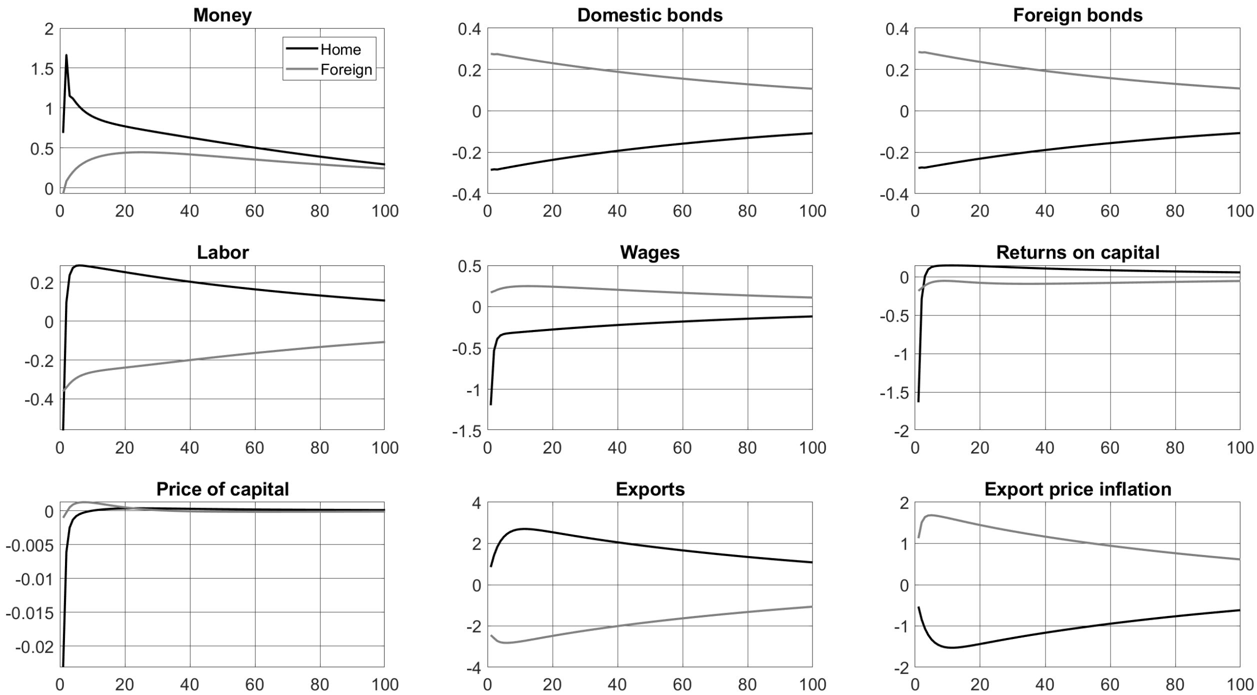

Figure C.1: Transition to the new equilibrium with CBDC without mitigating policies (other variables). **Notes**: Variables are reported in percentage deviations from to the steady-state with CBDC. The black line shows the transition in the home economy and the gray line in the foreign economy. The model is solved with global methods as in Equation (2.27). The CBDC is issued in the home country and there are no restrictions in place to limit CBDC demand.

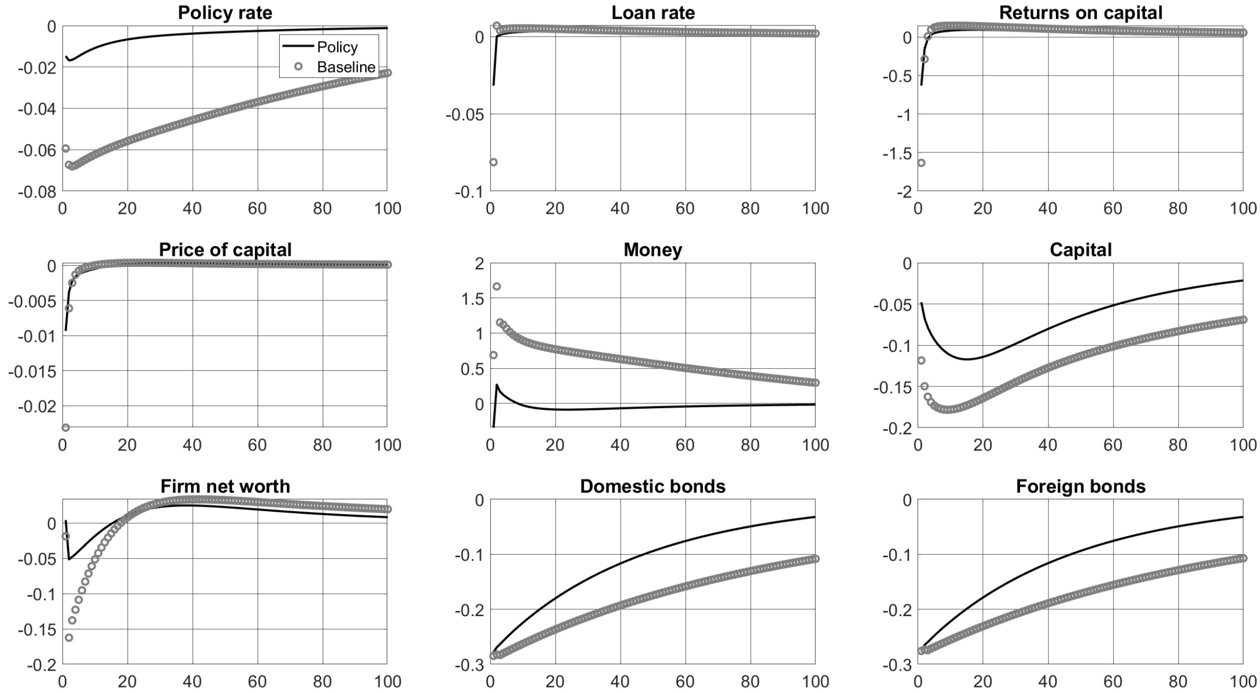

Figure C.2: Transition to the new equilibrium with CBDC with central bank balance sheet expansion (other variables).

Notes: Variables are reported in percentage deviations from the steady-state with CBDC. The black line shows the transition in the home economy under the occasionally binding constraint and the gray dots the unconstrained transition path. The model is solved with global methods as in Equation (2.27). The CBDC is issued in the home country and, during the transition period, the central bank purchases private-sector assets for the amount of CBDC demand above equilibrium.

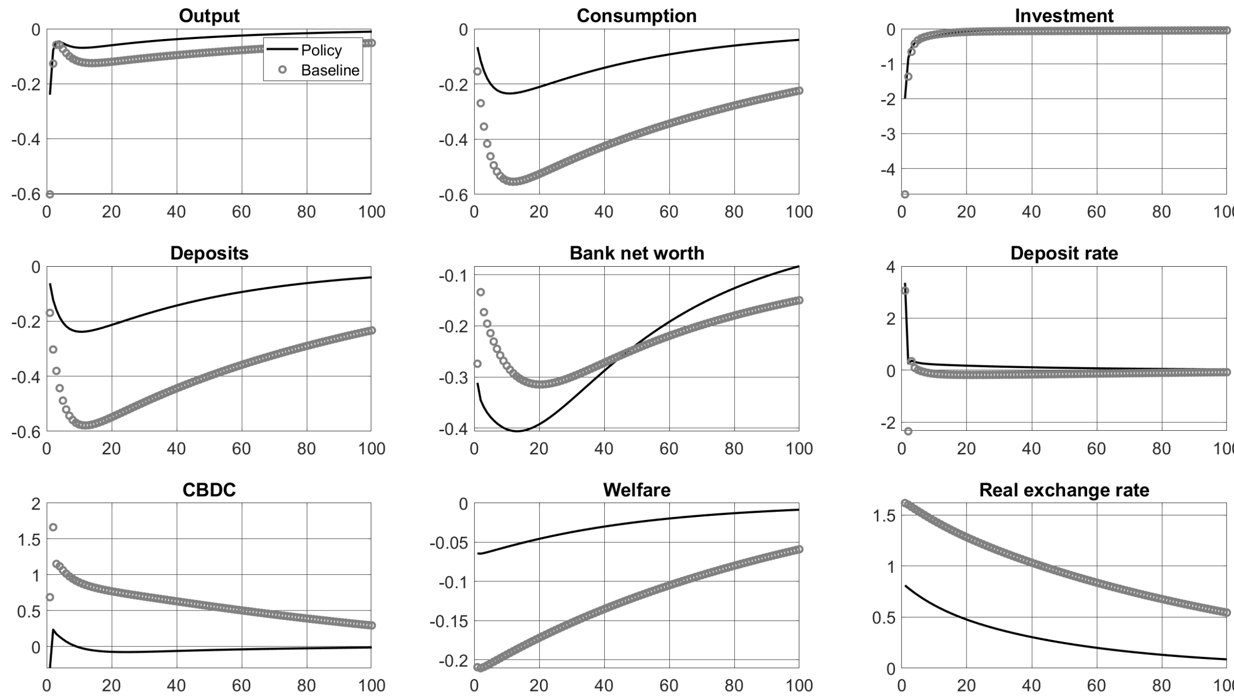

Figure C.3: Transition to the new equilibrium with CBDC under aggressive central bank balance sheet expansion.

Notes: Variables are reported in percentage changes relative to the steady-state with CBDC. The black line shows the transition in the home economy under the occasionally binding constraint and the gray dots the unconstrained transition path. The model is solved with global methods as in Equation (2.27). The CBDC is issued in the home country and, during the transition period, the central bank purchases private-sector assets equal to the amount of CBDC demanded 50% above steady-state.

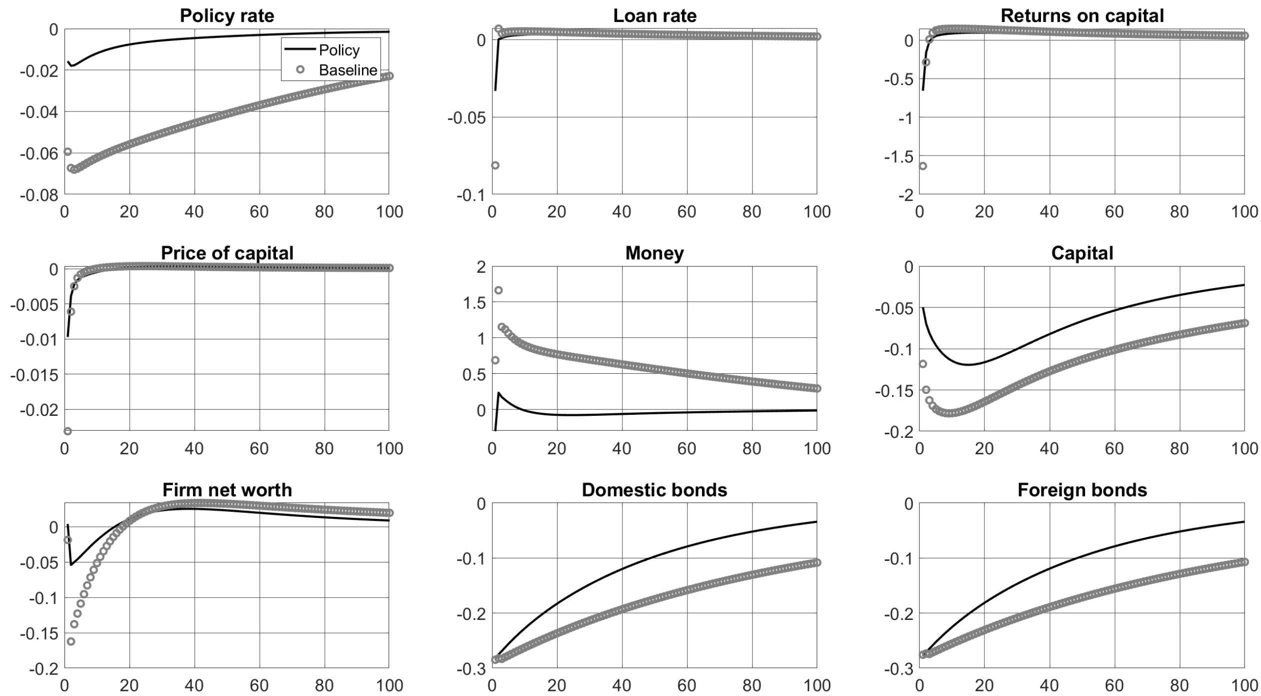

Figure C.4: Transition to the new equilibrium with CBDC under aggressive central bank balance sheet expansion (other variables).

Notes: Variables are reported in percentage deviations from the steady-state with CBDC. The black line shows the transition in the home economy under the occasionally binding constraint and the gray dots the unconstrained transition path. The model is solved with global methods as in Equation (2.27). The CBDC is issued in the home country and, during the transition period, the central bank purchases private-sector assets equal to the amount of CBDC demanded 50% above steady-state.

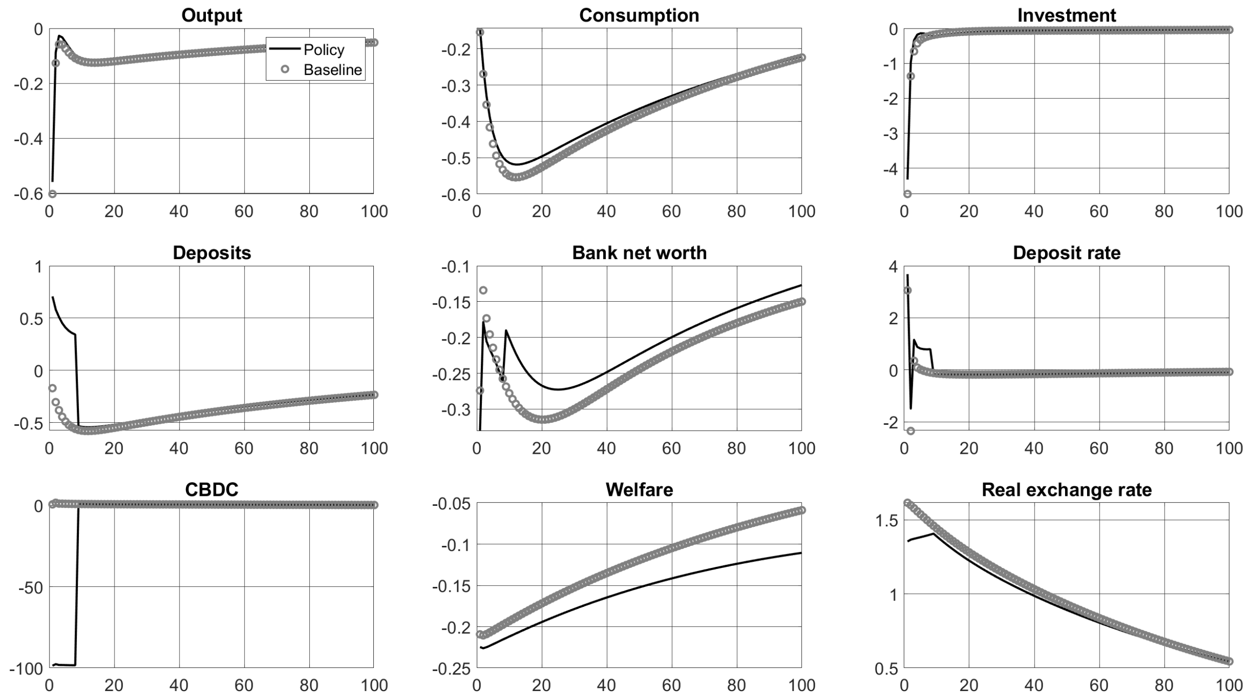

Figure C.5: Transition to the new equilibrium with CBDC when CBDC issuance is announced 12 periods (3 years) in advance.

Notes: Variables are reported in percentage deviations from the steady-state with CBDC. The black line shows the transition in the home economy under the CBDC announcement and the gray dots the baseline (unconstrained) transition path. The model is solved with global methods as in Equation (2.27) and under a linear approximation. The CBDC is issued in the home country and there are no policies during the transition.

C.2 Foreign economy

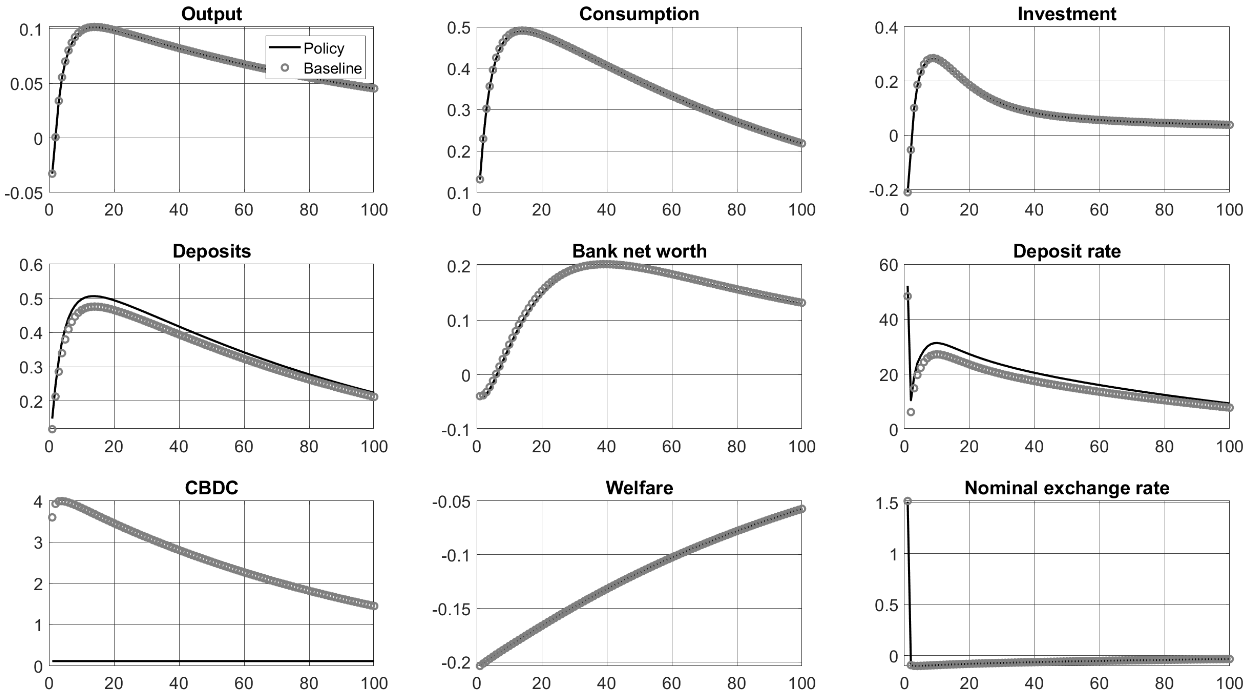

Figure C.6: Transition to new equilibrium with CBDC and soft holding limit calibrated to the steady-state level of CBDC demand – foreign economy.

Notes: Variables are reported in percentage deviations from the steady-state with CBDC. The black line shows the transition in the home economy under the occasionally-binding constraint and the gray dots the unconstrained transition path. The model is solved with global methods as in Equation (2.27). The CBDC is issued in the home economy and, during the transition period, supply of CBDC is defined as in Equation (2.22) where the limits $\bar{DC}$ and $\bar{DC}^*$ are set to the steady-state demand.

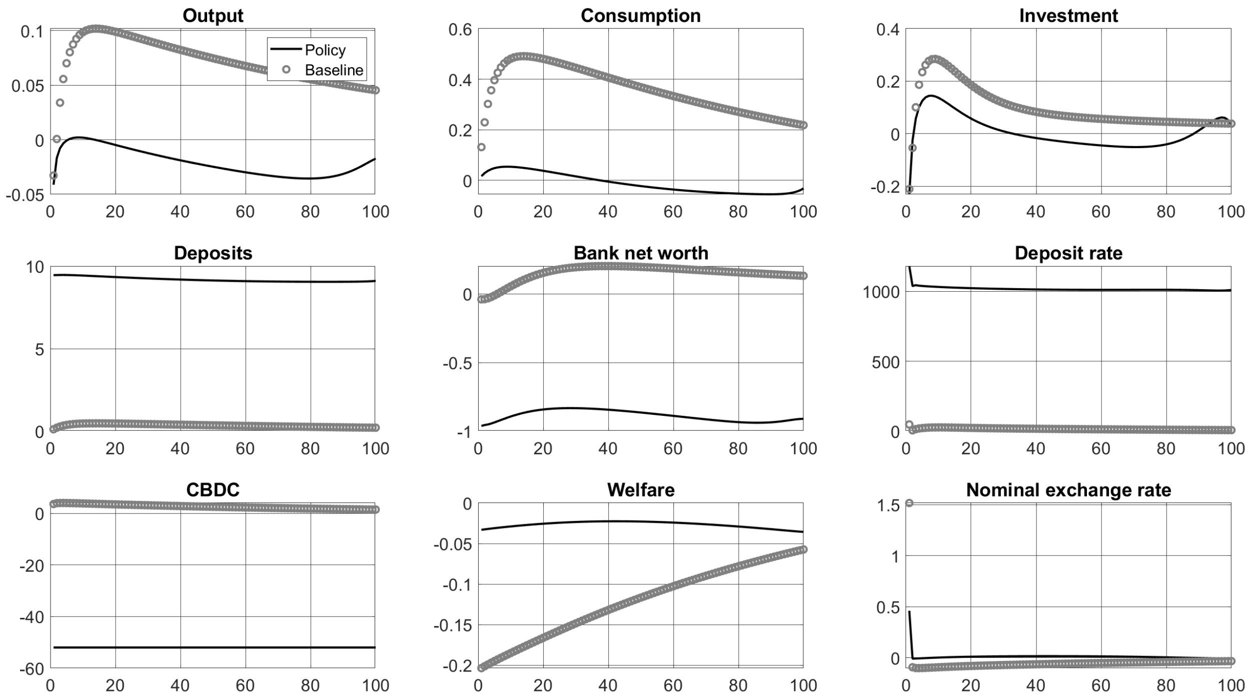

Figure C.7: Transition to new equilibrium with CBDC and holding limit calibrated to 50% of steady-state demand for CBDC – foreign economy.

Notes: Variables are reported in percentage deviations from the steady-state with CBDC. The black line shows the transition in the home economy under the occasionally binding constraint and the gray dots the unconstrained transition path. The model is solved with global methods as in Equation (2.27). The CBDC is issued in the home economy and, during the transition period, supply of CBDC is defined as in Equation (2.22) where the limits $\bar{DC}$ and $\bar{DC}^*$ are set to 50% of the steady-state demand. The limit is gradually reduced to the steady-state level.

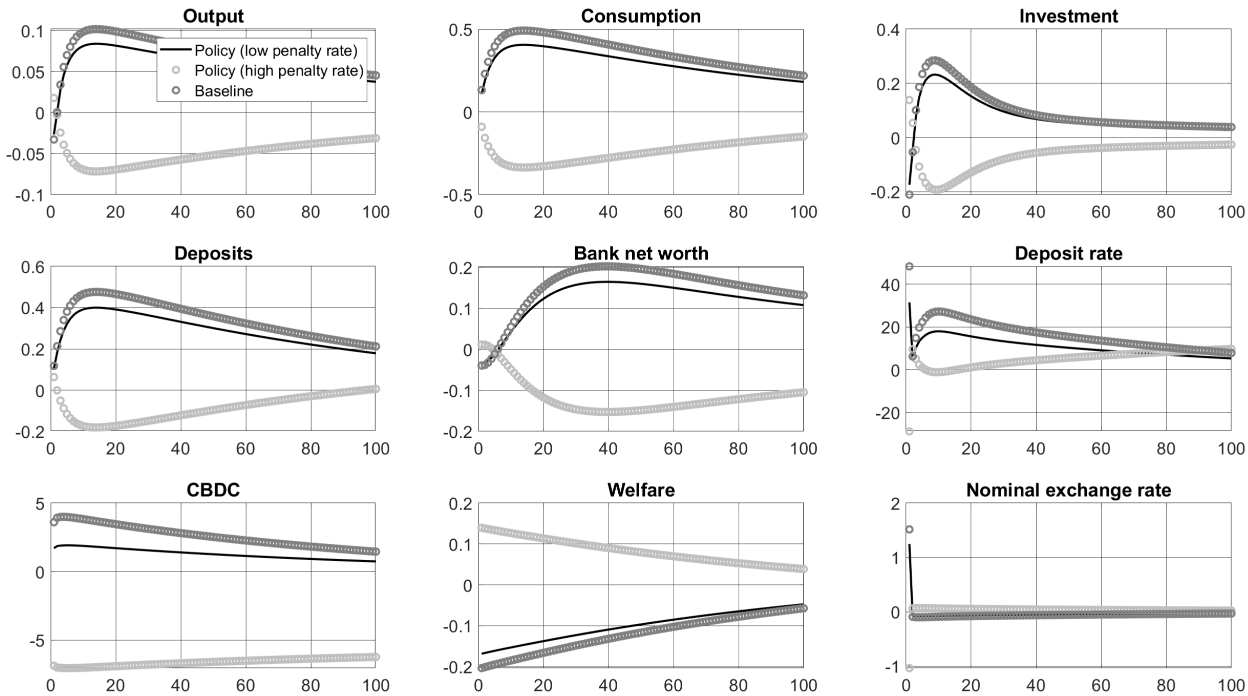

Figure C.8: Transition to new equilibrium with CBDC and tiered remuneration – foreign economy. **Notes**: Variables are reported in percentage deviations from the steady-state with CBDC. The black line shows the transition in the home economy under the occasionally binding constraint with low penalty rate (300 bps), the light gray dots the occasionally binding constraint with high penalty rate (500 bps) and the dark gray dots the unconstrained transition path. The model is solved with global methods as in Equation (2.27). The CBDC is issued in the home economy and, during the transition, a tiered remuneration scheme is applied as in Equation (2.24).

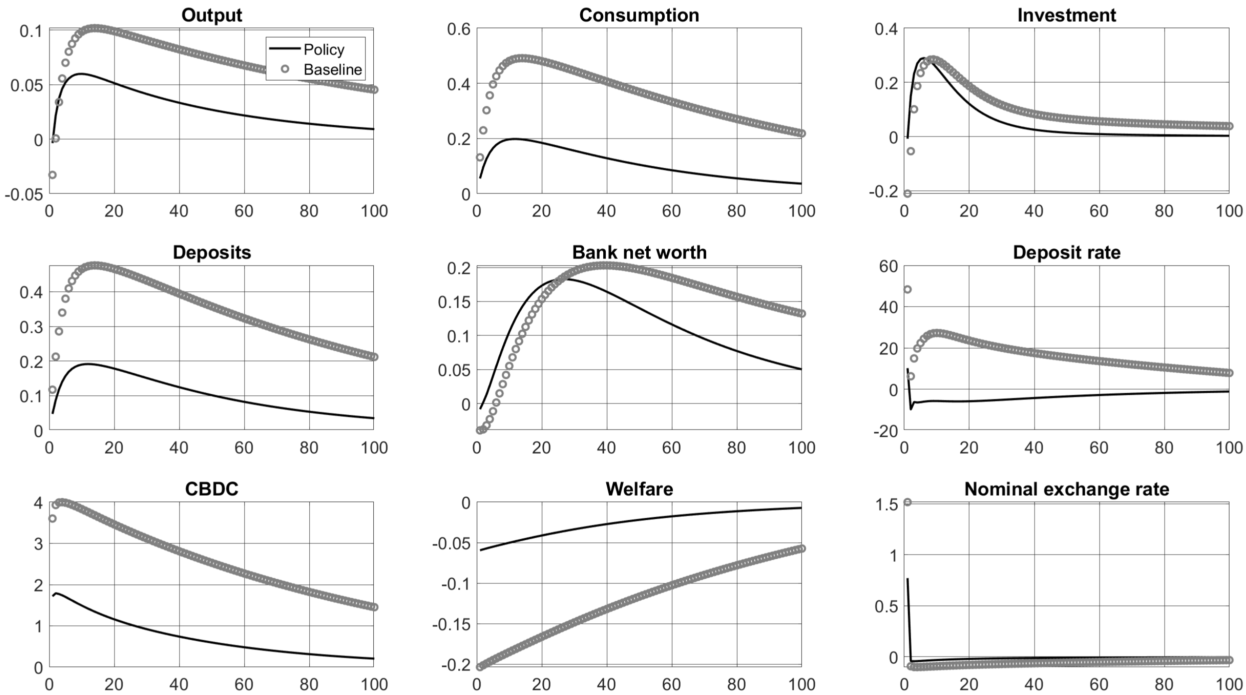

Figure C.9: Transition to new equilibrium with CBDC currencies with central bank balance sheet expansion – foreign economy.

Notes: Variables are reported in percentage deviations from the steady-state with CBDC. The black line shows the transition in the home economy under the occasionally-binding constraint and the gray dots the unconstrained transition path. The model is solved with global methods as in Equation (2.27). The CBDC is issued in the home economy and, during the transition, the central bank purchases private-sector assets for the amount of CBDC demand above equilibrium.

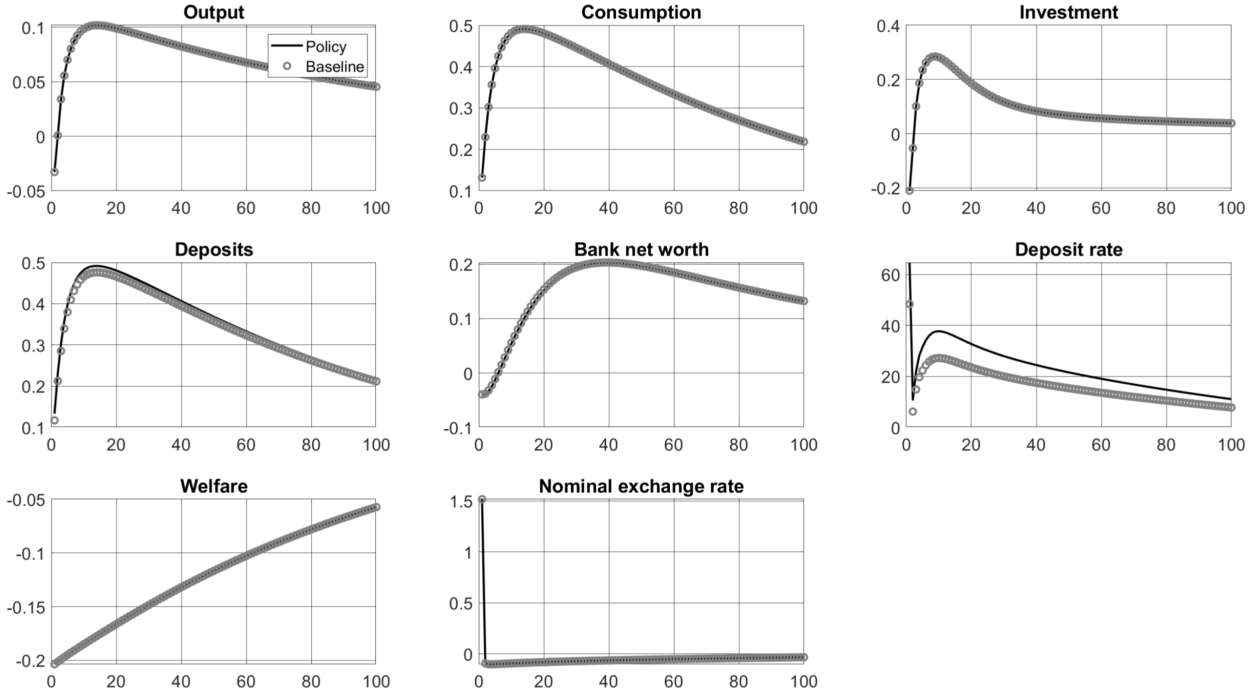

Figure C.10: Transition to the new equilibrium with CBDC with no access by foreigners – foreign economy.

Notes: Variables are reported in percentage deviations from the steady-state with CBDC. The black line shows the transition in the home economy if foreigners have no access to the CBDC and the gray dots the unconstrained transition path. The model is solved with global methods as in Equation (2.27). The CBDC is available in the home country.

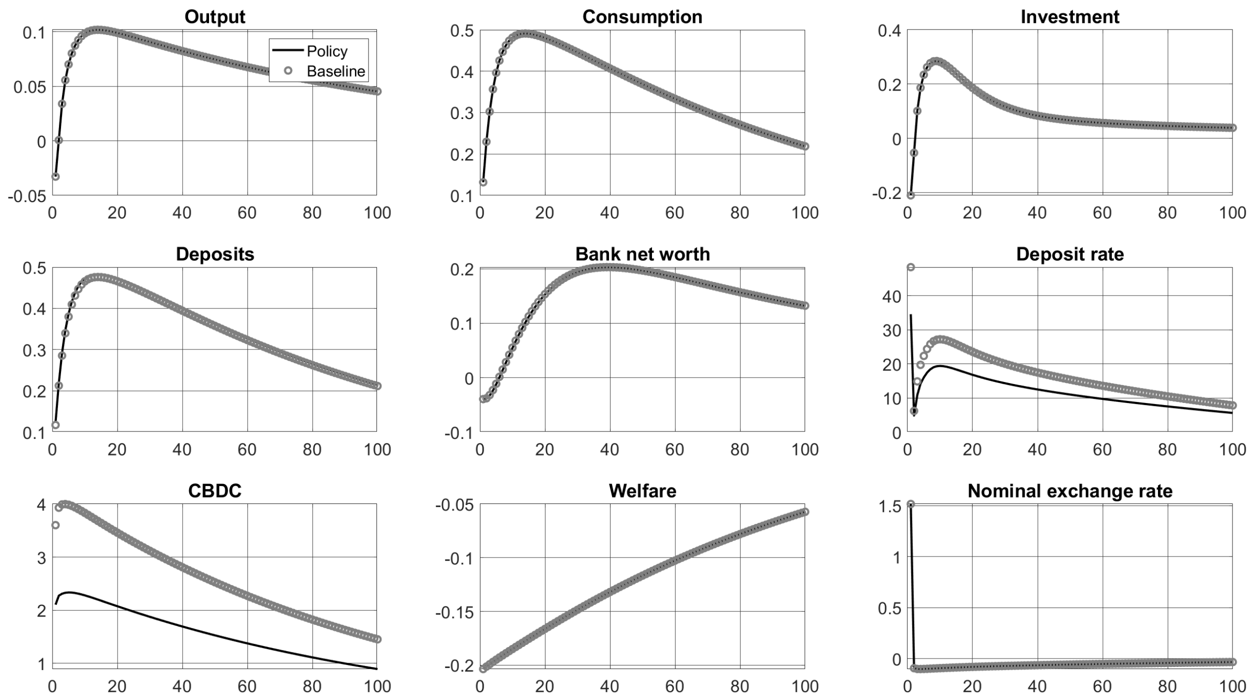

Figure C.11: Transition to the new equilibrium with CBDC with partial access by foreigners (higher cross-border transaction costs) – foreign economy.

Notes: Variables are reported in percentage deviations from the steady-state with CBDC. The black line shows the transition in the home economy when cross-border transaction costs ($\phi^{*,DC}$) are 50 times higher than in the baseline calibration and the gray dots shows the unconstrained transition path. The model is solved with global methods as in Equation (2.27).