# Firm-specific production factors in a DSGE model with Taylor price setting*

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**Abstract**

This paper compares the Calvo model with a Taylor contracting model in the context of the Smets-Wouters (2003) Dynamic Stochastic General Equilibrium (DSGE) model. In the Taylor price setting model, we introduce firm-specific production factors and discuss how this assumption can help to reduce the estimated nominal price stickiness. Furthermore, we show that a Taylor contracting model with firm-specific capital and sticky wage and with a relatively short price contract length of four quarters is able to outperform, in terms of empirical fit, the standard Calvo model with homogeneous production factors and high nominal price stickiness. In order to obtain this result, we need very large real rigidities either in the form of a huge (constant) elasticity of substitution between goods or in the form of an elasticity of substitution that is endogenous and very sensitive to the relative price.

*The views expressed in this paper are our own and do not necessarily reflects those of the National Bank of Belgium or the European Central Bank.

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# 1 Introduction

Following the theoretical work of Yun (1996) and Woodford (2003) and the empirical work of Gali and Gertler (1999) and Sbordone (1999), the New-Keynesian Phillips curve has become very popular in monetary policy analysis. In previous work (Smets and Wouters, 2003, 2004a,b), we estimated a Dynamic Stochastic General Equilibrium model (using euro area and US data) that embedded a hybrid version of the New-Keynesian Phillips curve. Overall, the estimated parameters, and in particular the degree of indexation and the elasticity of inflation with respect to its main driver, the real marginal cost, were very similar to those estimated by Gali and Gertler (1999) and Gali, Gertler and Lopez-Salido (2001) using a very different methodology. However, these estimates lead to a surprising and implausibly high estimated degree of nominal price stickiness. It corresponds to an average duration of prices not being re-optimised for more than 2 years. Clearly, this is not in line with existing micro evidence that suggests that on average prices are sticky for around 6 months to 1 year.[1,2] In this paper we focus on the possibility for firm-specific production factors to create real rigidities and subsequently help to reduce nominal price stickiness. Our contribution is to introduce firm-specific production factors into a general equilibrium model with both price and wage Taylor contracts (Taylor, 1980).

As a first step, we compare the Calvo specification with a standard Taylor contracting specification. Indeed, while analytically very tractable, the Calvo model has a number of implications that are less attractive. In particular, it implies that at any time there are some firms that have not adjusted their price optimally for a very long time. We analyse the differences in the impulse responses between the two price setting schemes. When making this comparison, we maintain the assumption that firms are price-takers in the factor markets, i.e. the labour and capital markets, and hence all firms face the same flat marginal cost curve. Not surprisingly, we find that the Taylor contracts need to be quite long in order to match the data as well as the Calvo scheme. We also show that the standard way of introducing a mark-up shock does not work very well with Taylor-type price setting.

In order to assess the role played respectively by nominal and real rigidities in pro-

[1] See the evidence in Bils and Klenow (2002) for the US and various papers produced in the context of the Eurosystem's Inflation Persistence Network for euro area countries (e.g. Aucremanne and Dhyne (2004), Neves et al. (2004)).

[2] However, one should be careful with using the micro-evidence to interpret the macro estimates. Because of indexation and a positive steady state inflation rate, all prices change all the time. However, only a small fraction of prices are set optimally. The alternative story for introducing a lagged inflation term in the Phillips curve based on the presence of rule-of-thumb price setters is more appealing from this perspective, as it does not imply that all prices change all the time. In that case, the comparison of the Calvo parameter with the micro evidence makes more sense. As the reduced form representations are almost identical, one could still argue that the estimated Calvo parameter is implausibly high.

ducing persistence in both price and wage inflation, we then estimate some variants of the model, introducing firm-specific production factors. As argued in Coenen and Levin (2004) for the Taylor model and Woodford (2003, 2005), Eichenbaum and Fischer (2004) and Altig *et al.* (2005) for the Calvo model, firm-specific capital lowers the elasticity of prices with respect to the real marginal cost for a given degree of price stickiness. We re-estimate the Taylor contracting models with firm-specific capital and/or firm-specific labour and analyse the impact of these assumptions on the empirical performance of the DSGE model and on the estimated contract length in the goods market. Our main findings are twofold.

First, in line with the previous literature we find that introducing firm-specific capital does lead to a fall in the estimated Taylor contract length in the goods market to a more reasonable 4 quarters. It also improves the empirical fit of the Taylor model, which in this case performs better than the estimated Calvo model. However, in the model with the best data fit the elasticity of substitution between goods of the various sectors is estimated to be improbably high. Furthermore, the corresponding price mark-up is estimated to be smaller than the fixed cost, so that profits are negative in steady state. Forcing the fixed cost to be equal to the price mark-up so that steady-state profits are zero leads to a significant deterioration of the empirical fit, while the estimated elasticity of substitution remains very large. Moving from the traditional Dixit-Stiglitz aggregator towards Kimball's (1995) generalized aggregator helps to solve both of those problems. In that case, the curvature parameter is estimated to be high, which is a sign that real rigidities are at work, but both the estimated elasticity of substitution and the cost of imposing the above-mentioned constraint are sharply reduced. These results are in line with Eichenbaum and Fischer (2004), Coenen and Levin (2004) and Altig *et al.* (2005). In this context, we also investigate the implications of the various models for the sector-specific supply and pricing decisions, which is easier to perform in a Taylor-contracting framework.

Second, we also analyse the impact on empirical performance of introducing sector-specific labour markets. Here the results are less promising in terms of reducing the estimated degree of nominal price stickiness. The reason is that sector-specific labour markets will only dampen the price impact of a change in demand for a given degree of nominal price stickiness, if the sector-specific labour markets are flexible and the sector-specific wage is responding strongly to changes in the demand for labour. Such wage flexibility is, however, incompatible with the empirical properties of aggregate wage behaviour.

The rest of the paper is structured as follows. First, we briefly review the estimated DSGE model of Smets and Wouters (2004b) with a special focus on the estimated degree of price stickiness and the sources of inflation variation. The focus is put on the inconsistency between the micro data and macro observation. Section 3 compares the

Calvo model with the standard Taylor-contracting model. Section 4 explores the impact of introducing firm-specific production factors. The concluding remarks are in Section 5.

# 2 Calvo price-setting in a linearised DSGE model

In this Section, we briefly describe the DSGE model that we estimate using euro area data. For a discussion of the micro-foundations of the model we refer to Smets and Wouters (2004b). Next, we review the main estimation results with regard to price setting and the sources of inflation variability.

## 2.1 The DSGE model

The DSGE model contains many frictions that affect both nominal and real decisions of households and firms. The model is based on Smets and Wouters (2004a). Households maximise a non-separable utility function with two arguments (goods and labour effort) over an infinite life horizon. Consumption appears in the utility function relative to a time-varying external habit variable. Labour is differentiated, so that there is some monopoly power over wages, which results in an explicit wage equation and allows for the introduction of sticky nominal wages à la Calvo (1983). Households rent capital services to firms and decide how much capital to accumulate taking into account capital adjustment costs.

The main focus of this paper is on the firms' price setting. A continuum of firms produce differentiated goods, decide on labour and capital inputs, and set prices. Following Calvo (1983), every period only a fraction of firms in the monopolistic competitive sector are allowed to re-optimise their price. This fraction is constant over time. Moreover, those firms that are not allowed to re-optimise, partially index their prices to the past inflation rate and the time-varying inflation target of the central bank. An additional important assumption is that all firms are price takers in the factor markets for labour and capital and thus face the same marginal cost. The marginal costs depend on wages, the rental rate of capital and productivity.

As shown in Smets and Wouters (2004a), this leads to the following linearised *inflation equation*:

$$
\begin{aligned}
\widehat{\pi}_t - \overline{\pi}_t &= \frac{\beta}{1+\beta\gamma_p}\left(E_t\widehat{\pi}_{t+1} - \overline{\pi}_t\right) + \frac{\gamma_p}{1+\beta\gamma_p}\left(\widehat{\pi}_{t-1} - \overline{\pi}_t\right) \qquad (1)\\
&\quad + \frac{1}{1+\beta\gamma_p}\frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{\xi_p}\widehat{s}_t + \eta_t^p \\
\widehat{s}_t &= \alpha\widehat{r}_t^k + (1-\alpha)\widehat{w}_t - \varepsilon_t^a - (1-\alpha)\gamma t \qquad (2)
\end{aligned}
$$

The deviation of inflation $\widehat{\pi}_t$ from the target inflation rate $\overline{\pi}_t$ depends on past and expected future inflation deviations and on the current marginal cost, which itself is a function of the rental rate on capital $\widehat{r}_t^k$, the real wage $\widehat{w}_t$ and the productivity process, that is composed of a deterministic trend in labour efficiency $\gamma t$ and a stochastic component $\varepsilon_t^a$, which is assumed to follow a first-order autoregressive process: $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$ where $\eta_t^a$ is an iid-Normal productivity shock. Finally, $\eta_t^p$ is an iid-Normal price mark-up shock.

When the degree of indexation to past inflation is zero ($\gamma_p = 0$ ), this equation reverts to the standard purely forward-looking New Keynesian Phillips curve. By assuming that all prices are indexed to the inflation objective in that case, this Phillips curve will be vertical in the long run. Announcements of changes in the inflation objective will be largely neutral even in the short run. This is based on the strong assumption that indexation habits will adjust immediately to the new inflation objective. With $\gamma_p > 0$, the degree of indexation to lagged inflation determines how backward looking the inflation process is or, in other words, how much structural persistence there is in the inflation process. The elasticity of inflation with respect to changes in the marginal cost depends mainly on the degree of price stickiness. When all prices are flexible ($\xi_p = 0$) and the price mark-up shock is zero, this equation reduces to the normal condition that in a flexible price economy the real marginal cost should equal one.

Equation (1) yields a direct link between the elasticity of inflation with respect to the marginal cost and the Calvo parameter. A weak reaction of inflation to the marginal cost implies a very high Calvo parameter. Both demand and supply shocks that affect the marginal cost will influence inflation only gradually as a consequence of the high price stickiness. However, the marginal cost is not directly observed and its definition is therefore open to discussion. It is clear from equation (1) that a smoother response of the marginal cost to these shocks might result in a lower estimate for the price stickiness. Variable capital utilisation should then help to obtain a low Calvo parameter since it mitigates the reaction of marginal cost to output fluctuations. However, Smets and Wouters (2004a) show that, empirically, this friction is not very important once one allows for the other frictions that smooth marginal costs such as nominal wage rigidities.[3]

The simple relation between the elasticity of inflation with respect to the marginal cost and the Calvo parameter, as appearing in equation (1), is only valid if all firms are producing at the same marginal cost. This marginal cost is homogenous and equal for all firms $j$ as capital is mobile between firms at each point in time and all firms can

[3] In the version of the model estimated in this paper, variable capital utilisation still appears but the adjustment cost has been given a looser prior than in Smets and Wouters (2003) (cf. appendix). This results in a much higher estimated adjustment cost. As a consequence the variable capacity utilisation plays virtually no role in the model presented in this paper. This is not necessarily a bad thing since allowing for a relatively insensitive marginal cost of changing the utilisation of capital substantially reduces the impact of introducing firm-specific capital.

hire labour at a given wage, determined in the aggregate labour market. This aggregate wide marginal cost is equal to the weighted sum of the wage cost and the rental rate of capital as displayed in equation (2). All firms $j$ will hire capital at the rental market rate independently of their relative price and corresponding output level, so that the capital-labour ratio is equal to the aggregate wide relative price of the production factors:

$$\frac{W_t L_{j,t}}{r_t^k \widetilde{K}_{j,t}} = \frac{1-\alpha}{\alpha} \forall j \in [0,1]$$

and all firms produce with the same marginal cost and capital-labour ratio.

It is important to note that in the empirical exercise wages are observed (in contrast to the rental rate on capital) and as a result the response of wages to all types of shocks is therefore restricted by the data. The smoother the reaction of wages to the different shocks the flatter the marginal cost curve and the lower will be the estimated price stickiness.

The rest of the linearised DSGE model is summarised in the appendix. In sum, the model determines nine endogenous variables: inflation, the real wage, capital, the value of capital, investment, consumption, the short-term nominal interest rate, the rental rate on capital and hours worked. The stochastic behaviour of the system of linear rational expectations equations is driven by ten exogenous shock variables. Five shocks arise from technology and preference parameters: the total factor productivity shock, the investment-specific technology shock, the preference shock, the labour supply shock and the government spending shock. Those shocks are assumed to follow an autoregressive process of order one. Three shocks can be interpreted as "cost-push" shocks: the price mark-up shock, the wage mark-up shock and the equity premium shock. Those are assumed to follow a white-noise process. And, finally, there are two monetary policy shocks: a permanent inflation target shock and a temporary interest rate shock.

### 2.2 Findings in the baseline model

The linearised DSGE model is estimated for the euro area using seven key macro-economic time series: output, consumption, investment, employment, real wages, prices and a short-term interest rate. The data are described in section 6.1 of the appendix. The full information Bayesian estimation methodology used for estimation is extensively discussed in Smets and Wouters (2003). Table 1 reports the estimates of the main parameters governing the hybrid New Keynesian Phillips curve and compares these estimates with those obtained by Gali *et al.* (2001) which use single-equation GMM methods to estimate a similar equation on the same euro area data set.[4]

[4] As there are many models estimated throughout the paper and since the Monte Carlo Markov Chain sampling method used to derive the posterior distribution of the parameters is extremely demanding in computer-time for such large scale models, the MCMC sampling algorithm has only been run for some

A number of observations are worth mentioning. First, the degree of indexation is rather limited. The parameter equals 0.18, which implies a coefficient on the lagged inflation rate of 0.15. As shown in Table 2, putting the degree of indexation equal to zero does not significantly modify the log data density of the model. Second, the degree of Calvo price stickiness is very large: each period 89 percent of the firms do not re-optimise their price setting. The average duration of non re-optimisation is therefore more than 2 years. This is implausibly high, but those results are very similar to the ones reported by Gali *et al.* (2001). Our estimates generally fall in the range of estimates reported by Gali *et al.* (2001), if they assume constant returns to scale as we do in our model (Table 1).

**Table 1: Comparison of estimated Philipps-curve parameters with Gali, Gertler and Lopez-Salido (GGL, 2001)**

| | SW | GGL (2001) (1) | GGL (2001) (2) |
|---|---|---|---|
| Structural parameters | | | |
| $\xi_p$ | 0.89 (0.01) | 0.90 (0.01) | 0.92 (0.03) |
| $\gamma_p$ | 0.18 (0.10) | 0.02 (0.12) | 0.33 (0.12) |
| $D$ | 9.0 | 10.0 | 12.8 |
| Reduced-form parameters | | | |
| $\gamma_f$ | 0.84 | 0.87 (0.04) | 0.68 (0.04) |
| $\gamma_b$ | 0.15 | 0.02 (0.12) | 0.27 (0.07) |
| $\lambda$ | 0.013 | 0.018 (0.012) | 0.006 (0.007) |

Notes: The GGL (2001) estimates are those obtained under the assumption of constant returns to labour under two alternative specifications. Strictly speaking, the structural parameters are not directly comparable as GGL use the inclusion of rule-of-thumb price setters (rather than indexation) as a way of introducing lagged inflation. $D$ stands for duration in numbers of quarters; $\gamma_f$ is the implied reduced-form coefficient on expected future inflation; $\gamma_b$ is the coefficient on lagged inflation and $\lambda$ is the coefficient on the real marginal cost.

Moreover, reducing the degree of Calvo price stickiness to more reasonable numbers such as 75 percent or an average duration of about 4 quarters reduces the log data density of the estimated model drastically (by about 76 as shown in Table 2). This is

models (see Appendix 6.4). The parameters and standard errors reported in all Tables are the estimated modes and their corresponding standard error. The log data density displayed is actually the Laplace approximation. It is shown in appendix 6.4 that it is very close to the modified harmonic mean for the model for which the latter has been computed.

similar to the findings in Smets and Wouters (2004a) for the US: the degree of price stickiness is one of the most costly frictions to remove in terms of the empirical fit of the DSGE model. This feature perfectly illustrates the puzzle we face. At the micro level, one observes that prices are re-optimized on average between every 6 month and one year, while at the macro level, inflation is shown to be very persistent. In the model with homogeneous production factors, the latter feature requires large nominal price stickiness which is contradictory with the micro observation.

**Table 2: Testing the nominal rigidities using the marginal likelihood**

| | *Baseline* | $\xi_p = 0.75$ | $\gamma_p = 0$ |
|---|---|---|---|
| log data density | -471.11 | -546.77 | -470.75 |
| $\rho_a$ | 0.991 | -0.997 | 0.990 |
| | (0.007) | (0.001) | (0.007) |
| $\sigma_a$ | 0.654 | 0.554 | 0.632 |
| | (0.094) | (0.056) | (0.087) |
| $\sigma_p$ | 0.207 | 0.288 | 0.233 |
| | (0.019) | (0.030) | (0.019) |
| $\xi_w$ | 0.712 | 0.502 | 0.714 |
| | (0.046) | (0.041) | (0.047) |
| $\xi_p$ | 0.891 | 0.75 | 0.887 |
| | (0.014) | (-) | (0.015) |
| $\gamma_w$ | 0.388 | 0.415 | 0.321 |
| | (0.197) | (0.229) | (0.199) |
| $\gamma_p$ | 0.178 | 0.010 | 0.00 |
| | (0.096) | (0.014) | (-) |

Notes: $\rho_a$: persistency parameter of the productivity shock, $\sigma_{a:}$ :std. err. of the productivity shock, $\sigma_p$ : std. err. of the price mark-up shock, $\xi_w$: Calvo wage stickiness parameter, $\xi_p$ : Calvo price stickiness parameter, $\gamma_p$: wage indexation parameter, $\gamma_p$ : price indexation parameter, $\rho_p$: persistency parameter of the persistent price mark-up shock.

# 3 Taylor versus Calvo model with mobile production factors

One unattractive feature of the Calvo price setting model is that some firms do not re-optimise their prices for a very long time.[5] As indicated by Wolman (2001), the resulting misalignments due to relative price distortions may be very large and this may have important welfare implications. The standard Taylor contracting model avoids this problem.[6] In this model firms set prices for a fixed number of periods and price setting is staggered over the duration of the contract, i.e. the number of firms adjusting their price is the same every period.[7] The Taylor model presents also the advantage that it requires an explicit modelling of (cohorts of) firms and households. This facilitates the analysis since linearisation is directly applicable. Furthermore, firm-specific capital and labour can directly be introduced and handled explicitly. The explicit modelling of the different firm types has the advantage that the individual output and price levels are also available. For the model to be realistic, it is important that the dispersion of output and prices accross sectors is realistic. Such a condition is difficult to check in a Calvo context where the complete distribution of prices and outputs is not available explicitly.

In order to be able to compare this price-setting model with the Calvo model discussed above, we also maintain the assumption of partial indexation to lagged inflation and the inflation objective. In this section, we also maintain the assumption of mobile production factors. The comparison of the Taylor contracting model with the Calvo model is a preliminary step to the next section which deals with the introduction of firm-specific labour and capital .

As discussed in Whelan (2004) and Coenen and Levin (2004), the staggered Taylor contracting model gives rise to the following linearised equations for the newly set optimal price and the general price index :

$$\widehat{p}_t^* = \frac{1}{\sum_{i=0}^{n_p-1}\beta^i}\left[\sum_{i=0}^{n_p-1}\beta^i\left(\widehat{s}_{t+i}+\widehat{p}_{t+i}\right)-\sum_{i=0}^{n_p-2}\left(\left(\gamma_p\widehat{\pi}_{t+i}+(1-\gamma_p)\overline{\pi}_{t+i+1}\right)\sum_{q=i+1}^{n_p-1}\beta^q\right)\right]+d\varepsilon_t^p \tag{3}$$

$$\widehat{p}_t = \frac{1}{n_p}\sum_{i=0}^{n_p-1}\left(\widehat{p}_{t-i}^*+\sum_{q=0}^{i-1}\left(\gamma_p\widehat{\pi}_{t-1-q}+(1-\gamma_p)\overline{\pi}_{t-q}\right)\right)+(1-d)\varepsilon_t^p \tag{4}$$

where $\widehat{s}_t$ is the marginal cost, $\beta$ is the discount factor, $\gamma_p$ is the degree of indexation to the lagged inflation rate, $\overline{\pi}_t$ is the inflation objective of the monetary authorities

[5] See, however, Levy et al. (2003) for an exception. The 5-nickel price of a bottle of coca cola has been fixed for a period of almost 80 years.

[6] Another alternative is the truncated Calvo model as analysed in Dotsey *et al.* (1999), Bakhshi *et al.* (2003) and Murchinson *et al.* (2004).

[7] See Coenen and Levin (2004) and Dixon and Kara (2005) for a generalisation of the standard Taylor contracting model where different firms may set prices for different lengths of time.

modelised as a random walk, $n_p$ is the duration of the contract, $d$ is a binary parameter ($d \in \{0,1\}$) and $\varepsilon_t^p = \rho_t^p \varepsilon_{t-1}^p + \eta_t^p$, with $\eta_t^p$ an i.i.d. shock. We experiment with two ways of introducing the price mark-up shocks in the Taylor contracting model. The first method ($d = 1$), is fully analogous with the Calvo model. We assume a time-varying mark-up in the optimal price setting equation, which introduces a shock in the linearised price setting equation (3) as shown above. The second method ($d = 0$) is somewhat more ad hoc. It consists of introducing a shock in the aggregate price equation (4).[8]

Similarly, we introduce Taylor contracting in the wage setting process. This leads to the following linearised equations for the newly set optimal wage and the average wage

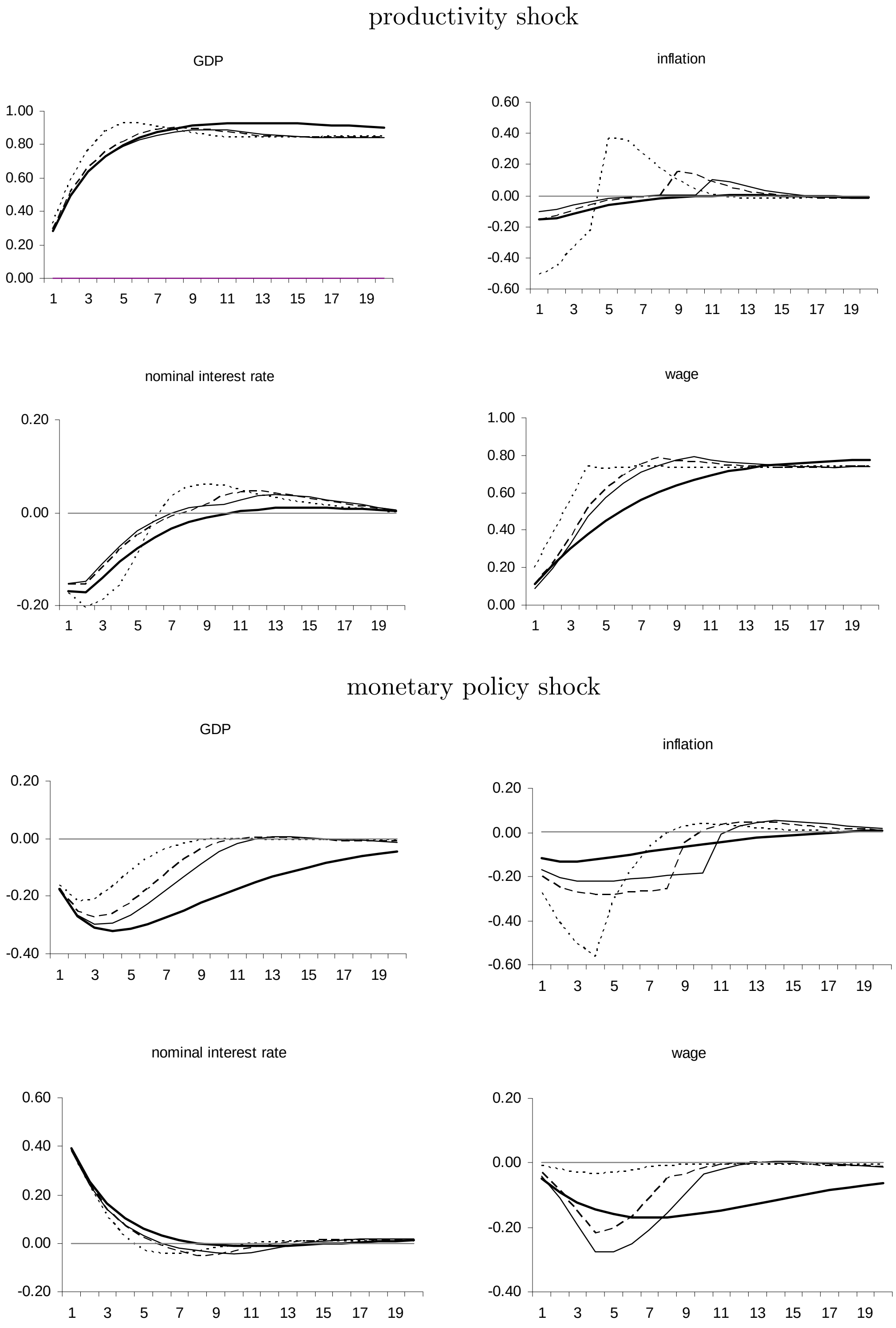
$$\begin{aligned}
\widehat{w}_t^* &= \frac{1}{\sum_{i=0}^{n_w-1}\beta^i}\left[\sum_{i=0}^{n_w-1}\beta^i\left(\sigma_l \widehat{l}_{i,t+i} + \frac{1}{1-h}\left(\widehat{c}_{t+i} - h\widehat{c}_{t+i-1}\right) - \varepsilon_{t+i}^l\right)\right. \\
&\quad \left. + \sum_{i=1}^{n_w-1}\left(\left(\widehat{\pi}_{t+i} - \gamma_w \widehat{\pi}_{t+i-1} - (1-\gamma_w)\overline{\pi}_{t+i}\right)\sum_{q=i}^{n_w-1}\beta^q\right)\right] + d\varepsilon_t^w && (5) \\
\widehat{w}_t &= \frac{1}{n_w}\left[\sum_{i=0}^{n_w-1}\widehat{w}_{i,t} + \widehat{p}_{t-i}\right] - \widehat{p}_t + (1-d)\varepsilon_t^w && (6) \\
&\text{with} \\
\widehat{w}_{i,t} &= \widehat{w}_{t-i}^* + \sum_{q=0}^{i-1}\left(\gamma_w \widehat{\pi}_{t-1-q} + (1-\gamma_w)\overline{\pi}_{t-q}\right) && (7) \\
\widehat{l}_{i,t+i} &= \widehat{l}_{t+i} - \frac{1+\lambda_w}{\lambda_w}\left[\widehat{w}_{i,t+i} + \widehat{p}_t - (\widehat{w}_{t+i} + \widehat{p}_{t+i})\right] && (8)
\end{aligned}$$

where $n_w$ is the duration of the wage contract, $\sigma_l$ represents the inverse elasticity of work effort with respect to real wage, $\widehat{l}_t$ is the labour demand described in (A6) (cf. model appendix) and $\widehat{l}_{i,t}$ is the demand for the labour supplied at nominal wage $\widehat{w}_{i,t}$ by the households who reoptimized their wage $i$ periods ago, $h$ is the habit parameter, $\widehat{c}_t$ is consumption, $\varepsilon_t^l = \rho_t^l \varepsilon_{t-1}^l + \eta_t^l$ and $\eta_t^l$ is an i.i.d. shock to the labour supply, $\gamma_w$ is the degree of indexation to the lagged wage growth rate, $\varepsilon_t^w = \rho_t^w \varepsilon_{t-1}^w + \eta_t^w$ and $\eta_t^w$ is an i.i.d. wage mark-up shock. Finally, $\lambda_w$ is the wage mark-up. Note that as we did for price shocks, wage shocks have been introduced in two different ways. This will be discussed in a forthcoming paragraph.

Figure 1 compares the impulse responses to respectively a productivity and a monetary policy shock in the baseline Calvo model and 4, 8 and 10-quarter Taylor-contracting models using the estimated parameters of the baseline Calvo model. In other words, in this case the parameters have not been re-estimated. We allow for variations in the length of the price contract but the wage contract length $n_w$ is fixed at four quarters. Generally speaking, two main observations can be made. First, typically the inflation

[8] This could be justified as a relative price shock to a flexible-price sector that is not explicitly modelled. Of course, such a shortcut ignores the general equilibrium implications (e.g. in terms of labour and capital reallocations).

**Figure 1: Selected impulse responses: Calvo versus Taylor contracts (baseline parameters)**




Legend: bold black line: baseline (Calvo) model; full line: 10-quarter Taylor price contract; dashed line: 8-quarter Taylor price contract; dotted line: 4-quarter Taylor price contract.

response in the Taylor contracting models is larger in size, but less persistent. The peak effect on inflation increases with the length of the contract. This is the case in spite of the fact that prices are partially indexed to past inflation. Conversely, the output and real wage responses are closer to the flexible price outcome under Taylor contracting. For example, in response to a monetary policy shock the response of output is considerably smaller in absolute value under Taylor contracting.

Second, as the duration of the Taylor contract lengthens, the impulse responses appear to approach the outcome under the Calvo model. However, one needs a very long duration (more than 10 quarters) in order to come close to the Calvo model. Even in that case, the inflation response changes sign quite abruptly after the length of the contract. This feature is absent in the Calvo specification. As discussed in Whelan (2004), in reduced-form inflation equations the reversal of the inflation response after the contract length is captured by a negative coefficient on lagged inflation once current and expected future marginal costs are taken into account.

The two ways of introducing price (resp. wage) shocks generate very different short run dynamics in response to such shocks, as shown in Figure 2. The right-hand column shows that introducing a persistent shock in the GDP deflator equation (i.e. $d = 0$) allows the Taylor-contracting model to mimic most closely the response to a mark-up shock in the baseline Calvo specification.[9]

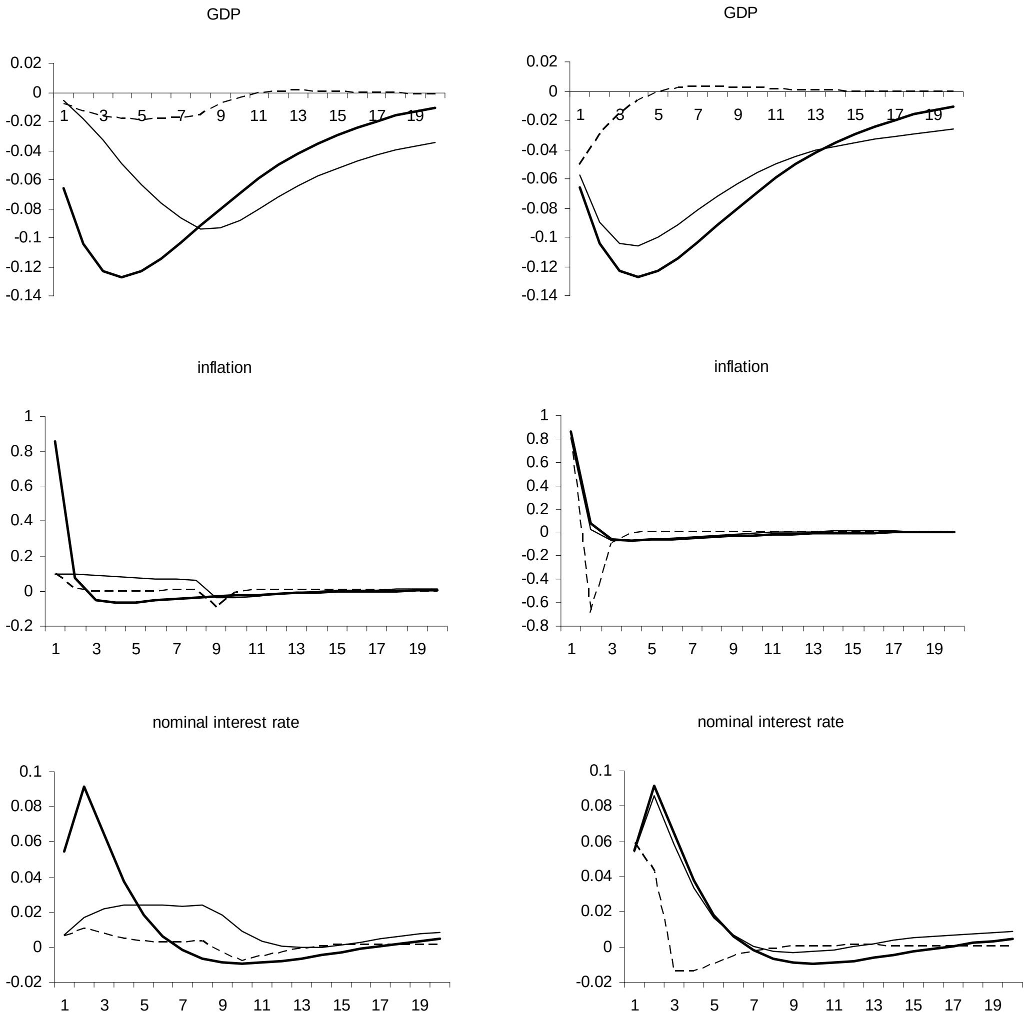
Table 3 reports selected findings from re-estimating the model with both price and wage Taylor contracts for various price contract lengths (again keeping the length of the wage contract fixed at four quarters). A number of results are worth highlighting. First, in line with the impulse responses shown in Figure 2, we find that the specification with the persistent price shock in the GDP price equation does best in terms of the log data density (see the third line in Table 3).[10] Second, comparing across various contract lengths (4, 8 and 10 quarters), it appears that the 10-quarter contract specification performs best (Table 3). Even in that case, however, the log data density is considerably lower than that of the Calvo model (the log difference is about 14). This probably reflects the fact that the model with homogeneous labour and capital does not generate enough persistence, as already displayed in Figure 1. Third, while most of the other parameters are estimated to be very similar, it is noteworthy that the estimated degree of indexation rises quite significantly to about 0.45 under Taylor contracting (from 0.18 in the baseline). Possibly, this reflects the need to overcome the negative dependence on past inflation in the standard Taylor contract.

[9] The same exercise could actually be run for a wage shock. Since it leads to similar conclusions we do not reproduce it here.

[10] Again, similar estimations could have been diplayed for the various specifications of the wage shock. We haven't since the exercise leads to similar conclusions. It is nevertheless important to note that, for all the estimations displayed in Table 3, we have considered a persistent wage shock in the average wage equation.

**Figure 2: Impulse response to a price shock in the 8-quarter Taylor model for different specifications of the price shock (baseline parameters)**

$d = 1$ : price shock in (3) $\qquad$ $d = 0$ : price shock in (4)




Legend: bold black line: baseline (Calvo) model; black line: 8-quarter Taylor contract with persistent price shock; dashed black line: 8-quarter Taylor contract with i.i.d. price shock.

**Table 3: Comparing the Calvo model with Taylor contracting models**

| | Calvo | 4-Q Taylor | 8-Q Taylor | 10-Q Taylor |
|---|---|---|---|---|
| | i.i.d. price shock in the optimal price setting equation | | | |
| log data density | $\boxed{-471.113}$ | -636.757 | -606.631 | -656.354 |
| | i.i.d. price shock in the GDP price equation | | | |
| log data density | - | -694.708 | -539.118 | -519.666 |
| | persistent price shock in the optimal price setting equation | | | |
| log data density | - | -653.987 | -619.145 | -595.155 |
| | persistent price shock in the GDP price equation | | | |
| log data density | - | $\boxed{-495.566}$ | $\boxed{-489.174}$ | $\boxed{-485.483}$ |
| $\rho_a$ | 0.991 | 0.980 | 0.982 | 0.962 |
| | (0.006) | (0.007) | (0.006) | (0.006) |
| $\sigma_a$ | 0.653 | 0.615 | 0.682 | 0.619 |
| | (0.093) | (0.068) | (0.085) | (0.076) |
| $\rho_p$ | 0 | 0.995 | 0.995 | 0.912 |
| | (-) | (0.004) | (0.004) | (0.016) |
| $\sigma_p$ | 0.207 | 0.406 | 0.323 | 0.277 |
| | (0.019) | (0.030) | (0.023) | (0.020) |
| $\rho_w$ | 0 | 0.973 | 0.966 | 0.881 |
| | (-) | (0.012) | (0.014) | (0.017) |
| $\sigma_w$ | 0.250 | 0.4386 | 0.453 | 0.461 |
| | (0.021) | (0.031) | (0.034) | (0.031) |
| $\gamma_w$ | 0.388 | 0.313 | 0.397 | 0.351 |
| | (0.197) | (0.166) | (0.205) | (0.206) |
| $\gamma_p$ | 0.178 | 0.859 | 0.463 | 0.436 |
| | (0.096) | (0.150) | (0.130) | (0.116) |
| $D_w$ | 3.5 Q | 4 Q | 4 Q | 4 Q |
| $D_p$ | 9.2 Q | 4 Q | 8 Q | 10 Q |

Note: $\rho_a$, $\rho_p$ and $\rho_w$ are the persistency parameters associated to the productivity, the price and the wage shock respectively; $\sigma_a$, $\sigma_p$ and $\sigma_w$ are the standard error of the productivity, the price and the wage shock respectively; $\gamma_w$ and $\gamma_p$ are respectively the wage and price indexation parameters; $D_w$ and $D_p$ are respectively the average length of the wage and the price contract.

# 4 Firm-specific production factors and Taylor contracts

So far the model includes all kinds of adjustment costs such as those related to the accumulation of new capital, to changes in prices and wages and to changes in capacity utilisation, but shifting capital or labour from one firm to another is assumed to be costless (see Danthine and Donaldson, 2002). The latter assumption is clearly not fully realistic. In this section we instead assume that production factors are sector specific, i.e. the cost of moving them across sectors is extremely high. Although this is also an extreme assumption, it may be more realistic. The objective is to investigate the implications of introducing this additional real rigidity on the estimated degree of nominal price stickiness and the overall empirical performance of the Taylor contracting model. As shown in Coenen and Levin (2004) for the Taylor model and Woodford (2003, 2005), Eichenbaum and Fischer (2004) and Altig *et al.* (2005) for the Calvo model, the introduction of firm-specific capital reduces the sensitivity of inflation with respect to its driving variables. Similarly, Woodford (2003, 2005) shows that firm-specific labour may also help reducing price variations and may lead to higher inflation persistence.

In the case of firm-specific factors, the key equations of the linearised model governing the decision of a firm belonging to the cohort $j$ (with $j \in [1, n_p]$) which reoptimises its price in period $t$ are given by:

$$\widehat{p}_t^*(j) = \frac{1}{\sum_{i=0}^{n_p-1} \beta^i} \left[ \sum_{i=0}^{n_p-1} \beta^i \left( \widehat{s}_{t+i}(j) + \widehat{p}_{t+i} \right) - \sum_{i=0}^{n_p-2} \left( \left( \gamma_p \widehat{\pi}_{t+i} + (1-\gamma_p)\overline{\pi}_{t+i+1} \right) \sum_{q=i+1}^{n_p-1} \beta^q \right) \right] \tag{3b}$$

$$\widehat{p}_t = \frac{1}{n_p} \sum_{i=0}^{n_p-1} \widehat{p}_t(j-i) + \varepsilon_t^p \tag{4b}$$

$$\widehat{s}_{t+i}(j) = \alpha \widehat{\rho}_{t+i}(j) + (1-\alpha)\widehat{w}_{t+i}(j) - \widehat{\varepsilon}_{t+i}^a - (1-\alpha)\gamma t \tag{9}$$

$$\widehat{Y}_{t+i}(j) = \widehat{Y}_{t+i} - \frac{1+\lambda_p}{\lambda_p} \left( \widehat{p}_{t+i}(j) - \widehat{p}_{t+i} \right) \tag{10}$$

$$\widehat{p}_{t+i}(j) = \widehat{p}_t^*(j) + \sum_{q=0}^{i-1} \left( \gamma_p \widehat{\pi}_{t-1-q} + (1-\gamma_p)\overline{\pi}_{t-q} \right) \tag{11}$$

$$\text{with } \frac{\partial \widehat{\rho}_{t+i}(j)}{\partial \widehat{Y}_{t+i}(j)} > 0 \text{ and } \frac{\partial \widehat{w}_{t+i}(j)}{\partial \widehat{Y}_{t+i}(j)} > 0 \tag{12}$$

where $\widehat{\rho}_t(j)$ is the "shadow rental rate of capital services",[11] and $\lambda_p$ is the price mark-up so that $\frac{1+\lambda_p}{\lambda_p}$ is the elasticity of substitution between goods. The main difference with equations (3) and (4) is that the introduction of firm-specific factors implies that firms no longer share the same marginal cost. Instead, a firm's marginal cost and its optimal price will depend on the demand for its output. A higher demand for its output implies that the firm will have a higher demand for the firm-specific input factors, which in turn

[11] Indeed, we left aside the assumption of a rental market for capital services. Each firm builds its own capital stock. The "shadow rental rate" of capital services is the rental rate of capital services such that the firm would hire the same quantity of capital services in an economy with a market for capital services as it does in the economy with firm-specific capital.

will lead to a rise in the firm-specific wage costs and capital rental rate. Because this demand will be affected by the pricing behavior of the firm's competitors, the optimal price will also depend on the pricing decisions of the competitors.

The net effect of this interaction will be to dampen the price effects of various shocks. Consider, for example, an unexpected demand expansion. Compared to the case of homogenous marginal costs across firms, the first price mover will increase its price by less because everything else equal the associated fall in the relative demand for its goods leads to a fall in its relative marginal cost. This, in turn, reduces the incentive to raise prices. This relative marginal cost effect is absent when factors are mobile across firms and, as a result, firms face the same marginal cost irrespective of their output levels. From this example it is clear that the extent to which variations in firm-specific marginal costs will reduce the amplitude of price variations will depend on the combination of two elasticities: *i)* the elasticity of substitution between the goods produced by the firm and those produced by its competitors, which will govern how sensitive relative demand for a firm's goods is to changes in its relative price (see equation (10); *ii)* the elasticity of the individual firms' marginal cost with respect to changes in the demand for its products (see equation (11)). With a Cobb-Douglas production function, the latter elasticity will mainly depend on the elasticity of the supply of the factors with respect to changes in the factor prices. In brief, the combination of steep firm-specific marginal cost curve and high demand elasticity will maximise the relative marginal cost effect and minimise the price effects, thereby reducing the need for a high estimated degree of nominal price stickiness.

Before turning to a quantitative analysis of these effects in the next sections, it is worth examining in somewhat more detail the determinants of the partial derivatives in equation (12) in each of the two factor markets (capital and labour). Consider first firm-specific capital. Given the one-period time-to-build assumption in capital accumulation, the firm-specific capital stock is given within the quarter. As a result, when the demand faced by the firm increases, production can only be adjusted by either increasing the labour/capital ratio or by increasing the rate of capital utilisation. Both actions will tend to increase the cost of capital services. It is, however, also clear that when the firm can increase the utilisation of capital at a constant marginal cost, the effect of an increase in demand on the cost of capital will be zero. In this case, the supply of capital services is infinitely elastic at a rental price that equals the marginal cost of changing capital utilisation and, as a result, the first elasticity in equation (12) will be zero. In the estimations reported below, the marginal cost of changing capital utilisation is indeed high, so that in effect there is nearly no possibility to change capital utilisation. Over time, the firm can adjust its capital stock subject to adjustment costs. This implies that the firm's marginal cost depends on its capital stock, which itself depends on previous pricing and investment decisions of the firm. As a result, also the capital stock, the value of capital and investment will be firm-specific. In the case of a Calvo model, Woodford (2005) and Christiano (2004) show how the linearised model can still be solved in terms of aggregate variables, without solving for the whole distribution of the capital stock over the different firms. This linearisation is however complicated and remains model specific. With staggered Taylor contracts, it is straightforward to model the cohorts of firms characterised by the same price separately. The key linearised equations governing the investment decision for a firm belonging to the $j$th cohort are then:

$$\widehat{K}_t(j) = (1-\tau)\widehat{K}_{t-1}(j) + \tau\widehat{I}_{t-1}(j) + \tau\varepsilon^I_{t-1} \tag{13}$$

$$\widehat{I}_t(j) = \frac{1}{1+\beta}\widehat{I}_{t-1}(j) + \frac{\beta}{1+\beta}E_t\widehat{I}_{t+1}(j) + \frac{1/\varphi}{1+\beta}\widehat{Q}_t(j) + \varepsilon^I_t \tag{14}$$

$$\widehat{Q}_t(j) = -(\widehat{R}_t - \widehat{\pi}_{t+1}) + \frac{1-\tau}{1-\tau+\overline{\rho}}E_t\widehat{Q}_{t+1}(j) + \frac{\overline{\rho}}{1-\tau+\overline{\rho}}E_t\widehat{\rho}_{t+1}(j) + \eta^Q_t \tag{15}$$

where $\widehat{K}_t(j)$, $\widehat{I}_t(j)$ and $\widehat{Q}_t(j)$ are respectively the capital stock, investment and the Tobin's Q for each of the firms belonging to the $jth$ price setting cohort. Parameter $\tau$ is the depreciation rate of capital and $\overline{\rho}$ is the shadow rental rate of capital discussed above, so that $\beta = 1/(1-\tau-\overline{\rho})$. Parameter $\varphi$ depends on the investment adjustment cost function.[12]

Consider next firm-specific monopolistic competitive labour markets. In this case each firm requires a specific type of labour which can not be used in other firms. Moreover, within each firm-specific labour market, we allow for Taylor-type staggered wage setting. The following linearised equations display how a worker belonging to the $f$th wage setting cohort (with $f \in [1, n_w]$) optimises its wage in period $t$ for the labour it rents to the firms of the $j$th price setting cohort (with $j \in [1, n_p]$):

$$\begin{aligned}\widehat{w}^*_t(f,j) &= \frac{1}{\sum_{i=0}^{n_w-1}\beta^i}\left[\sum_{i=0}^{n_w-1}\beta^i\left(\sigma_l\widehat{l}_{t+i}(f,j) + \frac{1}{1-h}\left(\widehat{c}_{t+i} - h\widehat{c}_{t+i-1}\right) - \varepsilon^l_{t+i}\right)\right. \\ &\quad \left. + \sum_{i=1}^{n_w-1}\left((\widehat{\pi}_{t+i} - \gamma_w\widehat{\pi}_{t+i-1} - (1-\gamma_w)\overline{\pi}_{t+i})\sum_{q=i}^{n_w-1}\beta^q\right)\right]\end{aligned}$$

$$\widehat{w}_t(j) = \frac{1}{n_w}\left[\sum_{i=0}^{n_w-1}\widehat{w}_t(f-i,j) + \widehat{p}_{t-i}\right] - \widehat{p}_t + \varepsilon^w_t \tag{6b}$$

$$\widehat{w}_{t+i}(f,j) = \widehat{w}^*_t(f,j) + \sum_{q=0}^{i-1}(\gamma_w\widehat{\pi}_{t-1-q} + (1-\gamma_w)\overline{\pi}_{t-q}) \tag{7b}$$

[12] As in the baseline model, there are two aggregate investment shocks: $\varepsilon^I_t$ which is an investment technology shock and $\eta^Q_t$ which is meant to capture stochastic variations in the external finance premium. The first one is assumed to follow an AR(1) process with an iid-Normal error term and the second is assumed to be iid-Normal distributed.

$$\widehat{l}_{t+i}(f,j) = \widehat{l}_{t+i}(j) - \frac{1+\lambda_w}{\lambda_w}\left(\widehat{w}_{t+i}(f,j) + \widehat{p}_t - (\widehat{w}_{t+i}(j) + \widehat{p}_{t+i})\right) \quad \text{(8b)}$$

$$\widehat{l}_t(j) = -\widehat{w}_t(j) + (1+\psi)\widehat{\rho}_t(j) + \widehat{K}_{t-1}(j) \quad \text{(A6b)}$$

It directly appears from these equations that there is now a labour market for each cohort of firms. Contrarily to the homogeneous labour setting, the labour demand of (cohort of) firm(s) $j$ (equation 16) directly affects the optimal wage choosen by the worker $f$ (equation 5b) and consequently the cohort specific average wage (6b). When $\gamma_w = 0$, real wages do not depend on the lagged inflation rate.[13]

Due to the staggered wage setting it is not so simple to see how changes in the demand for the firm's output will affect the firm-specific wage cost (equation (12)). A number of intuitive statements can, however, be made. First, higher wage stickiness as captured by the length of the typical wage contract will tend to reduce the response of wages to demand. As a result, high wage stickiness is likely to reduce the impact of firm-specific labour markets on the estimated degree of nominal price stickiness. In contrast, with flexible wages, the relative wage effect may be quite substantial, contributing to large changes in relative marginal cost of the firm and thereby dampening the relative price effects discussed above. Second, this effect is likely to be larger the higher the demand elasticity of labour (as captured by a lower labour market mark-up parameter) and the higher the elasticity of labour supply. Concerning the latter, if labour supply is infinitely elastic, wages will again tend to be very sticky and as a result relative wage costs will not respond very much to changes in relative demand even in the case of firm-specific labour markets.

### 4.1 Alternative models

In this section we illustrate the discussion above by displaying how the output, the marginal cost and the price of the first price-setting cohort respond to a monetary policy shock. We compare the benchmark model with mobile production factors (hereafter denoted MKL) with the following three models:

- a model with homogeneous capital and firm-specific labour market (hereafter denoted NML)
- a model with firm-specific capital, homogeneous labour (hereafter denoted NMK)
- a model with firm-specific capital and labour (hereafter denoted NMKL)

[13] Parameter $\psi$ is the inverse of the elasticity of the capital utilisation cost function.

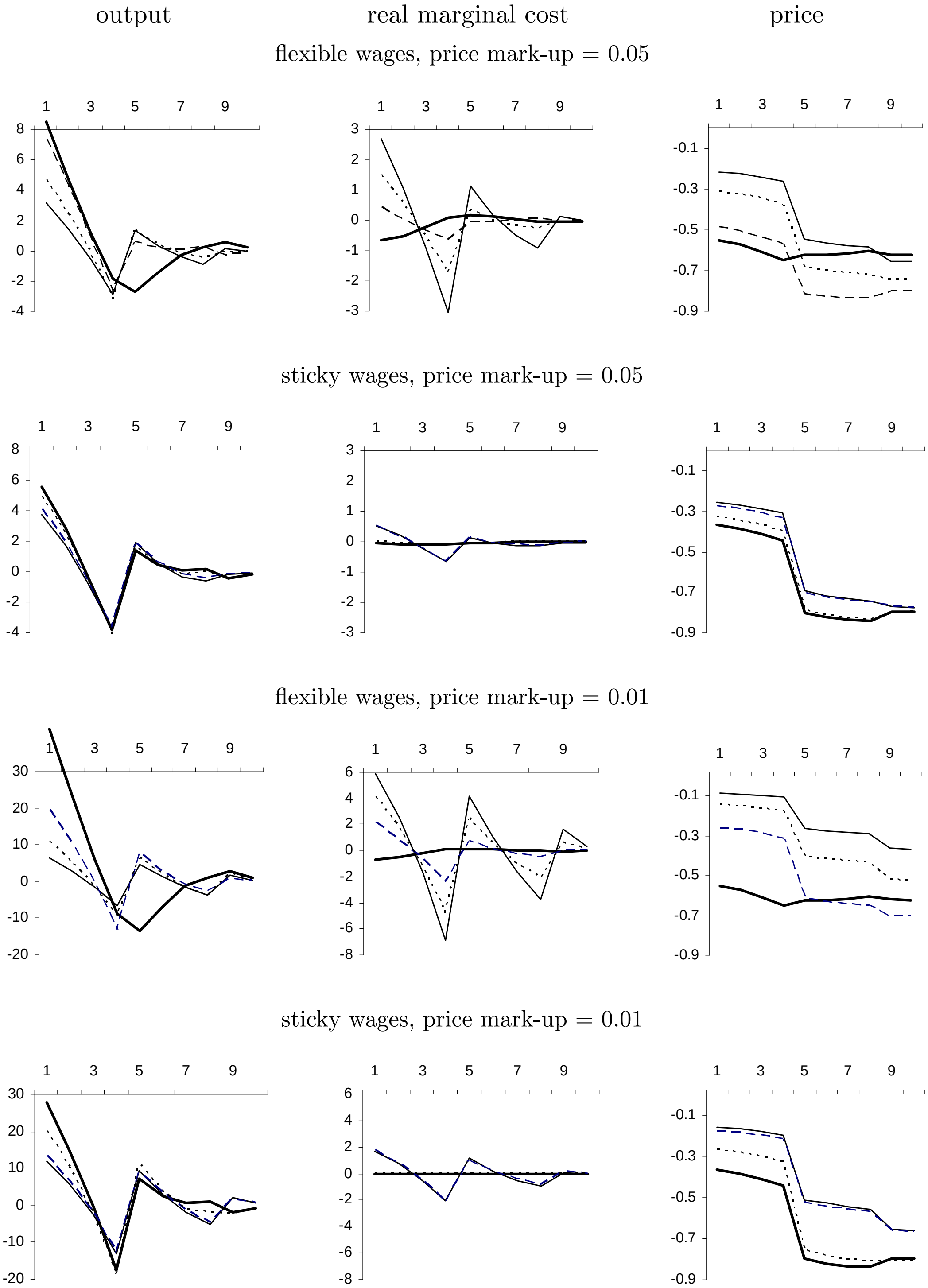
Moreover, for each of those models we consider four cases corresponding to flexible and sticky wages and low (0.01) and high (0.05) mark-ups in the goods market.[14] Figure 3 shows the responses of the cohort that is allowed to change its price in the period of the monetary policy shock. In this Figure we assume that the length of the price and wage contracts is 4 quarters. The rest of the parameters are those estimated for the benchmark Taylor model (MKL) with the corresponding contract length. Responses are displayed for the first 10 quarters following the shock, i.e. prices are re-optimised three times by the considered cohort in the time span considered, at periods 1, 5 and 9.

Several points are worth noting. First, introducing firm-specific factors always reduces the initial impact on prices and output, while it increases the impact on the marginal cost. As discussed, above with firm-specific production factors, price-setting firms internalise that large price responses lead to large variations in marginal costs and therefore lower their initial price response. Second, the introduction of firm-specific factors increases the persistence of price changes in particular when wages are flexible. While in the case of mobile production factors with flexible wages, the initial price decrease is partially reversed after four quarters, prices continue to decrease five and nine quarters after the initial shock when factors are firm-specific. Third, in the case with mobile factors, (MKL - bold black curve), it is clear that prices and marginal cost are not affected by changes in the demand elasticity, while the firm's output is very much affected. On the contrary, for all the models with at least one non-mobile production factor, price responses decrease while marginal cost variations increase with a higher demand elasticity.

Finally, as long as wages are considered to be flexible, firm-specific labour market is the device that leads to the largest reactions in marginal cost. It is also worth to remark that the combination of firm-specific labour market and firm-specific capital brings more reaction in the marginal cost than the respective effect of each assumption separately. However, as soon as wages become sticky, firm-specific labour market does not generate much more variability in marginal cost. In this case, it is striking that the responses of the NMK and NMKL models gets very close to each other.

[14] This corresponds to demand elasticities of 21 and 101 respectively. The latter is the one estimated by ACEL (2005). Furthermore, one needs rather high substitution elasticities to observe significant difference between the homogeneous marginal cost model and its firm-specific production factors counterparts. So, for demand elasticities below 10, there is nearly no difference between the the MKL model and the NML, NMK and NMKL ones. This indicates again the importance of a very elastic demand curve.

**Figure 3: The effect of a monetary policy shock on output, marginal cost and price of the first cohort in the 3 considered models**




Note: bold black curve: MKL; full curve: NMKL; dashed curve: NMK; dotted curve: NML

## 4.2 Estimation

In this section we re-estimate the model with firm-specific production factors to investigate the effects of this assumption on the empirical performance of the model. Sbordone (2002) and Gali *et al.* (2001) show that considering capital as a fixed factor non-mobile across firms does indeed reduce the estimated degree of nominal price stickiness in US data. In particular, it reduces the implied duration of nominal contracts from an implausibly high number of more than 2 years to a duration of typically less than a year. Altig *et al.* (2005) reach the same conclusion in a richer setup where firms endogenously determine their capital stock. We will extend this analysis to the case of firm-specific labour markets and test whether similar results are obtained in the context of Taylor contracts.

Table 4 reports the log data densities of the three models considered above and their flexible/sticky wages variants for various price contract lengths. A higher log data density implies a better empirical fit in terms of the model's one-step ahead prediction performance.

**Table 4: log data densities for the three models considered and their variants**

| | 2-Q | 4-Q | 6-Q | 8-Q |
|---|---|---|---|---|
| | | flexible wages | | |
| NML | -520.21 | -481.86 | -492.87 | -490.16 |
| NMKL | -484.92 | -479.56 | -481.87 | -485.23 |
| NMK | -486.50 | -480.68 | -482.16 | -481.97 |
| | sticky wages (4-quarter Taylor contract) | | | |
| NML | -512.50 | -490.19 | -484.72 | -480.54 |
| NMKL | -484.46 | -466.10 | -475.80 | -477.23 |
| NMK | -479.11 | -464.92 | -473.17 | -474.30 |

The following findings are noteworthy. First, in almost all cases, the data prefer the sticky wage over the flexible wage version. This is not surprising as sticky wages are better able to capture the empirical persistence in wage developments. In what follows, we therefore focus on the sticky wage models (lower part of Table 4). Second, with sticky wages the data prefers the model with firm-specific capital, but mobile labour. The introduction of firm-specific labour markets does not help the empirical fit of the model. The main reason for this result is that, as argued before, in order for firm-specific labour markets to help in explaining price and inflation persistence one needs a strong response of wages to changes in demand. But this is in contrast to the observed persistence in wage developments. On the other hand, as we do not observe the rental rate of capital, no such empirical constraint is relevant for the introduction of firm-specific capital. Finally, introducing firm-specific capital does indeed reduce the contract length that fits the data best. While the log data density is maximised at a contract length of 10 quarters (or even higher) in the case of homogeneous production factors, it is maximised at only four quarters when capital cannot move across firms. This confirms the findings of Gali *et al.* (2001) and Altig *et al.* (2005). Moreover, it turns out that the four-quarter Taylor contracting model with firm-specific capital performs even better than the baseline Calvo model.

In line with these results, in the rest of the paper we will focus on the model with firm-specific capital, homogeneous labour and sticky wages. Table 5 presents a selection of the parameters estimated for this model. Note that, in comparison to the case with homogeneous production factors, we also estimate the elasticity of substitution between the goods of the various cohorts. A number of findings are worth noting. First, allowing for sector-specific capital leads to a drop in the estimated degree of inflation indexation in the goods sector. In comparison with results displayed in Table 3, in this case the parameter drops back to the low level estimated for the Calvo model and does not appear to be significantly different from zero. Second, as discussed in Coenen and Levin (2004), one advantage of the Taylor price setting is that the price mark-up parameter is identified and therefore can be estimated. In contrast, with Calvo price setting, the model with firm-specific factors is observationally equivalent to its counterpart with homogeneous production factors. Table 5 shows that one needs a very high elasticity of substitution (or low mark-up) to match the Calvo model in terms of empirical performance. It is also interesting to note that the estimated price mark-up increases with the length of the price contract, showing the substitutability between nominal and real rigidities. Finally, the persistence parameter of the price shock significantly decreases with the length of the price contract.

Figure 4 displays the estimated reactions of some variables to a monetary policy shock and to a price shock for the 4 and 8-quarter price contract model together with those of the Calvo model. Comparing these responses with those presented in Figure 1 for the homogeneous production factors case, one directly observes that the degree of persistence after a monetary policy shock has very much increased. While in the homogeneous production factors case it was the model with the longer price contract that allowed to better reproduce the responses of the Calvo model to the price shock, under firm-specific capital it is the 4-quarter contract model which produces the responses closest to those of the Calvo model.

**Table 5: A selection of estimated parameters for the Taylor contracts models with firm-specific capital (NMK)**

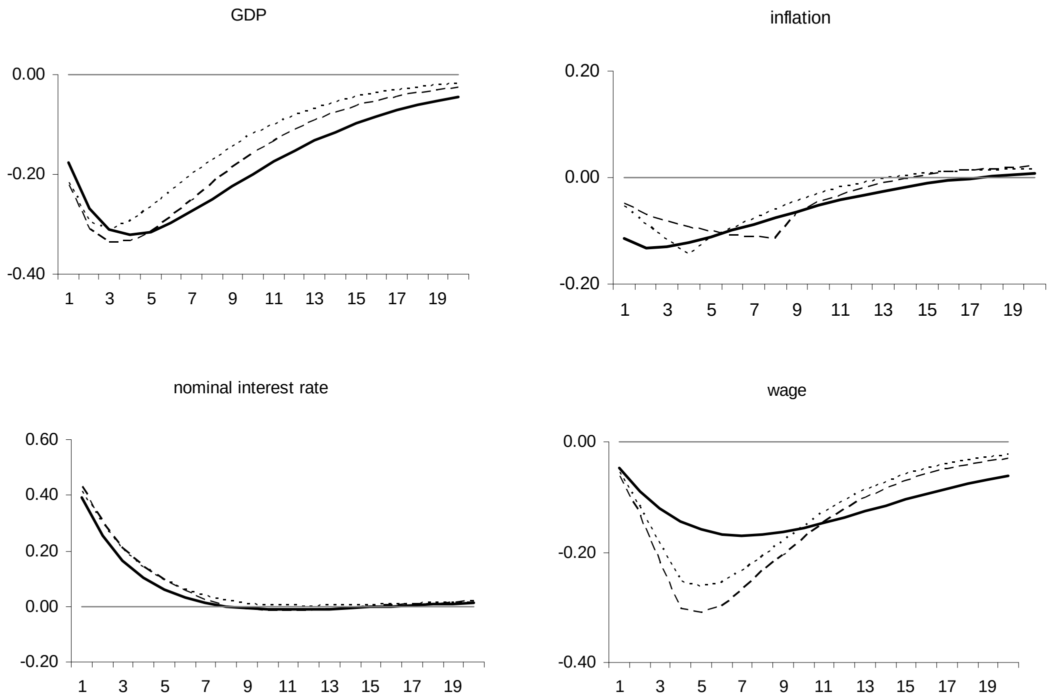
| | 2-Q Taylor | 4-Q Taylor | 6-Q Taylor | 8-Q Taylor |
|---|---|---|---|---|
| $\sigma_p$ | 0.216 | 0.225 | 0.232 | 0.230 |
| | (0.016) | (0.016) | (0.019) | (0.017) |
| $\rho_p$ | 0.997 | 0.979 | 0.863 | 0.802 |
| | (0.002) | (0.029) | (0.124) | (0.085) |
| $1+\phi$ | 1.616 | 1.515 | 1.522 | 1.520 |
| | (0.093) | (0.138) | (0.111) | (0.100) |
| $\lambda_p$ | 0.0008 | 0.004 | 0.008 | 0.016 |
| | (0.0003) | (0.0015) | (0.003) | (0.006) |
| $\gamma_p$ | 0.067 | 0.093 | 0.149 | 0.220 |
| | (0.070) | (0.077) | (0.094) | (0.102) |
| $\gamma_w$ | 0.403 | 0.463 | 0.547 | 0.436 |
| | (0.195) | (0.210) | (0.232) | (0.231) |

Note: $\rho_p$ is the persistency parameter associated to the price shock; $\sigma_p$ is the standard error of the price shock; $\gamma_w$ and $\gamma_p$ are respectively the wage and price indexation parameters; $\phi$ is the share of the fixed cost; $\lambda_p$ is the price mark-up.

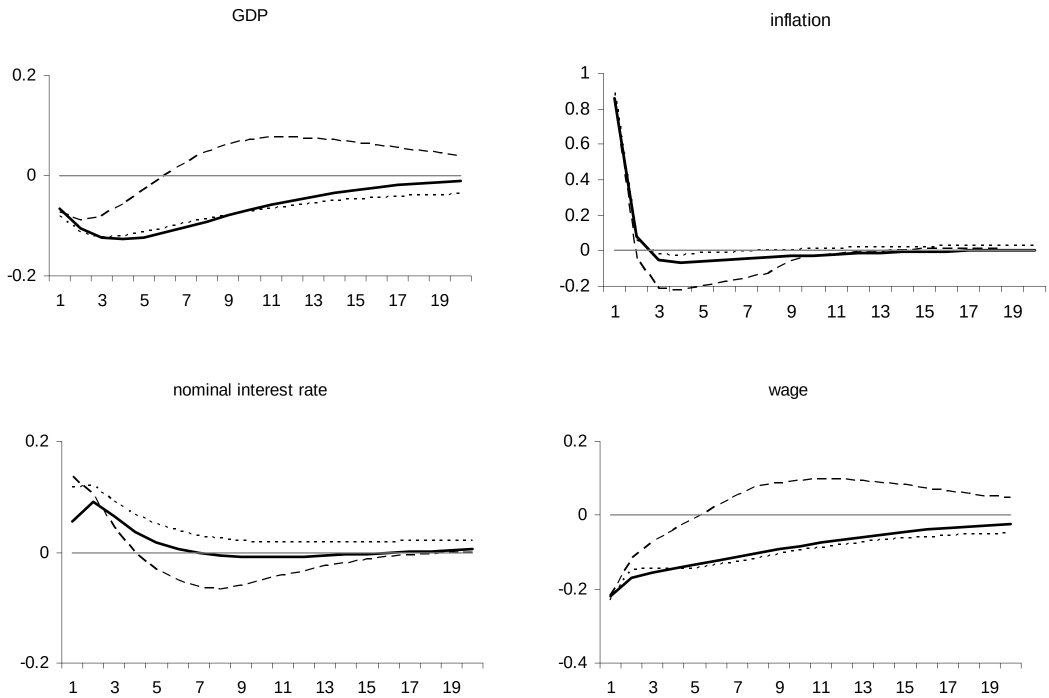
For the 4-quarter price contract model, the estimated parameter for the price mark-up is 0.004, which implies an extremely high elasticity of substitution of about 250. With such an elasticity, individual firms output are extremely sensitive to price variations. At this stage, it may be worth to run the same experiment as Altig *et al.* (2005) who compare the estimated contribution to output for firms after a monetary policy shock both in the model with firm-specific capital and with homogeneous capital. In order to be fair, we compare the homogeneous production factors model with the optimal contract length, i.e. 10 quarters, to the firm-specific capital model with the optimal length, i.e. 4 quarters. For both models, we use the corresponding estimated parameters. In the model with homogeneous capital the demand elasticity is not identified. Following Altig *et al.* (2005), we use the same elasticity as the one estimated in the firm-specific capital model. Figure 5 displays in the left column the contribution of each cohort to the overall output up to four periods after a monetary policy shock. In the right column, the relative prices are displayed for the same time span. For the homogeneous capital model, only five types of firms are displayed instead of 10. This is simply because all the firms that have not re-optimised their price after four periods do all share the same price and consequently face the same demand. These firms represent thus 60% of the firms in the economy.

**Figure 4: Selected impulse responses: Calvo versus Taylor contracts and non-mobile capital (estimated parameters)**

monetary policy shock




persistent price shock in equation ()

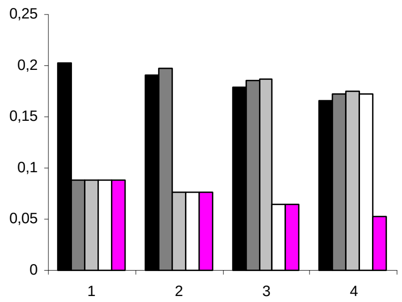


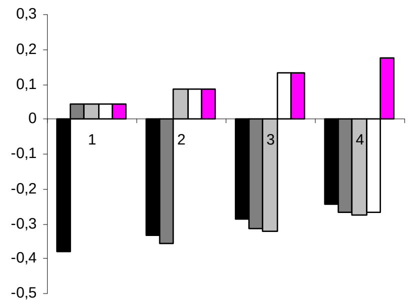

Legend: bold black line: baseline (Calvo) model; dashed line: 8-quarter Taylor price contract; dotted line: 4-quarter Taylor price contract.

**Figure 5: output shares and relative prices for the first 4 periods after a monetary policy shock in the homogeneous and firm-specific capital model (estimated parameters)**

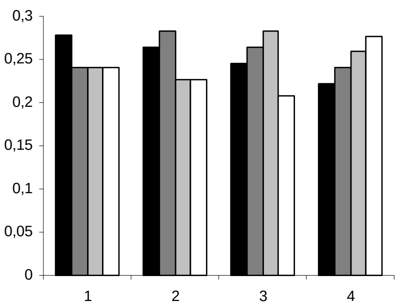
output shares in global GDP | relative prices

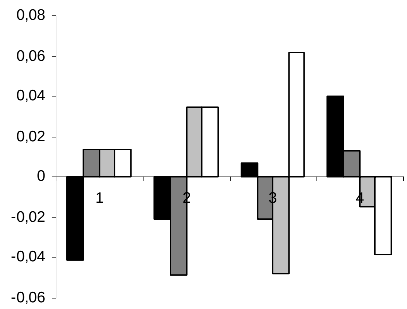
homogeneous production factors (10-quarter estimated model, table 3)




firm-specific capital (4-quarter estimated model, table 5)




Legend: columns from left to right are for cohort 1 to cohort 4 in the case of the 4-quarter model. In the homogeneous capital model, column 5 is for the six cohorts that have not yet had the opportunity to re-optimise their price.

Comparing the evolution of the relative prices in both models, we observe that they vary much more in the homogeneous factor model than in the firm-specific one. There are two reasons for this larger volatility: the fact that the marginal cost is independent of the output, and the length of the price contract. The corollary is a much larger variability in the market shares of the firms in the model with homogeneous capital. The first cohort to reset optimally its price doubles its share in the production. Even though this result is less extreme than the one presented in Altig *et al.* (2005),[15] such a high variability in

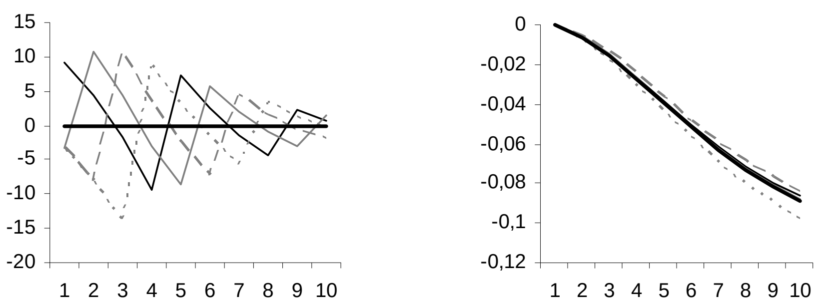
[15] In their model, with their erstimated parameters, at the fourth period after the monetary policy shock, 57% of the firms produce 180% of the global output, leaving the remaining firms with a negative output.

output shares following a monetary policy shock is empirically implausible. Reducing the contract length from 10 to 4 quarters while keeping all other parameters the same, the increase in market share of the first cohort would be 56% instead of 102%.[16] Similarly, reducing the huge elasticity of substitution would reduce these effects.

Finally, now that we have an estimated model with firm-specific capital, we may try to assess if the fact that investment enters the decision set of the firm is an essential feature of the model, or on the contrary if considering capital as a fixed factor gives a good approximation. Woodford (2005) explains graphically that "the implicit assumption of an exogenously evolving capital stock in derivations of the Phillips curve for models with firm-specific capital [...] appears not to have been a source of any great inaccuracy." This is rather intuitive since capital adjusts only slowly, so that labour is the main adjusting variable. In our setting, we cannot directly compare both models, but, for a given firm, we can display how both production factors react to a given shock. Figure 6 displays the estimated responses of capital stock and labour used by each one of the four cohorts up to 10 periods after a monetary policy shock. The movements in the firms' capital stock are small and very slow, while labour is much more volatile as it must adjust to the volatility of the individual firms output. This confirms that models where firms build their own capital stock are not much different from models where capital stock is fixed (Gali *et al.*, 2001, Sbordone, 2002, Eichenbaum and Fisher, 2004 and Coenen and Levin, 2004).

**Figure 6: reactions of the firms' capital stock and labour after a monetary policy shock (estimated 4-quarter NMK model)**

labour capital stock




Legend: bold black lines are for global variables, black lines for firms in cohort 1, gray lines for firms in cohort 2, gray dashed lines for firms in cohort 3 and gray dotted lines for firms in cohort 4.

[16] Of course, this remains rather large compared with the 12% increase obtained in the model with firm-specific capital.

### 4.3 Endogenous price mark-up

As discussed above, the estimates of the firm-specific capital model suggest a very high elasticity of substitution in the goods market or, equivalently, a very low mark-up. However, this implies that the estimated fixed cost in production ($1+\phi$ stands at 1.515) very much exceeds the profit margin, implying negative profits in steady state. In order to circumvent this problem, one may follow Altig *et al.* (2005) and impose the zero profit condition in steady state. The estimation result obtained for the 4-quarter price contract model is displayed in the first column of Table 6. The empirical cost of imposing the constraint is rather high, about 15 in log data density. Furthermore, the estimated demand elasticity remains very high at about 167. Note also that the constraint leads to a much larger estimation of the standard error of the productivity shock.

Following Eichenbaum and Fisher (2004), and Coenen and Levin (2004), an alternative solution is to consider an endogenous mark-up, whereby the optimal mark-up is a function of the relative price as in Kimball (1995). Replacing the Dixit-Stiglitz aggregator by the homogeneous-degree-one aggregator considered by Kimball (1995), the linearised optimal price equation (9) becomes

$$\begin{aligned} \widehat{p}_t^*(j) \quad = \quad & \frac{1}{\sum_{i=0}^{n_p-1}\beta^i}\left[\frac{1}{1+\lambda_p\cdot\epsilon}\sum_{i=0}^{n_p-1}\beta^i\widehat{s}_{t+i}(j)+\sum_{i=0}^{n_p-1}\beta^i\widehat{p}_{t+i}\right. \\ & \left.-\sum_{i=0}^{n_p-2}\left(\left(\gamma_p\widehat{\pi}_{t+i}+(1-\gamma_p)\overline{\pi}_{t+i+1}\right)\sum_{q=i+1}^{n_p-1}\beta^q\right)\right] \end{aligned} \tag{16}$$

where $\epsilon$ represents the deviation from the steady state demand elasticity following a change in the relative price, while $\lambda_p$ is the steady state mark-up:[17]

$$\epsilon = \left.\frac{\partial\left(\frac{1+\lambda_p(z)}{\lambda_p(z)}\right)}{\partial p^*}\cdot\frac{p^*}{\frac{1+\lambda_p(z)}{\lambda_p(z)}}\right|_{z=1} \tag{17}$$

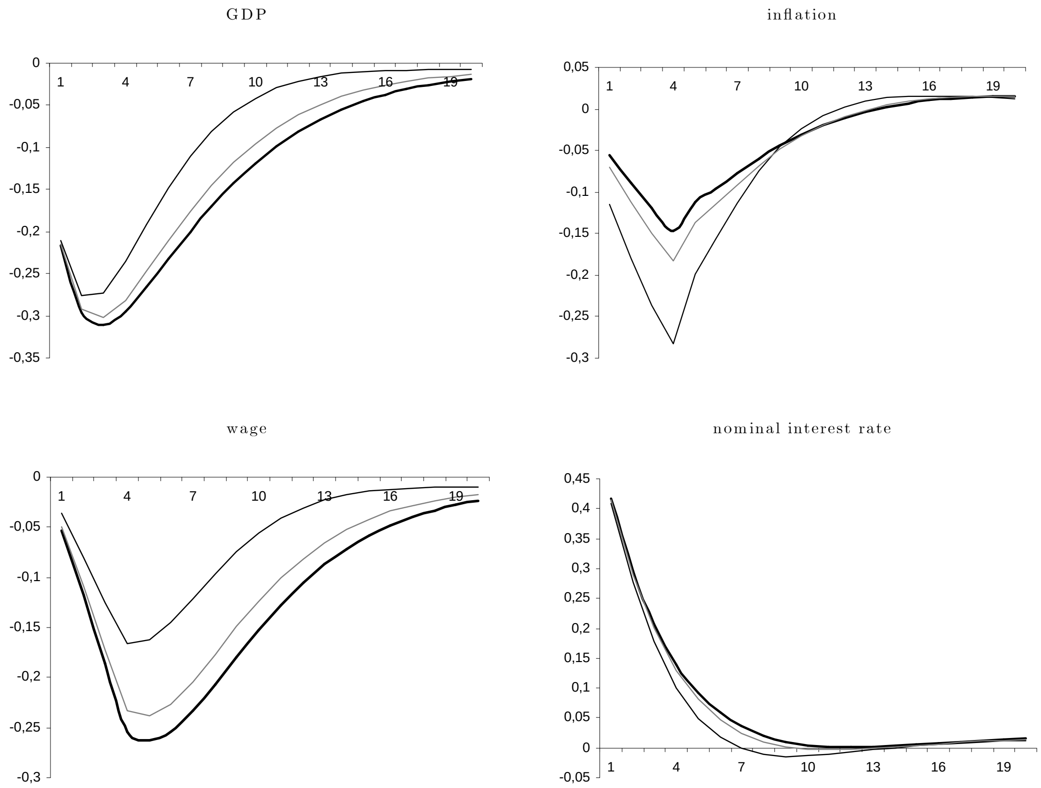
This elasticity plays the same role as the elasticity of substitution: the larger it is, the less the optimal price is sensitive to changes in the marginal cost. In this sense, having $\epsilon > 0$ can help to reduce the estimate for the demand elasticity to a more realistic level. In order to illustrate this mechanism, Figure 7 displays the reactions of global output, inflation, wage and interest rate after a monetary policy shock for a model with an endogenous price mark-up. As benchmark, we use the 4-quarter price contract model with constant price mark-up estimated in Table 6 and we compare it with the model integrating both the constraint on the mark-up and the endogenous price mark-up. For the latter model, we use the parameters estimated for the benchmark, except for the

[17] Of course, the Dixit-Stiglitz aggregator corresponds to the case where $\epsilon$ is equal to zero.

steady state mark-up, $\lambda_p$, which is fixed at 0.5, while different values are used for the curvature parameter $\epsilon$: 20 and 60.

**Figure 7: Assessing the substitutability between the steady state demand elasticity and the curvature parameter**

monetary policy shock




Legend: bold black line: baseline model; black line: $\lambda_p = 0.5$ and $\epsilon = 20$; gray line: $\lambda_p = 0.5$ and $\epsilon = 60$.

It is clear from Figure 7 that an endogenous price mark-up which is very sensitive to the relative price can produce the same effect on aggregate variables as a very small constant price mark-up. The next step is to re-estimate the NMK model with 4-quarter price and wage Taylor contracts but adding the modifications discussed above, i.e. imposing the price mark-up to equate the share of the fixed cost ($\phi = \lambda_p$) and allowing $\epsilon$ to be different from zero. The results are displayed in column 2 of Table 6. When the share of the fixed cost is forced to equate the mark-up, shifting from a final good production function with a constant price mark-up to one with a price mark-up declining in the relative price, the estimated steady state price mark-up becomes much larger, implying a demand elasticity of about 3. This helps to reduce the cost of the constraint and the log data density is improved by 11. The very high estimated curvature parameter $\epsilon$ (about 70) reveals the need for real rigidities.

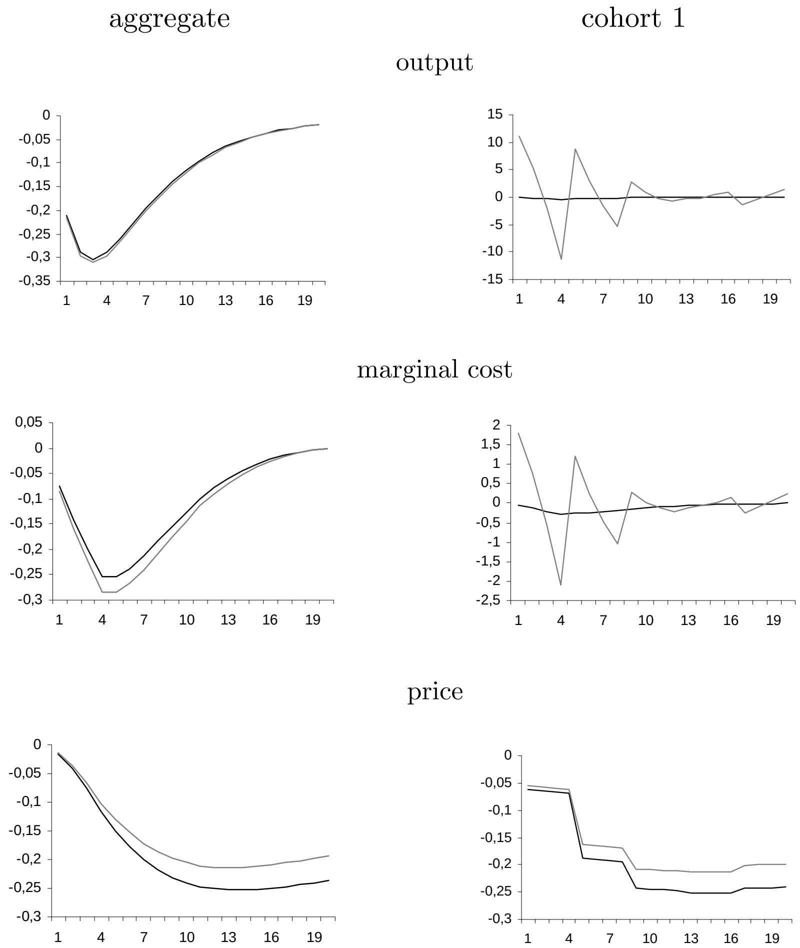
**Table 6: estimating models with constrained and/or endogeneous demand elasticity (some selected parameters)**

| | $\phi = \lambda_p$ and $\epsilon = 0$ | $\phi = \lambda_p$ and $\epsilon \neq 0$ |
|---|---|---|
| log data density | -479.671 | -468.344 |
| $\sigma_p$ | 0.208 | 0.178 |
| | (0.015) | (0.013) |
| $\rho_p$ | 0.829 | 0.539 |
| | (0.086) | (0.056) |
| $\sigma_a$ | 1.099 | 0.650 |
| | (0.153) | (0.088) |
| $\rho_a$ | 0.960 | 0.981 |
| | (0.011) | (0.007) |
| $\lambda_p = \phi$ | 0.006 | 0.489 |
| | (0.001) | (0.128) |
| $\frac{\epsilon}{1+\epsilon}$ | 0 | 0.986 |
| | - | (0.004) |

Note: $\rho_a$ and $\rho_p$ are the persistency parameter associated to the productivity and the price shock respectively; $\sigma_a$ and $\sigma_p$ are the standard error of the productivityand the price shock respectively; $\gamma_w$ and $\gamma_p$ are respectively the wage and price indexation parameters; $\phi$ is the share of the fixed cost; $\lambda_p$ and $\lambda_w$ are respectively the price and the wage mark-up; $\epsilon$ is the curvature parameter.

Of course, having a low demand elasticity leads to small variations in output shares for the different cohorts. Figure 8 draws the output, marginal cost and price reactions of the first cohort of price setting firms after a monetary policy shock for the constrained model with an endogeneous mark-up (Table 6, second column) and compares them with those of the unconstrained model with a constant mark-up (Table 5, second column) using their respective estimated parameters. The behaviour of the corresponding aggregate variables is displayed at the first column of the figure. The responses of the aggregate variables are very close to each other, but this outcome is obtained with very much different individual firms' behaviours. In the model with endogeneous mark-up, variations in firms' output and marginal cost are of the same scale as those of the aggregate output. The corollary is that the share of the firms in the global output is only very little affected by the shock.

**Figure 8: comparing responses to a monetary policy shock for the 4-quarter NMK model and its counterpart with constrained and endogeneous mark-up**




Legend: black line: 4-quarter NMK model with $\phi = \lambda_p$ and $\epsilon \neq 0$;
gray line: 4-quarter NMK model with $\phi \neq \lambda_p$ and $\epsilon = 0$.

# 5 Conclusion

In this paper we have introduced firm-specific production factors in a model with price and wage Taylor contracts. For this type of exercise, Taylor contracts present a twofold advantage over Calvo type contracts: *(i)* firm-specific production factors can be introduced and handled explicitly and *(ii)* the individual firm variables can be analysed explicitly. This allows a comparison of the implications of the various assumptions concerning the firm-specificity of production factors not only for aggregate variables, but also for cross-firm variability.

Our main results are threefold. First, in line with existing literature we show that introducing firm-specific capital reduces the estimated duration of price contracts from an implausible 10 quarters to an empirically more plausible 4 quarters. Such a modification improves the overall empirical performance of our DSGE model with Taylor contracts. Second, introducing firm-specific labour markets does not help in improving the empirical performance of the model. The main reason is that wages are sticky and therefore large variations in firm-specific wages which would prevent firms from changing their prices by a lot are empirically implausible. Overall, it thus appears that rigidities in the reallocation of capital across firms rather than rigidities in the labour market are a more plausible real friction for reducing the estimated degree of nominal price stickiness. Third, in order to obtain this outcome one needs a very high demand elasticity, larger than 250. This extreme elasticity implies implausibly large variations in the demand faced by the firms throughout the length of the contract. Furthermore, it implies that the share of the fixed cost is estimated to be larger than the mark-up, leading to negative profits in steady state. The cost of constraining the mark-up to equate the fixed cost is important but it is sharply reduced by replacing the constant mark-up by an endogenous one. The latter assumption implies a drastic reduction of the demand elasticity from 167 to 3 and a corresponding reduction in the volatility of output across firms. In this case, the need for important real rigidities becomes evident through a high estimated curvature parameter of the demand curve.

To compare the respective merits of the models with flat marginal cost (homogeneous production factors) and with increasing marginal cost (firm-specific production factors), it is important to remember what are the main conclusions emerging from micro data on firms pricing behaviour: price changes are at the same time frequent and large (cf. Bils and Klenow, 2002, Angeloni *et al.* (2004)). The model with flat marginal cost requires a lot of nominal stickiness to reproduce inflation persistency, together with large price changes. The introduction of increasing marginal cost leads to small price variations, so that one needs less nominal stickiness. It thus seems that, so far, neither the flat nor the increasing marginal cost models can satisfy simultaneously both stylized facts. Altig *et al.* (2005) favour the model with increasing marginal cost on the basis that it produces less extreme variations in output shares after an exogenous shock. We displayed that this outcome relies heavily on the price contract length and on the very large demand elasticity estimated for the increasing marginal cost model, so that we find it difficult to discriminate between both models on the basis of this argument. In order to solve the puzzle, it would certainly be useful to better assess the relationship between price, output and marginal cost at the firms level with the help of micro database. This will certainly help in the modelling of the marginal cost.

Note that in this paper and in contrast to Coenen and Levin (2004), we did not allow for heterogeneity in the contract length. This is often viewed as an alternative or a complementary track to reduce the average length of the price contract as long contracts could increase persistence more than proportionally to their share in the economy. Further research along these lines would be worthwhile (see also Dixon and Kara, 2004 and 2005).

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# 6 Appendix

## 6.1 Data appendix

All data are taken from the AWM database from the ECB (see Fagan *et al.*, 2000). Investment includes both private and public investment expenditures. The sample contains data from 1970Q2 to 2002Q2 and the first 15 quarters are used to initialize the Kalman filter. Real variables are deflated with their own deflator. Inflation is calculated as the first difference of the log GDP deflator. In the absence of data on hours worked, we use total employment data for the euro area. As explained in Smets and Wouters

(2003), we therefore use for the euro area model an auxiliary observation equation linking labour services in the model and observed employment based on a Calvo mechanism for the hiring decision of firms. The series are updated for the most recent period using growth rates for the corresponding series published in the Monthly Bulletin of the ECB. Consumption, investment, GDP, wages and hours/employment are expressed in 100 times the log. The interest rate and inflation rate are expressed on a quarterly basis corresponding with their appearance in the model (in the graphs the series are translated on an annual basis).

### 6.2 Model appendix

This appendix describes the other linearised equations of the Smets-Wouters model (2003-2004a).

Indexation of nominal wages results in the following *real wage equation*:

$$\begin{aligned}
\widehat{w}_t &= \frac{\beta}{1+\beta}E_t\widehat{w}_{t+1} + \frac{1}{1+\beta}\widehat{w}_{t-1} + \frac{\beta}{1+\beta}\left(E_t\widehat{\pi}_{t+1} - \overline{\pi}_t\right) \\
&\quad -\frac{1+\beta\gamma_w}{1+\beta}\left(\widehat{\pi}_t - \overline{\pi}_t\right) + \frac{\gamma_w}{1+\beta}\left(\widehat{\pi}_{t-1} - \overline{\pi}_t\right) \\
&\quad -\frac{1}{1+\beta}\frac{(1-\beta\xi_w)(1-\xi_w)}{\left(1+\frac{(1+\lambda_w)\sigma_l}{\lambda_w}\right)\xi_w}\left[\widehat{w}_t - \sigma_l\widehat{l}_t - \frac{1}{1-h}\left(\widehat{c}_t - h\widehat{c}_{t-1}\right) + \varepsilon_t^l\right] \\
&\quad +\eta_t^w \qquad \text{(A1)}
\end{aligned}$$

The real wage $\widehat{w}_t$ is a function of expected and past real wages and the expected, current and past inflation rate where the relative weight depends on the degree of indexation $\gamma_w$ to lagged inflation of the non-optimised wages. When $\gamma_w = 0$, real wages do not depend on the lagged inflation rate. There is a negative effect of the deviation of the actual real wage from the wage that would prevail in a flexible labour market. The size of this effect will be greater, the smaller the degree of wage stickiness ($\xi_w$), the lower the demand elasticity for labour (higher mark-up $\lambda_w$) and the lower the inverse elasticity of labour supply ($\sigma_l$ ) or the flatter the labour supply curve. $\varepsilon_t^l$ is a preference shock representing a shock to the labour supply and is assumed to follow a first-order autoregressive process with an iid-Normal error term: $\varepsilon_t^l = \rho_l\varepsilon_{t-1}^l + \eta_t^l$. In contrast, $\eta_t^w$ is assumed to be an iid-Normal wage mark-up shock.

The dynamics of *aggregate consumption* is given by:

$$\begin{aligned}
\widehat{c}_t &= \frac{h}{1+h}\widehat{c}_{t-1} + \frac{1}{1+h}E_t\widehat{c}_{t+1} + \frac{\sigma_c - 1}{\sigma_c(1+\lambda_w)(1+h)}(\widehat{l}_t - E_t\widehat{l}_{t+1}) \\
&\quad -\frac{1-h}{(1+h)\sigma_c}(\widehat{R}_t - E_t\widehat{\pi}_{t+1} + \varepsilon_t^b) \qquad \text{(A2)}
\end{aligned}$$

Consumption $\widehat{c}_t$ depends on the ex-ante real interest rate ($\widehat{R}_t - E_t\widehat{\pi}_{t+1}$) and, with external habit formation, on a weighted average of past and expected future consumption.

When $h = 0$, only the traditional forward-looking term is maintained. In addition, due to the non-separability of the utility function, consumption will also depend on expected employment growth $(E_t\widehat{l}_{t+1} - \widehat{l}_t)$. When the elasticity of intertemporal substitution (for constant labour) is smaller than one ($\sigma_c > 1$ ), consumption and labour supply are complements. Finally $\varepsilon_t^b$, represents a preference shock affecting the discount rate that determines the intertemporal substitution decisions of households. This shock is assumed to follow a first-order autoregressive process with an iid-Normal error term: $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$.

The *investment equation* is given by:

$$\widehat{I}_t = \frac{1}{1+\beta}\widehat{I}_{t-1} + \frac{\beta}{1+\beta}E_t\widehat{I}_{t+1} + \frac{1/\varphi}{1+\beta}\widehat{Q}_t + \varepsilon_t^I \tag{A3}$$

where $\varphi = \overline{S}''$depends on the adjustment cost function ($S$) and $\beta$ is the discount factor applied by the households. As discussed in CEE (2001), modelling the capital adjustment costs as a function of the change in investment rather than its level introduces additional dynamics in the investment equation, which is useful in capturing the hump-shaped response of investment to various shocks including monetary policy shocks. A positive shock to the investment-specific technology, $\varepsilon_t^I$, increases investment in the same way as an increase in the value of the existing capital stock $\widehat{Q}_t$. This investment shock is also assumed to follow a first-order autoregressive process with an iid-Normal error term: $\varepsilon_t^I = \rho_I \varepsilon_{t-1}^I + \eta_t^I$.

The corresponding $Q$ *equation* is given by:

$$\widehat{Q}_t = -(\widehat{R}_t - \widehat{\pi}_{t+1}) + \frac{1-\tau}{1-\tau+\overline{r}^k}E_t\widehat{Q}_{t+1} + \frac{\overline{r}^k}{1-\tau+\overline{r}^k}E_t\widehat{r}_{t+1}^k + \eta_t^Q \tag{A4}$$

where $\tau$ stands for the depreciation rate and $\overline{r}^k$ for the rental rate of capital so that $\beta = 1/(1-\tau+\overline{r}^k)$. The current value of the capital stock depends negatively on the ex-ante real interest rate, and positively on its expected future value and the expected rental rate. The introduction of a shock to the required rate of return on equity investment, $\eta_t^Q$, is meant as a shortcut to capture changes in the cost of capital that may be due to stochastic variations in the external finance premium. We assume that this equity premium shock follows an iid-Normal process. In a fully-fledged model, the production of capital goods and the associated investment process could be modelled in a separate sector. In such a case, imperfect information between the capital producing borrowers and the financial intermediaries could give rise to a stochastic external finance premium. Here, we implicitly assume that the deviation between the two returns can be captured by a stochastic shock, whereas the steady-state distortion due to such informational frictions is zero.

The *capital accumulation equation* becomes a function not only of the flow of investment but also of the relative efficiency of these investment expenditures as captured by the investment-specific technology shock:

$$\widehat{K}_t = (1-\tau)\widehat{K}_{t-1} + \tau\widehat{I}_{t-1} + \tau\varepsilon^I_{t-1} \tag{A5}$$

The equalisation of marginal cost implies that, for a given installed capital stock, *labour demand* depends negatively on the real wage (with a unit elasticity) and positively on the rental rate of capital:

$$\widehat{l}_t = -\widehat{w}_t + (1+\psi)\widehat{r}^k_t + \widehat{K}_{t-1} \tag{A6}$$

where $\psi = \frac{\psi'(1)}{\psi''(1)}$ is the inverse of the elasticity of the capital utilisation cost function.

The *goods market equilibrium condition* can be written as:

$$\begin{aligned} \widehat{Y}_t &= (1-\tau k_y - g_y)\widehat{c}_t + \tau k_y \widehat{I}_t + g_y \varepsilon^g_t && \text{(A7a)} \\ &= \phi\left[\alpha(\widehat{K}_{t-1} + \psi\widehat{r}^k_t) + (1-\alpha)(\widehat{l}_t + \gamma t\right] - (\phi-1)\gamma t && \text{(A7b)} \end{aligned}$$

where $k_y$ is the steady state capital-output ratio, $g_y$ the steady-state government spending-output ratio and $\phi$ is one plus the share of the fixed cost in production. We assume that the government spending shock follows a first-order autoregressive process with an iid-Normal error term: $\varepsilon^g_t = \rho_g\varepsilon^g_{t-1} + \eta^g_t$.

Finally, the model is closed by adding the following empirical *monetary policy reaction function*:

$$\begin{aligned} \widehat{R}_t = {} & \overline{\pi}_t + \rho(\widehat{R}_{t-1} - \overline{\pi}_{t-1}) + (1-\rho)\left[r_\pi(\widehat{\pi}_{t-1} - \overline{\pi}_{t-1}) + r_Y(\widehat{Y}_{t-1} - \widehat{Y}^p_{t-1})\right] + \\ & r_{\Delta\pi}\left[(\widehat{\pi}_t - \overline{\pi}_t) - (\widehat{\pi}_{t-1} - \overline{\pi}_{t-1})\right] + r_{\Delta Y}\left[(\widehat{Y}_t - \widehat{Y}^p_t) - (\widehat{Y}_{t-1} - \widehat{Y}^p_{t-1})\right] + \eta^R_t \end{aligned} \tag{A8}$$

The monetary authorities follow a generalised Taylor rule by gradually responding to deviations of lagged inflation from an inflation objective and the lagged output gap defined as the difference between actual and potential output. Consistently with the DSGE model, potential output is defined as the level of output that would prevail under flexible price and wages in the absence of the three "cost-push" shocks. The parameter $\rho$ captures the degree of interest rate smoothing. In addition, there is also a short-run feedback from the current changes in inflation and the output gap. Finally, we assume that there are two monetary policy shocks: one is a temporary iid-Normal interest rate shock ($\eta^R_t$) also denoted a monetary policy shock; the other is a permanent shock to the inflation objective ($\overline{\pi}_t$) which is assumed to follow a non-stationary process ($\overline{\pi}_t = \overline{\pi}_{t-1} + \eta^\pi_t$). The dynamic specification of the reaction function is such that changes in the inflation objective are immediately and without cost reflected in actual inflation and the interest rate if there is no exogenous persistence in the inflation process.

### 6.3 Description of the priors

Some parameters are fixed. They are principaly parameters related to the steady-state values of the state variables. The discount factor $\beta$ is calibrated at 0.99, corresponding with an annual steady-state reel interest rate of 4%. The depreciation rate $\tau$ is set at 0.025, so that the annual capital depreciation is equal to 10 percent. The steady state share of capital income is fixed at $\alpha = 0.24$.The share of steady-state consumption in total output is assumed equal to 0.6 and the share of steady-state investment to 0.22.

The priors on the other parameters are displayed in tables of the next appendix. The first column is the description of the parameter, the second its symbol. In the third column one find the prior distribution while the last two columns give respectively the prior mean and standard error. Most of the priors are the same as in Smets and Wouters (2003). However, an important difference is to note for the capacity utilisation adjustment cost parameter ($\psi$). Instead of estimating $cz = \frac{1}{\psi}$ with a prior [Normal 0.2 0.075], we now estimate $cz = \frac{1}{1+\psi}$ with a prior [beta 0.5 0.25], which actually corresponds to a much looser prior since it allows for values of the elasticity of the capital utilisation cost function between 0.1 and 10. Some new parameters appear: the price and wage mark-ups, which are given a rather loose prior of [beta 0.25 0.15), and the curvature parameter which is estimated via $eps = \frac{\epsilon}{1+\epsilon}$ with a prior of [beta 0.85 0.1]. The latter allows for values of parameter $\epsilon$ between 1.5 and 100.

For the rest, as in Smets and Wouters (2003), the persistency parameters are given a Normal prior distribution with a mean of 0.85 and a standard error of 0.10. The variance of the shocks are assumed to follow an inverted Gamma distribution with two degrees of freedom.

## 6.4 Parameter estimates for the main models

The Metropolis-Hastings algorithm has been run with 250 000 draws. Convergence is assessed with the help of Cumsum graphs.

NMK model, 4-quarter price contract, $\phi \neq \lambda_p$ and $\epsilon = 0$ (Table 5 column 2)

marginal likelihood :

Laplace approximation: -464.920

Modified harmonic mean: -463.902

| | Prior distribution | | | Estimated posterior mode and mean | | | | Posterior sample based | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | type | mean | st. error | mode | st. error | mean | st. error | 5% | 10% | 50% | 90% | 95% |
| **st. dev. of the shocks** | | | | | | | | | | | | |
| productivity shock | inv. gamm | 0.250 | 2 d.f. | 0.655 | 0.092 | 0.680 | 0.092 | 0.544 | 0.569 | 0.672 | 0.803 | 0.845 |
| inflation obj. shock | inv. gamm | 0.050 | 2 d.f. | 0.129 | 0.016 | 0.133 | 0.016 | 0.107 | 0.112 | 0.132 | 0.154 | 0.160 |
| cons. pref. shock | inv. gamm | 0.250 | 2 d.f. | 0.138 | 0.026 | 0.164 | 0.032 | 0.120 | 0.127 | 0.159 | 0.207 | 0.224 |
| gov. spend. shock | inv. gamm | 0.250 | 2 d.f. | 0.347 | 0.023 | 0.351 | 0.023 | 0.316 | 0.323 | 0.350 | 0.380 | 0.389 |
| lab. supl. shock | inv. gamm | 0.250 | 2 d.f. | 0.284 | 0.112 | 0.512 | 0.331 | 0.204 | 0.231 | 0.414 | 0.898 | 1.069 |
| investment shock | inv. gamm | 0.250 | 2 d.f. | 0.254 | 0.049 | 0.247 | 0.048 | 0.177 | 0.190 | 0.243 | 0.310 | 0.333 |
| interest rate shock | inv. gamm | 0.250 | 2 d.f. | 0.131 | 0.015 | 0.135 | 0.015 | 0.112 | 0.117 | 0.135 | 0.155 | 0.161 |
| equity premium shock | inv. gamm | 0.250 | 2 d.f. | 0.537 | 0.053 | 0.538 | 0.057 | 0.445 | 0.465 | 0.537 | 0.612 | 0.634 |
| price shock | inv. gamm | 0.250 | 2 d.f. | 0.225 | 0.016 | 0.227 | 0.018 | 0.199 | 0.205 | 0.225 | 0.250 | 0.257 |
| wage shock | inv. gamm | 0.250 | 2 d.f. | 0.441 | 0.035 | 0.449 | 0.035 | 0.395 | 0.406 | 0.447 | 0.495 | 0.512 |
| **persistency parameters** | | | | | | | | | | | | |
| productivity shock | beta | 0.850 | 0.100 | 0.979 | 0.009 | 0.979 | 0.008 | 0.964 | 0.968 | 0.979 | 0.988 | 0.990 |
| cons. pref. shock | beta | 0.850 | 0.100 | 0.922 | 0.019 | 0.914 | 0.016 | 0.885 | 0.892 | 0.915 | 0.934 | 0.938 |
| gov. spend. shock | beta | 0.850 | 0.100 | 0.992 | 0.009 | 0.984 | 0.010 | 0.965 | 0.971 | 0.986 | 0.995 | 0.997 |
| lab. supl. shock | beta | 0.850 | 0.100 | 0.882 | 0.087 | 0.855 | 0.098 | 0.668 | 0.718 | 0.874 | 0.965 | 0.976 |
| investmnet shock | beta | 0.850 | 0.100 | 0.997 | 0.003 | 0.991 | 0.007 | 0.977 | 0.982 | 0.993 | 0.998 | 0.999 |
| price shock | beta | 0.850 | 0.100 | 0.979 | 0.029 | 0.947 | 0.045 | 0.851 | 0.878 | 0.961 | 0.987 | 0.991 |
| wage shock | beta | 0.850 | 0.100 | 0.959 | 0.011 | 0.954 | 0.014 | 0.929 | 0.937 | 0.956 | 0.970 | 0.974 |
| **miscellaneous** | | | | | | | | | | | | |
| invest. adj. cost. | Normal | 4.000 | 1.500 | 6.261 | 1.029 | 6.221 | 1.025 | 4.620 | 4.930 | 6.177 | 7.585 | 7.986 |
| hsehold. rel.risk aversion | Normal | 1.000 | 0.375 | 2.083 | 0.285 | 1.956 | 0.282 | 1.485 | 1.594 | 1.960 | 2.311 | 2.413 |
| consumption habit | beta | 0.700 | 0.100 | 0.348 | 0.048 | 0.388 | 0.055 | 0.302 | 0.320 | 0.387 | 0.459 | 0.483 |
| labour utility | Normal | 2.000 | 0.750 | 0.892 | 0.648 | 1.267 | 0.597 | 0.459 | 0.583 | 1.179 | 2.070 | 2.382 |
| calvo employment | beta | 0.500 | 0.100 | 0.650 | 0.043 | 0.650 | 0.038 | 0.585 | 0.602 | 0.652 | 0.698 | 0.709 |
| indexation wage | beta | 0.500 | 0.250 | 0.463 | 0.210 | 0.511 | 0.191 | 0.190 | 0.257 | 0.513 | 0.764 | 0.827 |
| indexation price | beta | 0.500 | 0.250 | 0.093 | 0.077 | 0.113 | 0.065 | 0.024 | 0.035 | 0.103 | 0.201 | 0.233 |
| cap. util. adj. cost | beta | 0.500 | 0.250 | 0.834 | 0.113 | 0.867 | 0.070 | 0.744 | 0.772 | 0.873 | 0.955 | 0.971 |
| fixed cost | Normal | 1.250 | 0.125 | 1.515 | 0.138 | 1.482 | 0.104 | 1.313 | 1.349 | 1.482 | 1.616 | 1.654 |
| price markup | beta | 0.250 | 0.150 | 0.004 | 0.002 | 0.005 | 0.001 | 0.003 | 0.003 | 0.005 | 0.007 | 0.007 |
| wage markup | beta | 0.250 | 0.150 | 0.280 | 0.139 | 0.345 | 0.125 | 0.163 | 0.196 | 0.334 | 0.513 | 0.576 |
| curvature parameter | beta | 0.850 | 0.100 | | | | | | | | | |
| trend | Normal | 0.400 | 0.025 | 0.398 | 0.023 | 0.400 | 0.023 | 0.364 | 0.371 | 0.400 | 0.429 | 0.437 |
| **policy rule parameters** | | | | | | | | | | | | |
| r inflation | Normal | 1.500 | 0.100 | 1.536 | 0.083 | 1.556 | 0.081 | 1.429 | 1.454 | 1.553 | 1.661 | 1.695 |
| r d(inflation) | Normal | 0.300 | 0.100 | 0.172 | 0.045 | 0.183 | 0.046 | 0.107 | 0.124 | 0.183 | 0.242 | 0.259 |
| r lagged interest rate | beta | 0.750 | 0.050 | 0.868 | 0.017 | 0.861 | 0.018 | 0.829 | 0.837 | 0.862 | 0.883 | 0.889 |
| r output | beta | 0.125 | 0.050 | 0.114 | 0.027 | 0.106 | 0.027 | 0.066 | 0.074 | 0.104 | 0.142 | 0.153 |
| r d(output) | beta | 0.063 | 0.050 | 0.114 | 0.035 | 0.120 | 0.036 | 0.064 | 0.075 | 0.119 | 0.168 | 0.183 |

NMK model, 4-quarter price contract, $\phi = \lambda_p$ and $\epsilon \neq 0$ (Table 6 column 2)

marginal likelihood :

Laplace approximation: -468.344

Modified harmonic mean: -467.130

| | Prior distribution | | | Estimated posterior mode and mean | | | | Posterior sample based | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | type | mean | st. error | mode | st. error | mean | st. error | 5% | 10% | 50% | 90% | 95% |
| **st. dev. of the shocks** | | | | | | | | | | | | |
| productivity shock | inv. gamma | 0.250 | 2 d.f. | 0.650 | 0.088 | 0.659 | 0.089 | 0.528 | 0.552 | 0.651 | 0.778 | 0.820 |
| inflation obj. shock | inv. gamma | 0.050 | 2 d.f. | 0.130 | 0.017 | 0.130 | 0.017 | 0.102 | 0.108 | 0.129 | 0.152 | 0.159 |
| cons. pref. shock | inv. gamma | 0.250 | 2 d.f. | 0.144 | 0.024 | 0.168 | 0.033 | 0.124 | 0.132 | 0.163 | 0.211 | 0.229 |
| gov. spend. shock | inv. gamma | 0.250 | 2 d.f. | 0.347 | 0.023 | 0.350 | 0.023 | 0.314 | 0.321 | 0.349 | 0.380 | 0.389 |
| lab. supl. shock | inv. gamma | 0.250 | 2 d.f. | 0.286 | 0.115 | 0.511 | 0.289 | 0.205 | 0.234 | 0.420 | 0.934 | 1.125 |
| investment shock | inv. gamma | 0.250 | 2 d.f. | 0.250 | 0.048 | 0.249 | 0.051 | 0.177 | 0.189 | 0.243 | 0.316 | 0.342 |
| interest rate shock | inv. gamma | 0.250 | 2 d.f. | 0.130 | 0.015 | 0.137 | 0.016 | 0.112 | 0.117 | 0.136 | 0.158 | 0.165 |
| equity premium shock | inv. gamma | 0.250 | 2 d.f. | 0.538 | 0.054 | 0.536 | 0.057 | 0.442 | 0.463 | 0.535 | 0.608 | 0.628 |
| price shock | inv. gamma | 0.250 | 2 d.f. | 0.178 | 0.013 | 0.184 | 0.014 | 0.162 | 0.166 | 0.183 | 0.202 | 0.207 |
| wage shock | inv. gamma | 0.250 | 2 d.f. | 0.437 | 0.033 | 0.448 | 0.035 | 0.395 | 0.406 | 0.446 | 0.493 | 0.507 |
| **persistency parameters** | | | | | | | | | | | | |
| productivity shock | beta | 0.850 | 0.100 | 0.981 | 0.007 | 0.978 | 0.008 | 0.964 | 0.968 | 0.979 | 0.988 | 0.990 |
| cons. pref. shock | beta | 0.850 | 0.100 | 0.917 | 0.013 | 0.912 | 0.017 | 0.882 | 0.890 | 0.913 | 0.931 | 0.936 |
| gov. spend. shock | beta | 0.850 | 0.100 | 0.994 | 0.006 | 0.983 | 0.013 | 0.959 | 0.967 | 0.986 | 0.996 | 0.997 |
| lab. supl. shock | beta | 0.850 | 0.100 | 0.892 | 0.093 | 0.850 | 0.101 | 0.656 | 0.707 | 0.870 | 0.963 | 0.975 |
| investmnet shock | beta | 0.850 | 0.100 | 0.996 | 0.004 | 0.990 | 0.008 | 0.975 | 0.980 | 0.992 | 0.998 | 0.998 |
| price shock | beta | 0.850 | 0.100 | 0.539 | 0.056 | 0.544 | 0.060 | 0.443 | 0.467 | 0.547 | 0.617 | 0.639 |
| wage shock | beta | 0.850 | 0.100 | 0.956 | 0.013 | 0.954 | 0.015 | 0.927 | 0.935 | 0.956 | 0.971 | 0.975 |
| **miscellaneous** | | | | | | | | | | | | |
| invest. adj. cost. | Normal | 4.000 | 1.500 | 6.327 | 1.032 | 6.376 | 1.034 | 4.753 | 5.078 | 6.335 | 7.733 | 8.140 |
| hsehold. rel.risk aversion | Normal | 1.000 | 0.375 | 2.111 | 0.261 | 1.983 | 0.283 | 1.510 | 1.618 | 1.987 | 2.344 | 2.447 |
| consumption habit | beta | 0.700 | 0.100 | 0.356 | 0.049 | 0.388 | 0.053 | 0.303 | 0.321 | 0.386 | 0.458 | 0.480 |
| labour utility | Normal | 2.000 | 0.750 | 1.124 | 0.614 | 1.250 | 0.578 | 0.440 | 0.565 | 1.178 | 2.033 | 2.313 |
| calvo employment | beta | 0.500 | 0.100 | 0.643 | 0.040 | 0.641 | 0.039 | 0.574 | 0.590 | 0.643 | 0.690 | 0.702 |
| indexation wage | beta | 0.500 | 0.250 | 0.562 | 0.210 | 0.533 | 0.193 | 0.205 | 0.274 | 0.537 | 0.788 | 0.846 |
| indexation price | beta | 0.500 | 0.250 | 0.121 | 0.096 | 0.156 | 0.085 | 0.034 | 0.052 | 0.146 | 0.274 | 0.313 |
| cap. util. adj. cost | beta | 0.500 | 0.250 | 0.812 | 0.080 | 0.844 | 0.073 | 0.719 | 0.747 | 0.848 | 0.938 | 0.958 |
| fixed cost | Normal | 1.250 | 0.125 | | | | | | | | | |
| price markup | beta | 0.250 | 0.150 | 0.489 | 0.098 | 0.530 | 0.100 | 0.370 | 0.403 | 0.526 | 0.660 | 0.701 |
| wage markup | beta | 0.250 | 0.150 | 0.288 | 0.117 | 0.332 | 0.122 | 0.152 | 0.184 | 0.319 | 0.495 | 0.551 |
| curvature parameter | beta | 0.850 | 0.100 | 0.986 | 0.004 | 0.984 | 0.006 | 0.973 | 0.977 | 0.984 | 0.990 | 0.991 |
| trend | Normal | 0.400 | 0.025 | 0.399 | 0.022 | 0.395 | 0.022 | 0.359 | 0.367 | 0.395 | 0.424 | 0.432 |
| **policy rule parameters** | | | | | | | | | | | | |
| r inflation | Normal | 1.500 | 0.100 | 1.535 | 0.081 | 1.552 | 0.080 | 1.424 | 1.452 | 1.551 | 1.655 | 1.686 |
| r d(inflation) | Normal | 0.300 | 0.100 | 0.175 | 0.044 | 0.185 | 0.046 | 0.110 | 0.126 | 0.184 | 0.243 | 0.260 |
| r lagged interest rate | beta | 0.750 | 0.050 | 0.866 | 0.017 | 0.862 | 0.018 | 0.830 | 0.838 | 0.863 | 0.884 | 0.890 |
| r output | beta | 0.125 | 0.050 | 0.115 | 0.026 | 0.112 | 0.027 | 0.070 | 0.078 | 0.110 | 0.147 | 0.158 |
| r d(output) | beta | 0.063 | 0.050 | 0.117 | 0.034 | 0.128 | 0.037 | 0.070 | 0.081 | 0.126 | 0.176 | 0.192 |