Macroeconomic and interest rate volatility under alternative monetary operating procedures

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Abstract

The current financial crisis has led to the emergence of large risk premia in interbank markets and to a widening spread between risky and riskless interest rates. This suggests that it matters what rate a central bank chooses for setting monetary policy. We examine the volatility of macroeconomic variables and of the yield curve if monetary policy is formulated in terms of the central bank's riskless repo rate or in terms of a risky short-term or long-term money market rate. When financial shocks are large, gearing policy to money market rates yields lower macroeconomic volatility. If policy is set under commitment, using a longer-term market rate appears to yield the lowest macroeconomic volatility, while relying on a short-term market rate for formulating policy seems most attractive under discretion.

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1 Introduction

Monetary operating procedures have attracted little attention in the literature on monetary policy.[1] One reason for this is that different short-term interest rates typically are closely related, so that it does not matter exactly how a central bank implements policy. In the current financial crisis, however, the linkages between different short-term rates have changed fundamentally, raising the issue how alternative approaches to implement monetary policy impact on the economy.[2] In this paper, we examine the effects of various choices of operating procedure on the volatility of inflation, the output gap and the term structure of money market rates.

We distinguish between the policy rate and the repo rate. The *policy rate* is the interest rate that is used to define the monetary policy stance. This rate is typically changed in a series of subsequent steps of 25 basis points or multiples thereof. The *repo rate* is the instrument employed by the central bank to put its monetary policy decisions into effect. The policy rate and the repo rate are often one and the same, but they need not be. As we see below, this potential difference is what motivates this paper.

We focus on three operating procedures that are intended to capture the main features of those of the Bank of England, the Federal Reserve and the Swiss National Bank. The Bank of England formulates monetary policy in terms of Bank Rate, which is the repo rate at which the Bank is willing, against eligible collateral, to lend funds to commercial banks. The standard maturity of these repo transactions, which are essentially risk free, is one week. In the case of the Federal Reserve, monetary policy is formulated in terms of the federal funds rate, which is the rate at which commercial banks lend uncollateralised overnight funds to one another. Thus, the US monetary policy rate is a market rate at the very short end of the maturity spectrum that incorporates default risk. The Federal Reserve influences the level of the federal funds rate in its monetary policy implementation through repo transactions with mainly overnight and two-week maturity. The Swiss National Bank, finally, formulates monetary policy in terms of a target range for three-

[1] Exceptions are Bindseil [5] and Borio [7]. Borio and Nelson [8] discuss monetary operations during the financial crisis.

[2] For an analysis of the crisis, see e.g. Buiter [9] and Taylor and Williams [44].

month libor, which is a rate for uncollateralised funds on the interbank market.[3] The Swiss National Bank implements its policy using repos of typically one week maturity.[4]

Figure 1 shows the policy interest rates of the three central banks and one-week and three-month libor.[5] With the onset of the crisis in August 2007, interest rates became more variable in all three economies. Volatility was highest in the last quarter of 2008, after which point central banks' attempts to calm markets were increasingly successful.[6] One interesting feature is that after the onset of the crisis, three-month rates in GBP and USD displayed initially more volatility than one-week rates. For CHF rates, the opposite is true. It thus appears that the term structure is calmest at the maturity of the policy interest rate.[7] Depending on whether the business cycle responds to a short-term or a long-term rate, a risk-free or a risky rate, the choice of the policy rate thus seems important for the transmission of financial market shocks to the real economy. This suggests that conventional macroeconomic models that use only one interest rate to describe the economy fail to capture important aspects of the current financial crisis.

This paper studies the volatility of inflation and the output gap as well as of the yield curve in a setup that distinguishes between the central bank's repo rate and money market interest rates. Monetary policy can be formulated with respect to either the central bank's repo rate, a short-term or a long-term money market rate. The optimal rule for the repo rate, which is the only variable the central bank has full control over, differs across operating procedures since central banks smooth their respective policy rate and thus

[3] In a similar setup, the Bank of Canada had a target for the three-month Treasury bill rate until January 1996. See Borio [7].

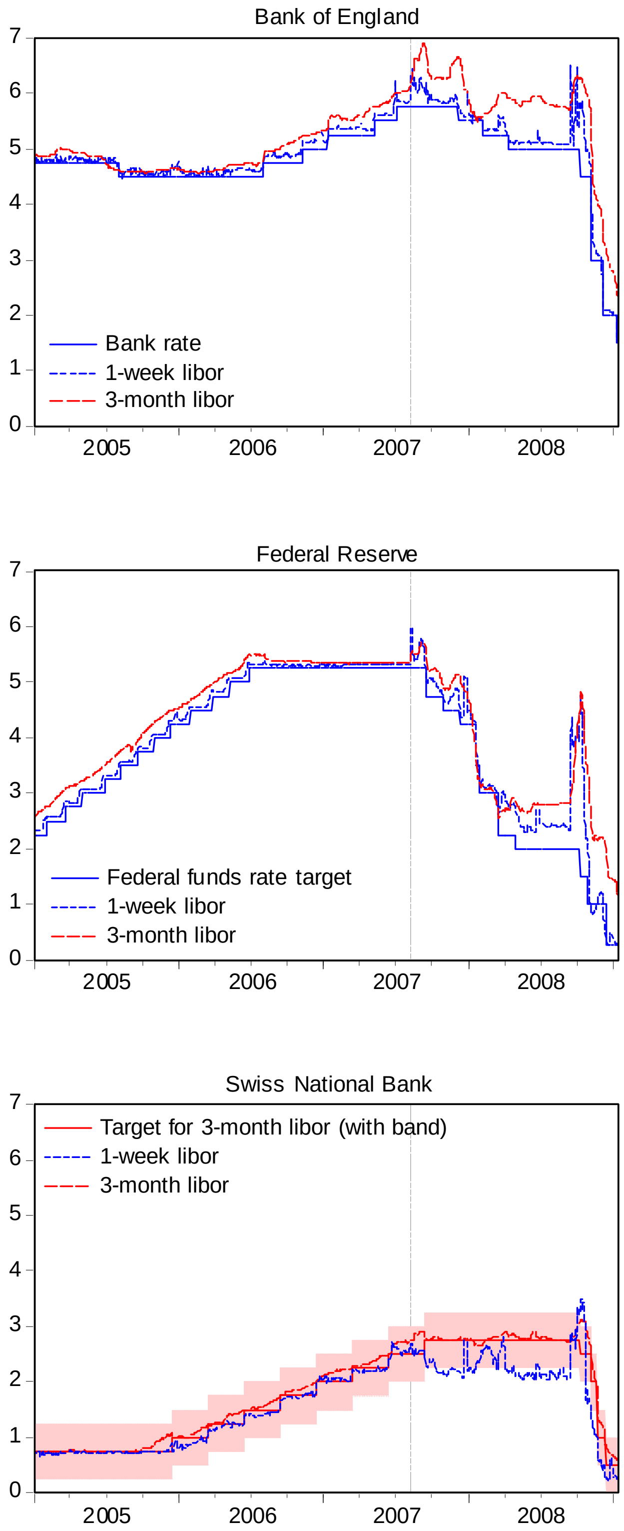
[4] On the implementation of monetary policy in Switzerland, see Jordan and Kugler [24] and Jordan, Ranaldo and Söderlind [25]. Taylor [43] discusses the SNB's policy during the financial crisis.

[5] For the SNB the mid-point of the target band, which is typically communicated in the policy decision, is also shown.

[6] Bernanke [3] distinguishes between three sets of measures central banks have been using to achieve this: the traditional provision of liquidity to sound banks; the direct provision of liquidity to key credit market participants; and the purchase of long-term securities. For a detailed discussion of central bank measures, see also the report by the Committee on the Global Financial System [10].

[7] Flemming [19] refers to the phenomenon that the yield curve is most stable around the rate the central bank is perceived to control as "pivotting."

Figure 1: Interest rates January 2005 to January 2009



Note: Interest rate data. The onset of the crisis in August 2007 marked by dashed line.

have slightly different objectives.[8] We find that the choice of operating procedure does not matter much for macroeconomic volatility when markets are tranquil, though yield curve volatility is smaller for operating procedures relying on a short-term rate. When financial market are in turmoil, macroeconomic volatility is smallest if the central bank formulates monetary policy in terms of the long-term money market rate.

Several recent papers also distinguish between different interest rates. Cúrdia and Woodford [12] model the spread between borrowing and lending rates and consider how the central bank might want to react to movements in the credit spread. Goodfriend and McCallum [22] model an interbank policy interest rate, a risk-free rate and collateralised and uncollateralised market rates in an economy with a banking sector and discuss the responses of the different rates to shocks. Finally, Martin and Milas [29] consider a spread between the monetary policy rate and an economically relevant borrowing rate and discuss how monetary policy in the UK appears to have responded to movements in market rates.

A number of older papers assume that the interest rate controlled by the central bank does not matter directly in the IS curve. Eijffinger, Schaling and Verhagen [14], Fendel [18], Lansing and Trehan [27] and Svensson [39] let a longer-term rate, which obeys the expectations hypothesis, matter for the output gap, and solve for an optimal rule for the shorter-term rate. Conversely, Kulish [26] and McGough, Rudebusch and Williams [30] let the shorter-term rate enter the IS curve and analyse different reaction functions for the longer-term interest rate, which, however, are not derived optimally.

It should be noted that using the short-term rate in the IS curve is compatible with the New Keynesian literature. By contrast, policymakers often argue that a longer-term rate, which is given by the expected future path of the short-term rate and a time-varying risk premium, impacts on economic activity.[9] For instance, Federal Reserve Chairman

[8] The literature has offered a number of explanations for interest rate smoothing, ranging from reducing financial market volatility (Blinder [6], Cukierman [11] and Goodfriend [21]) and a larger impact on expectations (Goodhart [23] and Woodford [45]) to uncertainty (Orphanides [32], Rudebusch [33], Sack [36], Sack and Wieland [37] and Smets [38]), omitted variables (Rudebusch [34], English, Nelson and Sack [15] and Gerlach-Kristen [20]) and reputation concerns on policymakers' part (Goodhart [23]).

[9] Rudebusch, Sack and Swanson [35] present empirical evidence for a link from decreased term premia to higher economic activity. This could be seen as evidence that longer-term, rather than shortest-term

Ben Bernanke states that

> "[t]he Fed controls very short-term interest rates quite effectively, but the long-term rates that really matter for the economy depend not on the current short-term rate but on the whole trajectory of future short-term rates expected by market participants." (Bernanke [4], p. 6)

In this paper, we use the standard New Keynesian model as baseline case. As a robustness check, we also consider a model that uses a longer-term rate in the IS curve and show that the results are essentially unchanged.

The paper is organised as follows. Section 2 introduces the model. In contrast to the literature, we model the risk premia on market interest rates as dynamic variables rather than as constants. We present for each of the three operating procedures the optimal rule under commitment for the repo rate, which is the rate the central bank controls and which it uses when implementing its monetary policy decisions. In solving the model a technical difficulty arises if the central bank formulates monetary policy in terms of the longer-term market rate since this rate depends on the path of expected future repo rates. Thus, minimising the central bank's loss function requires knowledge of the future path of the optimal repo rule which, however, implies knowledge about the solution of the optimisation problem. We present an algorithm that deals with this problem. Section 3 simulates the model using estimated empirical pre-crisis and crisis dynamics for the risk premia. We present the optimal repo rules for the different operating procedures, impulse responses and measures of the macroeconomic and the yield curve volatility. Section 4 presents robustness tests. We study how the volatilities depend on the assumed degree of forward-lookingness in the economy, who they change if monetary policy is discretionary and what happens if a longer-term market rate matters in the IS curve. Section 5 concludes.

interest rates matter for the output gap.

2 The model

We study optimal monetary policy implementation in a standard model consisting of a New Keynesian Phillips curve and a consumption Euler equation. We first describe the economy and then discuss the monetary policy problem and derive the optimal implementation rule for the repo rate, which is the interest rate the central bank controls.

2.1 The economy

The hybrid New Keynesian Phillips curve is given by

$$\pi_t = a_\pi E_t \pi_{t+1} + (1 - a_\pi)\pi_{t-1} + a_y y_t + u_{\pi,t}, \tag{1}$$

where π_t is the inflation rate, y_t the output gap, a_π a parameter reflecting the degree of forward-lookingness in the price setting behaviour of firms and a_y a composite parameter capturing the discount rate and the frequency of price adjustments (see Woodford [46]). The exogenous inflation shock, $u_{\pi,t}$, is assumed to follow an AR(1) process

$$u_{\pi,t} = \rho_\pi u_{\pi,t-1} + \sigma_\pi e_{\pi,t} \tag{2}$$

with $0 < \rho_\pi < 1$ and $e_{\pi,t} \sim N(0,1)$.

The log-linearised consumption Euler equation is given by

$$y_t = b_y E_t y_{t+1} + (1 - b_y) y_{t-1} - b_r (i_{1,t} - E_t \pi_{t+1} - \mu_{1,r}) + u_{y,t}, \tag{3}$$

where $i_{1,t}$ denotes the nominal money market interest rate with maturity of one period, $\mu_{1,r}$ the equilibrium one-period market rate, which is to be defined below, and $u_{y,t}$ is an exogenous demand shock which evolves according to

$$u_{y,t} = \rho_y u_{y,t-1} + \sigma_y e_{y,t} \tag{4}$$

with $0 < \rho_y < 1$ and $e_{y,t} \sim N(0,1)$. We assume that the central bank and the private sector have access to the same information about the economy and form rational expectations.

The money market interest rate $i_{1,t}$ deviates from the one-period repo interest rate i_t used by the central bank for the implementation of monetary policy by a risk premium $\theta_{1,t}$, so that

$$i_{1,t} = i_t + \theta_{1,t}. \tag{5}$$

This risk premium reflects counterparty risk that arises in a market transaction compared to a transaction with the central bank.[10] It is assumed to follow an AR(1) process of the form

$$\theta_{1,t} = \theta_1 + \rho_1 \theta_{1,t-1} + \varepsilon_{1,t}, \tag{6}$$

where $\varepsilon_{1,t} \sim N(0, \sigma_1^2)$. Two points are worth noting. First, the AR(1) structure imples that the risk premium can turn negative. While this is unrealistic, it keeps the model compact.[11] Second, the risk premium does not depend on the state of the economy. This assumption as well is made for tractability.

Finally, longer-term money market interest rates of maturity j are under the expectations hypothesis given by

$$i_{j,t} = \frac{1}{j} E_t \sum_{k=0}^{j-1} i_{t+k} + \tau_j + \theta_{j,t}, \tag{7}$$

where τ_j denotes a constant term premium and $\theta_{j,t}$ the j-period risk premium, which we assume follows

$$\theta_{j,t} = \theta_j + \rho_j \theta_{j,t-1} + \varepsilon_{j,t} \tag{8}$$

with $\varepsilon_{j,t} \sim N\left(0, \sigma_j^2\right)$.[12] The risk innovations $\varepsilon_{\cdot,t}$ are correlated across maturities. We define their covariance matrix $\varepsilon_t \varepsilon_t'$ as $C e_t e_t' C'$, where e_t is a vector of white noise errors and CC' contains the variances and covariances. A detailed exposition of the state space representation of the model is provided in Appendix A.

[10] Michaud and Upper [31] offer a detailed discussion of the recent evolution of the risk premium in a number of economies and analyse what factors drive the premium. They find that liquidity seems to matter at high frequencies, while default risk appears to have a slower impact.

[11] Similarly, we do not impose a zero lower bound on interest rates.

[12] An alternative to assuming the validity of the expectations hypothesis is to derive expressions for longer-term interest rates in a micro-based model with frictions. See Amisano and Tristani [1] and Atkeson and Kehoe [2].

2.2 The monetary policy problem

Monetary policy is conducted under commitment and in a timeless perspective (see Svensson and Woodford [42] and Woodford [46]).[13] We want to solve the model for the optimal rule for the repo rate i_t, which is the only interest rate the central bank has full control over.

The central bank's period loss function is given by

$$L_{p,t} = \frac{1}{2} Y_t' \Lambda_p Y_t, \tag{9}$$

and the intertemporal loss function by

$$\pounds_{p,0} = E_0 \sum_{t=0}^{\infty} (1-\delta)\delta^t L_{p,t}, \tag{10}$$

where Y_t are the goal variables, δ is the discount factor and Λ_p the matrix of goal weights, which differs between operating procedures. Under the first procedure, monetary policy is formulated with the repo rate i_t. We refer to this approach as the repo rate operating procedure (RR procedure). Alternatively, policy can be formulated with the one-period money market rate $i_{1,t}$ but implemented with i_t. This we call the short-term money market rate (SMR) procedure. Under the long-term money market rate (LMR) procedure, finally, policy is formulated with the three-period money market rate $i_{3,t}$ and again implemented with i_t.

The central bank minimises variations in inflation and the output gap under all three operating procedures. Moreover, policymakers smooth the monetary policy rate, i.e. i_t, $i_{1,t}$ or $i_{3,t}$, respectively. The set of all potential goal variables then is

$$Y_t = \left[\begin{array}{ccccc} \pi_t & y_t & \Delta i_t & \Delta i_{1,t} & \Delta i_{3,t} \end{array} \right]'.$$

The off-diagonal elements of Λ_p are zero for all operating procedures, while the diagonal is given by $\left[\begin{array}{ccccc} \lambda_\pi & \lambda_y & \lambda_i & 0 & 0 \end{array} \right]$ under the RR procedure, by $\left[\begin{array}{ccccc} \lambda_\pi & \lambda_y & 0 & \lambda_i & 0 \end{array} \right]$ under the SMR procedure and by $\left[\begin{array}{ccccc} \lambda_\pi & \lambda_y & 0 & 0 & \lambda_i \end{array} \right]$ under the LMR procedure, where λ_π is the weight attached to the goal of stabilising inflation, λ_y the weight for stabilising the output gap and λ_i the weight for interest rate smoothing.

[13] Section 4.2 discusses as a robustness check policy under discretion.

Thus, the only difference between the procedures is the policy rate and the fact that this rate is smoothed. Given the observation that monetary policy in practice tends to be changed gradually with no obvious attempts being made to smooth movements of interest rates at other maturities, this assumption is not far-fetched.

To determine the optimal rule for the central bank's repo rate, we define a dual period loss function $\widetilde{\widetilde{L}}_{p,t}$, which depends on $L_{p,t}$ and the structure of the economy as given by equations (1) to (8), and rewrite the central bank's overall loss function as a dual saddlepoint problem of the form

$$\max_{\{\gamma_t\}_{t\geq 0}} \min_{\{\pi_t, y_t, i_t\}_{t\geq 0}} E_0 \sum_{t=0}^{\infty} (1-\delta)\delta^t \widetilde{\widetilde{L}}_{p,t}, \tag{11}$$

where $\gamma_t = [\ \gamma_t^{PC} \quad \gamma_t^{IS}\ ]'$ are the Lagrange multipliers for equations (1) and (3). This function can be optimised using the standard linear quadratic regulator approach. Appendix A discusses this in detail.

Before proceeding, a technical difficulty with the LMR procedure should be noted. Here, policymakers smooth the policy rate $i_{3,t}$, which depends on the expected path of the repo rate over the periods t to $t+2$. The repo rate, however, is the variable for which we solve the optimisation problem. Thus, in defining the loss function for the LMR procedure, we should know the optimal reaction function for i_t that minimises this very loss function. We solve this problem by guessing an initial reaction function and then iterating until convergence is achieved.[14]

3 Simulations

In this section, we simulate the model to assess how the optimal repo rule and the volatility of the macroeconomy and the yield curve differ under the three operating procedures. We also study how the results change if risk premia follow the pattern observed during the financial crisis before turning to robustness tests in Section 4.

[14] Alternatively, one could proxy $i_{3,t}$ in a first-difference equation as an infinitely long rate with rapidly declining weights, which would allow the application of standard solution techniques. We thank Paul Söderlind for making this point.

We assume that the parameters describing the macroeconomy and central bank's preferences are unaffected by the financial crisis. The coefficients in the Phillips curve, the consumption Euler equation and their respective shock processes are set to $a_\pi = b_y = 0.8$, $a_y = 0.2$, $b_r = 0.5$, $\rho_\pi = \rho_y = 0.8$ and $\sigma_\pi = \sigma_y = 0.5$. Central bank preferences are specified by the weights in the period loss function put on inflation and output gap stabilisation and policy rate smoothing $\lambda_\pi = \lambda_y = \lambda_i = 1$ and by the discount factor $\delta = 0.99$. Only the parameters capturing the dynamics of market interest rates are allowed to vary between pre-crisis and crisis simulations. In particular, we use the coefficients presented in Tables 4 and 5 in Appendix B, which show that the volatility of risk premium shocks increased and that their comovement across maturities rose. Finally, we think of the time periods as months.

3.1 Optimal repo rules

Table 1 shows the optimal repo rules for the three operating procedures and the pre-crisis and the crisis regime. The upper panel shows the reaction functions obtained if the time-series behaviour of the risk premia is given by the pre-crisis parameters. The RR procedure calls for a essentially no response to past inflation and a very small reaction to the past output gap (values of 0.00 and 0.06, respectively), but a stronger reaction to innovations in the Phillips and IS curves (0.33 and 0.74). There is considerable interest rate smoothing (0.54) and a negative response to shocks in the risk premium (-0.17), reflecting that monetary policy accommodates such shocks in part. Past movements in the one-month risk premium and the level of the three-month rate and changes in the current three-month risk premium evoke no reaction, and the response to the Lagrange multipliers for the PC and the IS curves, which capture the importance of future interest rate setting, is small (-0.01 and 0.09).

The SMR procedure yields identical reaction function coefficients except for the response to financial shocks. These are fully absorbed in the current period (reaction coefficient of -1.00). Since monetary policy is set in terms of $i_{1,t}$, it is this rate, rather than the repo rate i_t, which is smoothed. Consequently, the past risk premium has the same reaction coefficient as the past repo rate (0.54).

Table 1: Optimal reaction functions under commitment

Reaction to	π_{t-1}	y_{t-1}	i_{t-1}	$u_{\pi,t}$	$u_{y,t}$	$\theta_{1,t}$	$\theta_{3,t}$	$\theta_{1,t-1}$	γ_{t-1}^{PC}	γ_{t-1}^{IS}
Pre-crisis simulation										
RR procedure	0.00	0.06	0.54	0.33	0.74	-0.17	0.00	0.00	-0.01	0.09
SMR procedure	0.00	0.06	0.54	0.33	0.74	-1.00	0.00	0.54	-0.01	0.09
LMR procedure	0.01	0.32	0.00	1.17	1.96	-0.82	-0.07	0.00	-0.01	0.59
Crisis simulation										
RR procedure	0.00	0.06	0.54	0.33	0.74	-0.22	0.00	0.00	-0.01	0.09
SMR procedure	0.00	0.06	0.54	0.33	0.74	-1.00	0.00	0.54	-0.01	0.09
LMR procedure	0.01	0.32	0.00	1.17	1.96	-0.88	-0.10	0.00	-0.01	0.59

Note: Repo rate reaction function coefficients for different operating procedures. RR/SMR/LMR procedure stands for repo rate/short-term/long-term money market rate procedure.

The LMR procedure yields quite different coefficients in the reaction function. The intuitive reason for this is that this procedure attaches more weight to future developments since the three-month market rate depends on the expected future path of the repo rate. As a consequence, the responses to inflation, the output gap and their innovations are stronger (reaction coefficients of 0.01, 0.32, 1.17 and 1.96, respectively). Since under the LMR procedure, the three-month market rate rather than the repo rate is smoothed, the past repo rate has no impact on today's i_t. The one-month risk premium is to a large extent absorbed (-0.82), and there is a weak negative response to an increase in the three-month risk premium. Finally, the response coefficients for the Lagrange multipliers is larger than for the other procedures (0.09 and 0.59), reflecting the fact that the future course of the economy is more important for interest rate setting if a longer-term rate, which reflects the expected future path of the repo rate, is smoothed.

The lower panel of Table 1 shows the coefficients of the three optimal repo rules if the time-series pattern of the risk premia is calibrated on the crisis-period estimates. Due to certainty equivalence, only the change in the autocorrelation of the risk premia matters in the computation of the new reaction functions. Correspondingly, most reaction coefficients are unchanged. For the RR procedure, the response to the current one-month risk premium becomes stronger (-0.22 instead of -0.17). The reason for this is that the risk premium displays more autocorrelation (in Table 4 in Appendix B ρ_1 equals in the crisis sample 0.37, compared with 0.20 in the pre-crisis sample), so that an innovation in $\theta_{1,t}$ has a more protracted impact and a larger effect on inflation and the output gap. The SMR procedure uses exactly the same reaction function as before the crisis, thus still fully absorbing risk premium shocks at the one-month horizon, while the coefficient on $\theta_{1,t}$ increases in absolute terms under the LMR procedure (from -0.82 to -0.88).

3.2 Impulse responses

To evaluate how the choice of monetary operating procedure affects the macroeconomic dynamics, we present in Figure 2 impulse responses for the pre-crisis specification and in Figure 3 those for the crisis regime. The first two columns show the impact of a one-standard deviation in the inflation and the output gap shock respectively, and the last column shows the effect of a shock to the one-month risk premium. The first two plots in that column show that a financial shock affects inflation and the output gap most under the RR procedure since the repo rate is not cut enough to absorb this shock completely, implying that the one-month market rate rises. In contrast, under the SMR and the LMR procedures, absorption is complete or close to complete and as a result, the one-month market rate is essentially unaffected. The three-month rate declines on impact under these two operating procedures, reflecting the reduction in the repo rate.

The impulse responses to inflation and output gap shocks are identical under the RR and the SMR procedures, as suggested by their reaction functions. An increase in inflation in $t = 0$ is undone slowly by tighter monetary policy, which causes the output gap to turn negative. This leads to a loosening of monetary policy, and the market rates follow the path of the repo rate. The impulse responses for the LMR procedure shows that inflation deviates slightly more from target than under the two other procedures, while the output gap response is smaller. A positive output gap shock triggers a higher repo rate, which under the RR and SMR procedures causes the output gap and inflation to turn negative for some time. Under the LMR procedure, the repo rate is increased so strongly that there is virtually no response of inflation and the output gap to the shock. Under all operating procedures the three-month rate increases in response to an output gap shock, mirroring the one-month market rate.

Under the crisis regime, the risk premium shock is larger. We observe a stronger response of the repo rate to third shock under the RR procedure. The reactions under the SMR and the LMR procedures are in relation to the size of the shock essentially the same as in Figure 2. One striking difference between the two figures is that in the crisis regime, the stronger correlation between the one-month and the three-month risk shocks lets the three-month interest rate increase toghether with the one-month risk shock, which it did not in Figure 3.

3.3 Macroeconomic and interest rate volatility

The comparison of the models thus far does not allow a conclusion as to which operating procedure is preferable in terms of welfare. As a crude proxy, we therefore next compute the volatility of the macroeconomy and of the yield curve under the different operating procedures.[15] Figure 4 shows the simulated values of

$$\textit{inflation volatility}_p = \frac{1}{T}\sum_{t=1}^{T}\pi_{p,t}^2, \tag{12}$$

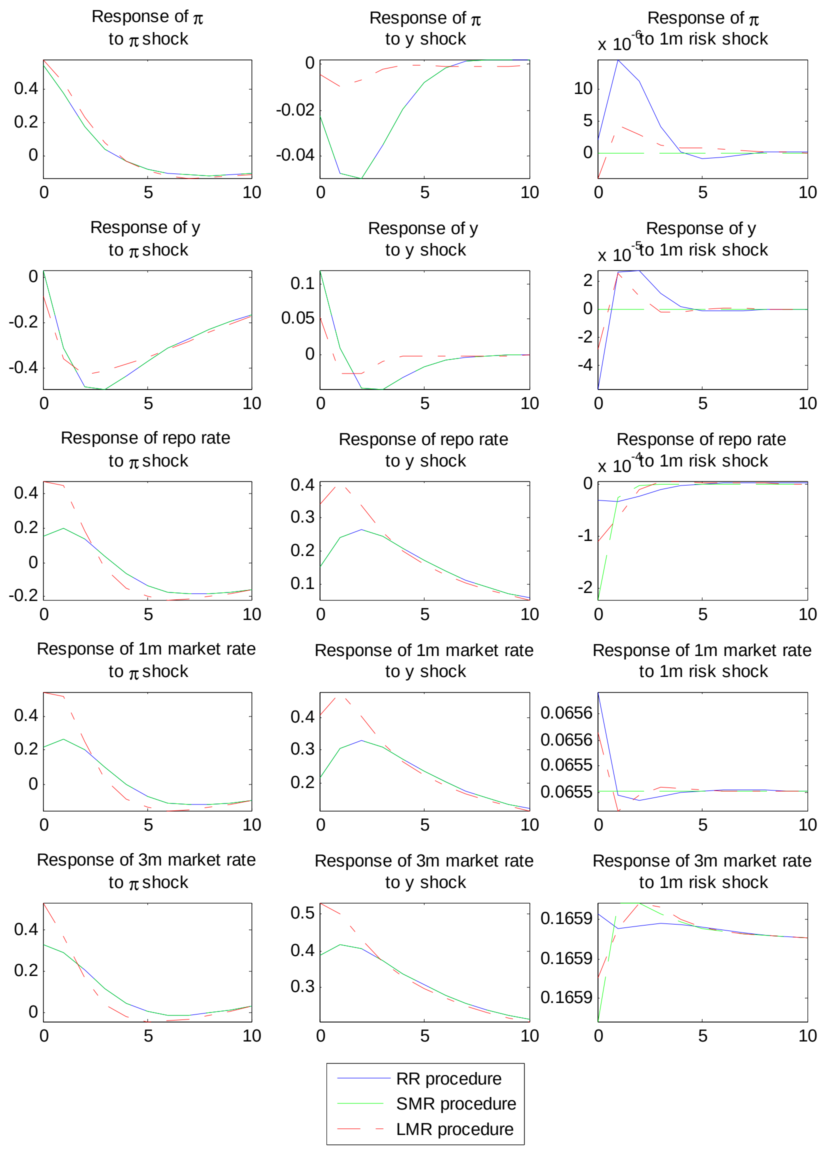
and

$$\textit{output gap volatility}_p = \frac{1}{T}\sum_{t=1}^{T}y_{p,t}^2$$

with $T = 10,000$ and $\pi_{p,t}$ and $y_{p,t}$ denoting the realisations of inflation and the output gap under operating procedure p. As can be seen in the left plot of Figure 4, the volatility of the macroeconomic variables is very similar for the two operating procedures using a one-month rate and slightly lower for the LMR procedure. This result is due to the more aggressive response to macroeconomic shocks under this procedure, which arises because policymakers smooth the three-month rate and thus are willing to adjust the one-month repo rate quickly. The downside of this approach is that the strong reactions

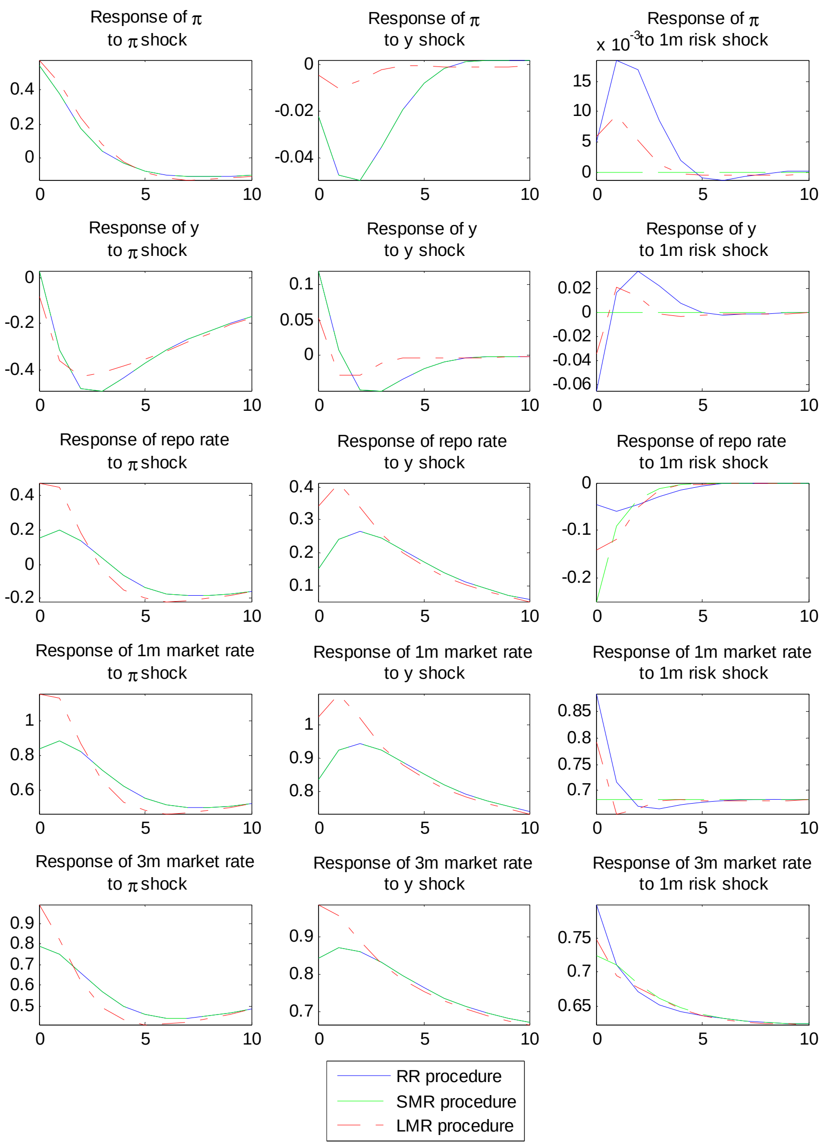
[15] We present inflation, output gap and yield curve volatility separately rather than the central bank loss function since the latter differs across procedures with respect to the interest rate smoothing term.

Figure 2: Impulse responses under commitment, pre-crisis simulation



Note: RR/SMR/LMR procedure stands for repo rate/short-term/ long-term money market rate procedure.

Figure 3: Impulse responses under commitment, crisis simulation



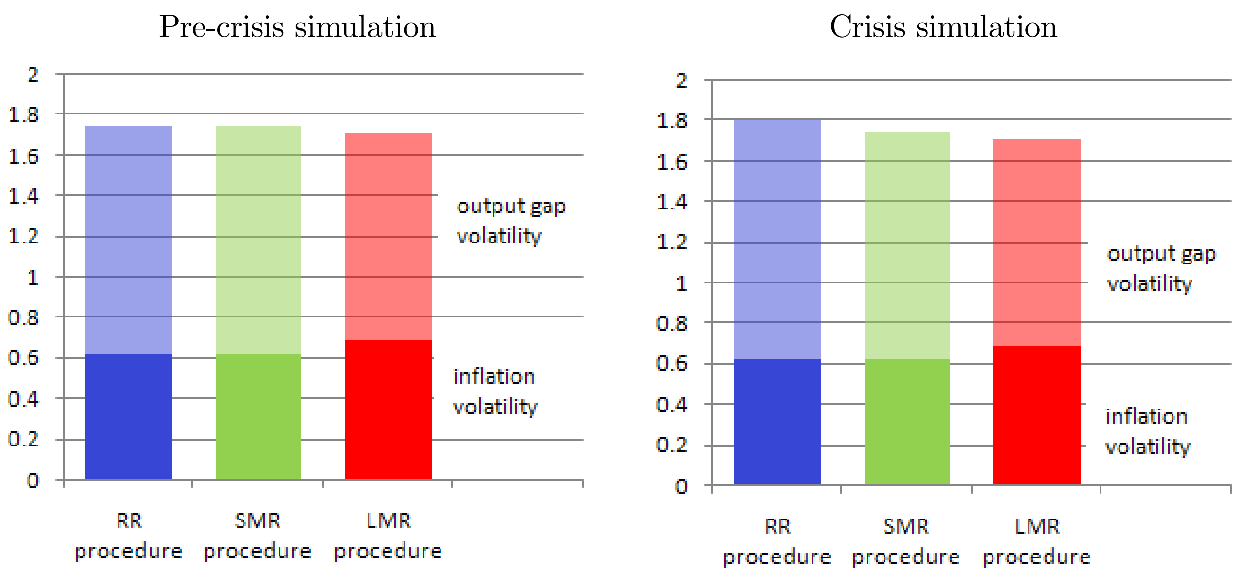
Note: RR/SMR/LMR procedure stands for repo rate/short-term/ long-term money market rate procedure.

to macroeconomic shocks cause a high volatility at the short end of the term structure. This is seen in the left plot of Figure 5, which shows the simulated volatility of market interest rates with maturity $j = 1, ..., 11$ months

$$interest\ rate\ volatility_{j,p} = \frac{1}{T} \sum_{t=1}^{T} i_{j,p,t}^2.$$

Under the RR and the SMR procedures, the yield curve displays little volatility. This pattern is due to the fact that movements in longer-term rates depend on i_t, which evolves smoothly in times of financial calm if policy is formulated with a short-term rate. The yield curve is more volatile under the LMR procedure, especially at the short end, which reflects the more aggressive adjustments of the repo rate to shocks.

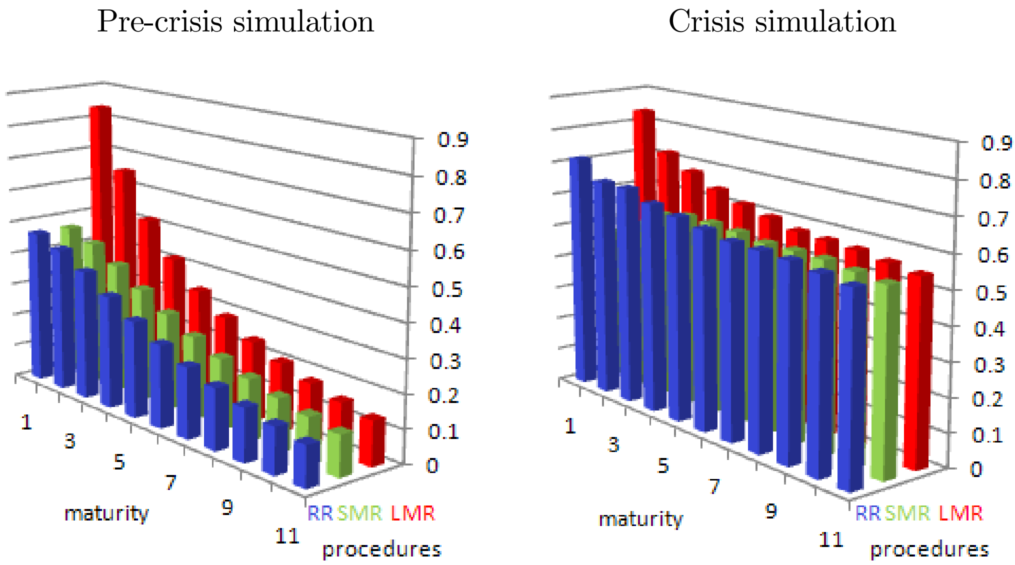
Figure 4: Macroeconomic volatility under commitment



Note: Simulations with 10,000 draws. RR/SMR/LMR procedure
stands for repo rate/short-term/long-term money market rate procedure.

The right plots of Figures 4 and 5 show the same analysis for the case in which the crisis parameters for the risk premia are used. The RR procedure yields the highest macroeconomic volatility since the now larger movements in the risk premium are not fully absorbed. The LMR procedure again performs best from a macroeconomic perspective. Interest rate volatility rises for all operating procedures and is lowest for the SMR

Figure 5: Volatility of market interest rates under commitment



Note: Simulations with 10,000 draws. RR/SMR/LMR procedure stands for repo rate/short-term/long-term money market rate procedure.

procedure. Under the RR procedure, the impact of risk shocks on the macroeconomy makes repo rate adjustments necessary and increases the volatility of the yield curve. Under the LMR procedure, short-term rates are again adjusted most aggressively.

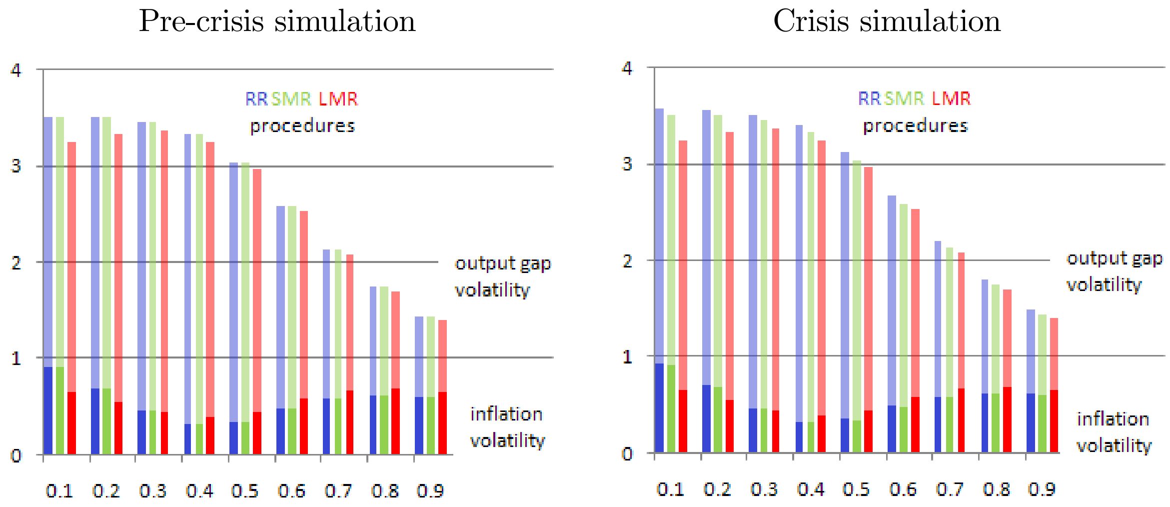
4 Robustness checks

This section studies how important a number of underlying assumptions are for the ranking of the three operating procedures. We argued above that the LMR procedure attaches more weight to the future since expected future repo rates determine today's longer-term money market rates, one natural test to consider is how the results change if we alter the degree of forward-lookingness in the economy. Another obvious question is how the conclusions are changed if policy is assumed to be discretionary rather than set under commitment. Finally, we assume that economic activity depends on a longer-term market rate, which is a widely held view in central bank circles.

4.1 Degree of forward-lookingness

As first robustness check, we study how the degree of forward-lookingness impacts on the ranking of the different operating procedures under commitment. The extent to which the economy is forward-looking is given by the coefficients a_π and b_y in the Phillips and IS curve. In the baseline simulation, we had set $a_\pi = b_y = 0.8$. We now increase these coefficients step-wise from 0.1 to 0.9. Figure 6 plots the resulting macroeconomic volatility. The left plot shows the results with the pre-crisis parameters, the right plot the losses simulated with the crisis parameters. The findings show that the results reported above are robust the the choice of forward-lookingness. The LMR procedure again performs best in terms of macroeconomic volatility in all cases, the RR and SMR procedures are equivalent in the pre-crisis simulation and the RR procedure yields more volatility in the crisis simulation.

Figure 6: Macroeconomic volatility as a function of forward-lookingness



Note: Simulations with 10,000 draws, $a_\pi = b_y$ increasing from 0.1 to 0.9. RR/SMR/LMR procedure stands for repo rate/short-term/long-term money market rate procedure.

4.2 Discretionary policy

As second robustness check, we assume that monetary policy is set under discretion rather than under commitment. Appendix C spells out the details of this version of the model.

It could be argued that discretion, which implies a reoptimisation of monetary policy at each point in time, is more attractive in times of crisis. To explore how interest rate setting and the volatility of inflation, the output gap and the yield curve are affected if policy is discretionary, we present in Table 2 the simulated reaction functions and in Figures 7 and 8 the macroeconomic and yield curve volatilities.

Table 2: Optimal reaction functions under discretion

Reaction to	π_{t-1}	y_{t-1}	i_{t-1}	$u_{\pi,t}$	$u_{y,t}$	$\theta_{1,t}$	$\theta_{3,t}$	$\theta_{1,t-1}$
Pre-crisis simulation								
RR procedure	0.05	0.10	0.33	2.55	1.16	-0.28	0.00	0.00
SMR procedure	0.05	0.10	0.33	2.55	1.16	-1.00	0.00	0.33
LMR procedure	0.14	0.37	0.00	5.29	2.94	-0.94	0.00	0.00
Crisis simulation								
RR procedure	0.05	0.10	0.32	2.55	1.16	-0.37	0.00	0.00
SMR procedure	0.05	0.10	0.32	2.55	1.16	-1.00	0.00	0.33
LMR procedure	0.14	0.37	0.00	5.29	2.94	-0.97	-0.01	0.00

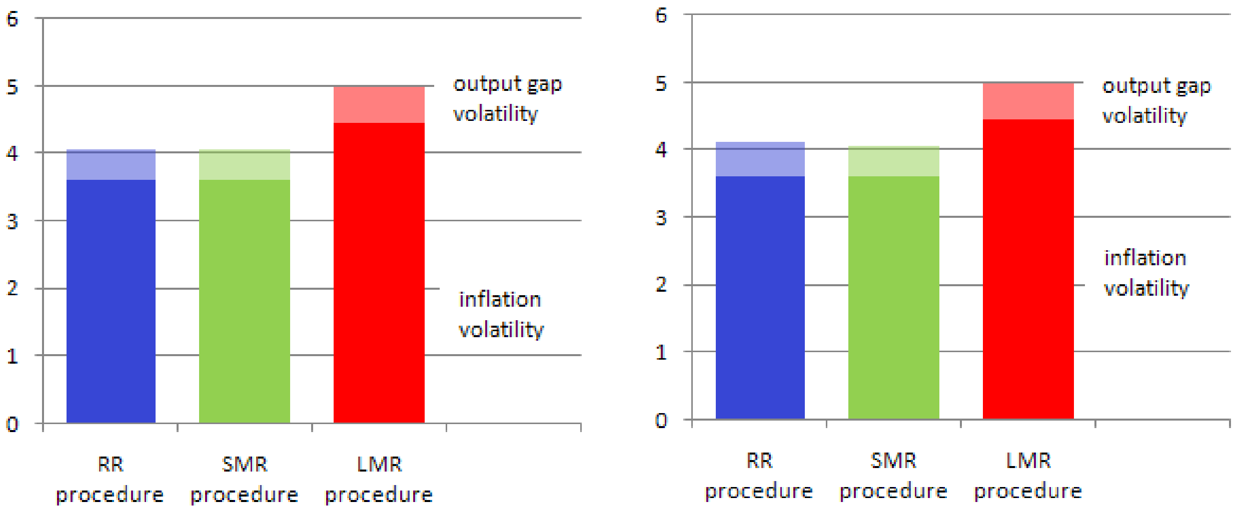
Note: Repo rate reaction function coefficients for different operating procedures. RR/SMR/LMR procedure stands for repo rate/short-term/long-term money market rate procedure.

Compared with Table 1, policy again responds little to past inflation and the past output gap. The reaction to the current inflation and output gap shock is larger. Under the SMR procedure, the repo rate again absorbs movements in the one-month risk premium, and the LMR procedure now does almost the same. The response under the RR procedures also is stronger than under commitment. Interest rate smoothing, by contrast, is less pronounced. This shift in coefficient reflects the stabilisation bias discussed in Dennis and Söderström [13] and Woodford [45]. In an economy with forward-looking agents, interest rate smoothing stabilises expectations and thereby reduces overall macroeconomic volatility. If policy is discretionary, policymakers do not follow this optimal gradual response but stabilise the output gap more and inflation less.

This is clearly visible in Figure 7, which also shows that the overall macroeconomic

Figure 7: Macroeconomic volatility under discretion

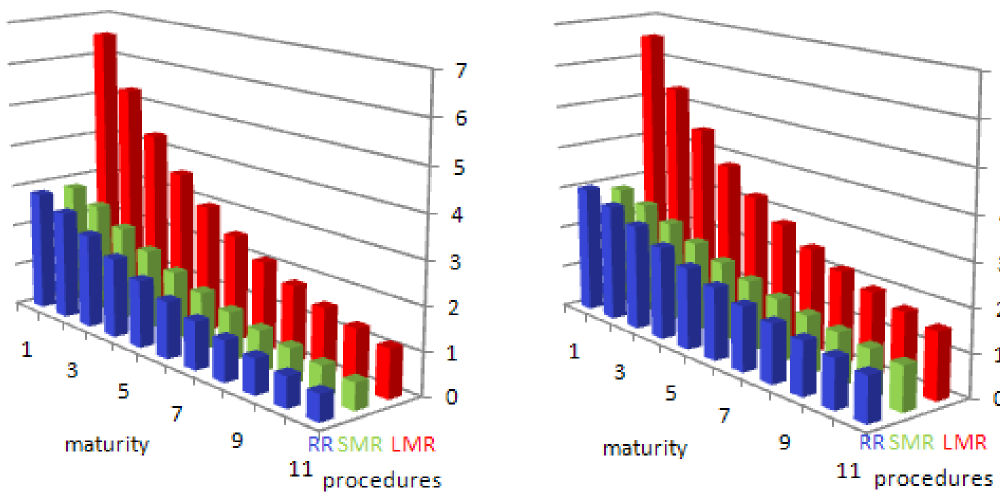
Pre-crisis simulation Crisis simulation



Note: Simulations with 10,000 draws. RR/SMR/LMR procedure stands for repo rate/short-term/long-term money market rate procedure.

Figure 8: Volatility of market interest rates under discretion

Pre-crisis simulation Crisis simulation



Note: Simulations with 10,000 draws. RR/SMR/LMR procedure stands for repo rate/short-term/long-term money market rate procedure.

volatility is higher than in the commitment case presented in Figure 4. The LMR procedure yields the highest volatility. This result seems due to the fact that under discretion, policymakers are less well able to impact on the public's expectations. Since those matter most for a policy that uses a longer-term interest rate, such an approach is unattractive in this setup.[16] Interest rate volatility, shown in Figure 8, is much higher under discretion than under commitment, increasing roughly tenfold compared with Figure 5.

4.3 Longer-term rate in IS curve

As third robustness check, we replace the one-period market rate in the IS curve with the three-period rate. The adjustments necessary in the model to capture this change, which many policymakers would argue describes the transmission mechanism better, is discussed in detail in Appendix D.[17] Table 3 shows the reaction coefficients and Figure 9 plots the macroeconomic volatility simulated for this version of the model.

Compared with the reaction function of the baseline model, we find that all three procedures respond to shocks in the three-month risk premium. Shocks in the one-month risk premium triger a response only under the SMR procedure.[18]

Macroeconomic volatility is for all procedures higher than if the one-month rate enters in the IS curve. In the pre-crisis simulation, the LMR procedure yields the lowest volatility, while the SMR procedure is marginally more successful in the crisis simulation.

[16] If the economy is very backward looking, so that expectations do not matter much, the LMR procedure also performs best under discretion.

[17] We also performed robustness checks using the eleven-month rate in the IS curve and the average of all rates from one to eleven months maturity. The conclusions are the same, and details are available upon request.

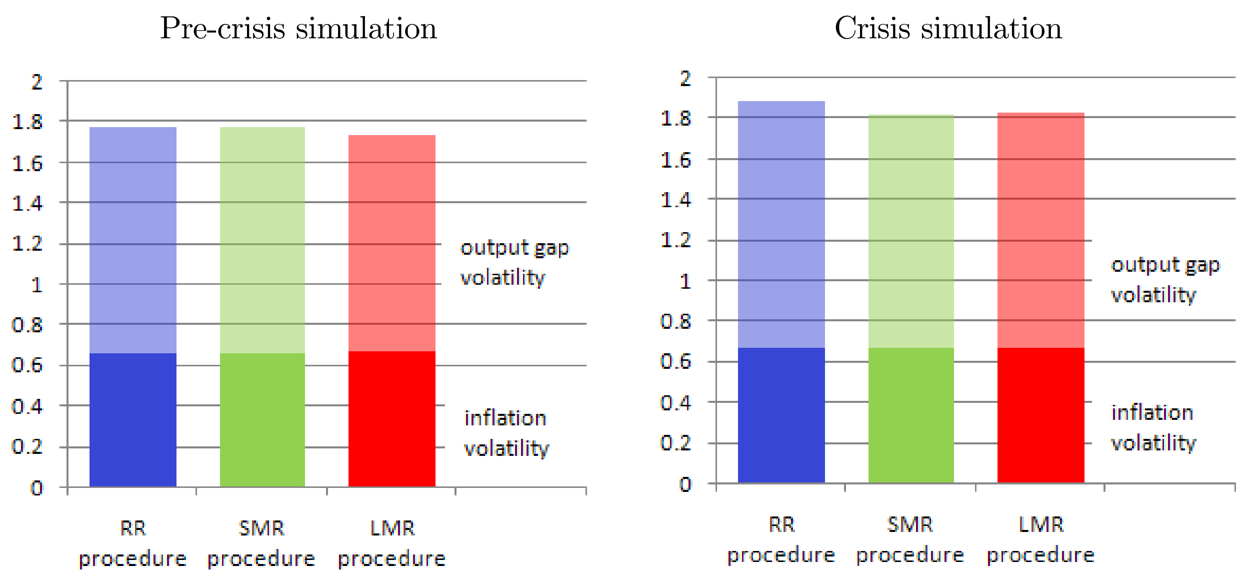
[18] We now also observe interest rate smoothing of the repo rate for the LMR procedure. This, however, is due to the assumption that a small weight is attached also to this goal, an assumption we need to make to achieve convergence in the simulations and that is discussed in Appendix D

Table 3: Optimal reaction functions with 3-month rate in IS curve

Reaction to	π_{t-1}	y_{t-1}	i_{t-1}	$u_{\pi,t}$	$u_{y,t}$	$\theta_{1,t}$	$\theta_{3,t}$	$\theta_{1,t-1}$	γ^{PC}_{t-1}	γ^{IS}_{t-1}
					Pre-crisis simulation					
RR procedure	0.00	0.05	0.68	0.08	0.82	0.00	-0.33	0.00	0.00	0.07
SMR procedure	0.00	0.05	0.68	0.08	0.82	-0.95	-0.33	0.68	0.00	0.07
LMR procedure	0.00	0.17	0.54	0.23	1.44	0.00	-0.62	0.00	0.00	0.30
					Crisis simulation					
RR procedure	0.00	0.05	0.68	0.08	0.82	0.00	-0.29	0.00	0.00	0.07
SMR procedure	0.00	0.05	0.68	0.08	0.82	-0.92	-0.29	0.68	0.00	0.07
LMR procedure	0.00	0.17	0.54	0.23	1.44	0.00	-0.55	0.00	0.00	0.30

Note: Repo rate reaction function coefficients for different operating procedures. RR/SMR/LMR procedure stands for repo rate/short-term/long-term money market rate procedure.

Figure 9: Macroeconomic volatility with 3-month rate in IS curve



Note: Simulations with 10,000 draws assuming commitment. RR/SMR/LMR procedure stands for repo rate/short-term/long-term money market rate procedure.

5 Conclusions

This paper studies how the choice of operating procedure influences the volatility of the macroeconomy and the yield curve during times of financial calm and crisis. We consider one operating procedure that formulates policy in terms of the central bank's repo rate, which is essentially free of default risk, one that uses a short-term money market rate and one that sets policy with reference to a longer-term money market rate. The results suggest that the performance of the three operating procedures in terms of macroeconomic volatility does not differ greatly when financial markets are tranquil, but that the long-rate approach causes more interest rate volatility at the short end of the yield curve. Macroeconomic performance differs when financial shocks are large. In that situation, operating procedures that formulate policy in terms of a market rate yield better macroeconomic outcomes than a setup that uses the central bank's repo rate for policy formulation. If policy is set under commitment, using a longer-term market rate appears to be the best strategy, while relying on a short-term market rate for formulating policy seems most attractive under discretion.

A State space representation

A.1 Setting up the model

To cast the model presented in Section 2 in state space form, we define a vector X_t of predetermined variables as

$$X_t = [\ 1 \quad \pi_{t-1} \quad y_{t-1} \quad i_{t-1} \quad u_{\pi,t} \quad u_{y,t} \quad \theta_{1,t} \quad \theta_{3,t} \quad \theta_{1,t-1} \quad i_{3,t-1}\ ]',$$

where $n_X = 10$.[19] Moreover, we define an expanded vector of state variables $\widetilde{X}_t = [\ X_t \quad \Xi_{t-1}\ ]'$, where $\Xi_t = [\ \Xi_t^{PC} \quad \Xi_t^{IS}\ ]'$ contains the Lagrange multipliers for the Phillips and the IS curve. We then write the equation for the predetermined variables as

$$\widetilde{X}_{t+1} = \widetilde{A}_{10} + \widetilde{A}_{11}\widetilde{X}_t + \widetilde{A}_{12}x_t + \widetilde{B}_1 i_t + Ce_{t+1},$$

where $x_t = [\ \pi_t \quad y_t\ ]'$, $n_{\widetilde{X}} = 12$, $n_x = 2$, $n_i = 1$,

$$e_{t+1} = [\ 0 \quad 0 \quad 0 \quad 0 \quad e_{\pi,t+1} \quad e_{y,t+1} \quad e_{1,t+1} \quad e_{3,t+1} \quad 0 \quad 0 \quad 0 \quad 0\ ]',$$

$$\widetilde{A}_{10} = [\ 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \theta_1 \quad \theta_3 \quad 0 \quad \tau_3 \quad 0 \quad 0\ ]',$$

$$\widetilde{A}_{11} = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&\rho_\pi&0&0&0&0&0&0&0 \\ 0&0&0&0&0&\rho_y&0&0&0&0&0&0 \\ 0&0&0&0&0&0&\rho_1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&\rho_3&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ f_{i,p}^{(3)} \\ 0 \\ 0 \end{bmatrix},$$

[19] In the simulations, we include $\theta_{1,t}$ to $\theta_{11,t}$ and $i_{1,t-1}$ to $i_{11,t-1}$ in X_t. This allows us to compute the volatility of the yield curve. To keep the matrices presented here compact, we leave out these additional variables.

$$\widetilde{A}_{12} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}',$$

$$\widetilde{B}_1 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}'$$

and

$$\widetilde{C}\widetilde{C}' = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_\pi^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_y^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_1^2 & \sigma_{13} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{13} & \sigma_3^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The row vector $f_{i,p}^{(3)}$ in $\widetilde{A}_{11}$ defines the 3-period market rate as a function of $\widetilde{X}_t$. To compute this vector, it is useful to state that the optimal repo rate is given by

$$i_t = f_{i,p}\widetilde{X}_t, \tag{13}$$

where $f_{i,p}$ contains the reaction coefficients of the repo rate to the predetermined variables under operating procedure p. The expected repo rate three periods ahead is given by

$$E_t i_{t+3} = f_{i,p} E_t \widetilde{X}_{t+3}. \tag{14}$$

Since expectations regarding $\widetilde{X}_{t+3}$ are optimally formed as linear projections

$$E_t \widetilde{X}_{t+3} = M_p^3 \widetilde{X}_t,$$

with M_p a function of $f_{i,p}$ (and defined further below), equation (14) can be written as

$$E_t i_{t+3} = f_{i,p} M_p^3 \widetilde{X}_t.$$

The three-period interest rate then is given by the expectations hypothesis as

$$i_{3,t} = \frac{1}{3}\sum_{k=0}^{2} f_{i,p} M_p^k \widetilde{X}_t + \tau_3 + \theta_{3,t} = f_{i,p}^{(3)} \widetilde{X}_t + \tau_3 + \theta_{3,t}.$$

The dynamics of the forward-looking variables is captured by

$$E_t H x_{t+1} = \widetilde{A}_{21}\widetilde{X}_t + A_{22} x_t + B_2 i_t,$$

where

$$H = \begin{bmatrix} a_\pi & 0 \\ b_r & b_y \end{bmatrix}, \; \widetilde{A}_{21} = \begin{bmatrix} 0 & -(1-a_\pi) & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -b_r\mu_{1,r} & 0 & -(1-b_y) & 0 & 0 & -1 & b_r & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_{22} = \begin{bmatrix} 1 & -a_y \\ 0 & 1 \end{bmatrix} \text{ and } B_2 = \begin{bmatrix} 0 \\ b_r \end{bmatrix}.$$

To link the goal variables Y_t to the other variables in the model, we define

$$Y_t = \widetilde{D}_p \begin{bmatrix} \widetilde{X}_t & \widetilde{i}_t \end{bmatrix}', \tag{15}$$

where $n_Y = 5$, $\widetilde{i}_t = [\; x_t \;\; \gamma_t \;\; i_t \;]'$, which is the vector the control variables of the model, and where $\gamma_t = \begin{bmatrix} \gamma_t^{PC} & \gamma_t^{IS} \end{bmatrix}' = \Xi_t$ are Lagrange multipliers that account for the dynamics of the forward-looking variables. Matrix $\widetilde{D}_p$ is given by

$$\widetilde{D}_p = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \tau_3 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

$$+ \begin{bmatrix} 0_{(n_Y-1)\times n_{\widetilde{X}}} & 0_{(n_Y-1)\times(2n_x+n_i)} \\ f_{i,p}^{(3)} & 0_{1\times(2n_x+n_i)} \end{bmatrix}$$

We thus can rewrite the period loss function

$$\begin{aligned} \widetilde{L}_{p,t} &= \frac{1}{2} Y_t' \Lambda_p Y_t = \frac{1}{2}\begin{bmatrix} \widetilde{X}_t & \widetilde{i}_t \end{bmatrix} \widetilde{D}_p' \Lambda_p \widetilde{D}_p \begin{bmatrix} \widetilde{X}_t & \widetilde{i}_t \end{bmatrix}' \\ &= \frac{1}{2}\begin{bmatrix} \widetilde{X}_t & \widetilde{i}_t \end{bmatrix} \widetilde{W}_p \begin{bmatrix} \widetilde{X}_t & \widetilde{i}_t \end{bmatrix}'. \end{aligned}$$

A.2 Optimisation

We solve the model using the dual saddlepoint approach discussed in Marcet and Marimon [28]. We follow Svensson and Williams [42] and define the dual period loss function as

$$\begin{aligned}\widetilde{\widetilde{L}}_{p,t} &= \widetilde{L}_{p,t} + \Xi_t'(Hx_{t+1} - \widetilde{A}_{21}\widetilde{X}_t - A_{22}x_t - B_2 i_t) \\ &= \widetilde{L}_{p,t} + \Xi_t'(-\widetilde{A}_{21}\widetilde{X}_t - A_{22}x_t - B_2 i_t) + \tfrac{1}{\delta}\Xi_{t-1}' H x_t \\ &= \widetilde{L}_{p,t} + \gamma_t'(-\widetilde{A}_{21}\widetilde{X}_t - A_{22}x_t - B_2 i_t) + \tfrac{1}{\delta}\Xi_{t-1}' H x_t\end{aligned} \tag{16}$$

where the second equality comes from the definition $\Xi_{-1} = 0$. Using equation (9), equation (16) can be rewritten as

$$\begin{aligned}\widetilde{\widetilde{L}}_{p,t} &= \widetilde{L}_{p,t} + \gamma_t'(-\widetilde{A}_{21}\widetilde{X}_t - A_{22}x_t - B_2 i_t) + \tfrac{1}{\delta}\Xi_{t-1}' H x_t \\ &= \tfrac{1}{2}\begin{bmatrix} \widetilde{X}_t & \widetilde{i}_t \end{bmatrix} \widetilde{\widetilde{W}}_p \begin{bmatrix} \widetilde{X}_t & \widetilde{i}_t \end{bmatrix}',\end{aligned} \tag{17}$$

where

$$\widetilde{\widetilde{W}}_p = \widetilde{W}_p + \begin{bmatrix} 0 & 0 & 0 & -\widetilde{A}_{21}' & 0 \\ 0 & 0 & \frac{1}{\delta}H & 0 & 0 \\ 0 & \frac{1}{\delta}H' & 0 & -A_{22}' & 0 \\ -\widetilde{A}_{21} & 0 & -A_{22} & 0 & -B_2 \\ 0 & 0 & 0 & -B_2' & 0 \end{bmatrix}.$$

Equation (17) is the quadratic loss function in the optimal regulator problem. The linear transition equation for the predetermined variables is given by

$$\widetilde{X}_{t+1} = \widetilde{A}_{11}\widetilde{X}_t + \widetilde{B}\widetilde{\widetilde{i}}_t + Ce_{t+1}, \tag{18}$$

with

$$\widetilde{B} = \left(\begin{bmatrix} \widetilde{A}_{12} & \widetilde{B}_1 \end{bmatrix} + \begin{bmatrix} 0_{n_X \times n_x} & 0_{n_X \times n_x} & 0_{n_X \times n_i} \\ 0_{n_x \times n_x} & I_{n_x \times n_x} & 0_{n_x \times n_i} \end{bmatrix} \right)$$

where the identity matrix captures $\Xi_t = \gamma_t$. The value function $V_p(\widetilde{X}_t)$ of the saddlepoint problem is quadratic,

$$V_p(\widetilde{X}_t) = [(1-\delta)\widetilde{X}_t' V_p \widetilde{X}_t + \delta\omega_p],$$

where ω_p is a scalar. The Bellman equation can therefore be written as

$$(1-\delta)\widetilde{X}_t' V_p \widetilde{X}_t + \delta\omega_p = (1-\delta) \max_{\{\gamma_t\}_{t\geq 0}} \min_{\{x_t, i_t\}_{t\geq 0}} \left\{ \widetilde{\widetilde{L}}_{p,t} + \delta E_t \left[\widetilde{X}_{t+1}' V_p \widetilde{X}_{t+1} + \frac{\delta}{1-\delta}\omega_p \right] \right\}.$$

Iterating over the resulting Riccati equation yields the optimal solution

$$\widetilde{i}_t = F_p \widetilde{X}_t, \tag{19}$$

where

$$F_p = -(R_p + \delta \widetilde{B}' V_p \widetilde{B})^{-1} (N_p' + \delta \widetilde{B}' V_p \widetilde{A}_{11}),$$

$$M_p = \widetilde{A}_{11} + \widetilde{B} F_p$$

and

$$V_p = Q_p + \delta \widetilde{A}_{11}' V_p \widetilde{A}_{11} - (N_p' + \delta \widetilde{B}' V_p \widetilde{A}_{11})' (R_p + \delta \widetilde{B}' V_p \widetilde{B})^{-1} (N_p' + \delta \widetilde{B}' V_p \widetilde{A}_{11})$$

where

$$\widetilde{\widetilde{W}}_p = \begin{bmatrix} Q_p & N_p \\ N_p' & R_p \end{bmatrix}$$

is partitioned conformably with $\widetilde{X}_t$ and $\widetilde{i}_t$.

The optimal rule $f_{i,p}$ for the repo rate is given as the last line in equation (19) and corresponds to equation (13). Inflation, which is the first element of x_t, is given by

$$\pi_t = f_{\pi,p} \widetilde{X}_t,$$

where $f_{\pi,p}$ is the first row of F_p. To derive the optimal repo rate rule, we choose starting values for F_p and iterate until convergence (the exact starting value does not appear to matter).

B Stylised facts

Since we are interested in analysing the effect of a crisis-induced change in the behaviour of these premia on the volatility of inflation, the output gap and the yield curve, it is important to make realistic parameter assumptions in the simulations. This appendix studies how the time series behaviour of term and risk premia in the US has changed with the onset of the financial crisis.[20]

[20] The changes reported here are similar to those in Switzerland and the UK. Those results are available from the authors upon request.

Table 4: Estimates for term and risk premia

	Pre-crisis				Crisis			
j	τ_j	θ_j	ρ_j	$\sigma_j \times 10^{-2}$	τ_j	θ_j	ρ_j	σ_j
1	-	0.05***	0.20	1.52	-	0.43*	0.37	0.53
2	0.05	0.03***	0.54***	1.15	-0.03	0.41*	0.55**	0.49
3	0.10	0.03**	0.67***	0.86	-0.06	0.44*	0.59**	0.49
4	0.14	0.03**	0.66***	0.96	-0.08	0.39*	0.65***	0.45
5	0.18	0.02**	0.72***	0.91	-0.09	0.36*	0.70***	0.42
6	0.21	0.02**	0.73***	0.82	-0.10	0.33*	0.75***	0.38
7	0.24	0.03**	0.67***	0.79	-0.10	0.31*	0.77***	0.37
8	0.26	0.03**	0.71***	0.81	-0.10	0.29*	0.78***	0.36
9	0.27	0.03**	0.72***	0.81	-0.10	0.27*	0.80***	0.35
10	0.29	0.02**	0.73***	0.89	-0.09	0.26	0.81***	0.34
11	0.30	0.03**	0.71***	0.93	-0.08	0.25	0.82***	0.33

Note: Average term premium τ_j and regression output for equation (8). Term premium defined as difference between j-month and one-month OIS rate, risk premium as difference between j-month libor and j-month OIS rate. Pre-crisis data span January 2005 to July 2007, and crisis data August 2007 to January 2009. */**/*** denotes significance at the one/five/ten percent level.

We consider monthly averages of daily interest rate data spanning January 2005 to January 2009 and interest rate maturities of up to eleven months. The crisis subsample begins in August 2007. As a first step, we compute the term premium τ_j attached to different interest rate maturities as the average difference between the OIS rate of maturity j and the one-month OIS rate. Table 4 shows that the term premia increase with maturity in the pre-crisis subsample, but turn negative in the crisis subsample.

Table 4 also shows the estimates of an autoregressive process for the risk premium $\theta_{j,t}$, which has been constructed as the difference between j-month libor and the j-month OIS rate. We fit equation (8) for the two subsamples.[21] As can be seen, the risk premium had a significant constant, displayed mean reversion and was subject to small innovations in the pre-crisis sample (note that the values for σ_j have been multiplied by $\times 10^{-2}$ for this subsample to make the table more easily readable). During the financial crisis, the constant ceases to be significant and the standard errors are larger by a factor of fifty.

[21] It should be noted that equation (8) implies that $\theta_{j,t}$ could turn negative, which is unrealistic. We ignore this for simplicity.

Table 5: Pre-crisis correlation matrices for risk premium shocks

	Pre-crisis sample										
j	1	2	3	4	5	6	7	8	9	10	11
1	1										
2	0.31	1									
3	0.36	0.69	1								
4	0.21	0.71	0.85	1							
5	0.11	0.62	0.68	0.88	1						
6	0.13	0.71	0.67	0.78	0.86	1					
7	0.17	0.60	0.68	0.73	0.78	0.90	1				
8	0.10	0.60	0.58	0.66	0.74	0.86	0.95	1			
9	0.01	0.56	0.51	0.58	0.69	0.82	0.88	0.97	1		
10	0.01	0.51	0.45	0.51	0.64	0.80	0.86	0.96	0.98	1	
11	-0.05	0.46	0.36	0.43	0.63	0.75	0.82	0.93	0.95	0.97	1
	Crisis sample										
j	1	2	3	4	5	6	7	8	9	10	11
1	1										
2	0.99	1									
3	0.98	1.00	1								
4	0.99	0.99	1.00	1							
5	0.98	0.99	0.99	1.00	1						
6	0.98	0.99	0.99	0.99	1.00	1					
7	0.98	0.99	0.98	0.99	1.00	1.00	1				
8	0.98	0.99	0.98	0.99	1.00	1.00	1.00	1			
9	0.98	0.98	0.98	0.99	1.00	1.00	1.00	1.00	1		
10	0.97	0.98	0.98	0.99	0.99	1.00	1.00	1.00	1.00	1	
11	0.97	0.98	0.98	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1

Note: Correlations of risk premium innovations $\varepsilon_{\cdot,t}$. Pre-crisis data span January 2005 to July 2007, crisis data August 2007 to January 2009.

It thus appears that the risk premia used to be affected by small shocks, but have been exposed to much larger innovations during the crisis.

Another important change during the financial crisis concerns the correlation of the shocks that drive the risk premia of different maturities, $cor(\varepsilon_{j,t}, \varepsilon_{k,t})$ for $j \neq k$. The results reported in Table 5 indicate that these correlations have increased sharply during the crisis, suggesting that the innovations in this subsample are mainly driven by a common component. In fact, the first principal component computed from the covariance matrix of the risk premia explains 74.9% of all variations in the first subsample and 96.2% in the second subsample. The parameters reported in Tables 4 and 5 will be are in Section 3.

C Discretionary monetary policy

To derive the optimal repo rules under discretion, we define A_{10} as the first n_X elements of $\widetilde{A}_{10}$, A_{11} as the first n_X rows and columns of $\widetilde{A}_{11}$, A_{12} as the first n_X rows of $\widetilde{A}_{12}$, A_{21} as the first n_X columns of $\widetilde{A}_{21}$, B_1 as the first n_X elements of $\widetilde{B}_1$, CC' as the first n_X rows and columns of $\widetilde{C}\widetilde{C}'$ and D_p as $\widetilde{D}_p$ without the columns referring to Ξ_{t-1} and γ_t. We then write the period loss function as

$$L_{p,t} = \frac{1}{2} \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix}' W_p \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix} \tag{20}$$

with

$$W_p = D_p' \Lambda_p D_p.$$

Under discretion, the repo rate i_t is chosen to minimise equation (20) subject to

$$\begin{bmatrix} X_{t+1} \\ E_t H x_{t+1} \end{bmatrix} = \begin{bmatrix} A_{10} \\ 0 \end{bmatrix} + \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X_t \\ x_t \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} i_t + \begin{bmatrix} C \\ 0 \end{bmatrix} e_{t+1,} \tag{21}$$

$$i_{t+1} = F_{p,t+1} X_{t+1} \tag{22}$$

and

$$x_{t+1} = G_{p,t+1} X_{t+1}, \tag{23}$$

where $F_{p,t+1}$ and $G_{p,t+1}$ are determined in the optimisation in the next period and are assumed to be known today. Taking expectations, combining equations (21) to (23) and solving for x_t yields

$$x_t = \overline{A}_{p,t} X_t + \overline{B}_{p,t} i_t \tag{24}$$

with

$$\overline{A}_{p,t} = (A_{22} - HG_{p,t+1}A_{12})^{-1}(HG_{p,t+1}A_{11} - A_{21})$$

and

$$\overline{B}_{p,t} = (A_{22} - HG_{p,t+1}A_{12})^{-1}(HG_{p,t+1}B_1 - B_2).$$

From this follows that

$$X_{t+1} = \widehat{A}_{p,t} X_t + \widehat{B}_{p,t} i_t + Ce_{t+1}$$

with

$$\widehat{A}_{p,t} = A_{11} + A_{12}\overline{A}_{p,t}$$

and

$$\widehat{B}_{p,t} = B_1 + A_{12}\overline{B}_{p,t}.$$

Using equation (24) in equation (20) yields

$$L_{p,t} = \frac{1}{2} \begin{bmatrix} X_t \\ i_t \end{bmatrix}' \begin{bmatrix} Q_{p,t} & N_{p,t} \\ N'_{p,t} & R_{p,t} \end{bmatrix} \begin{bmatrix} X_t \\ i_t \end{bmatrix},$$

where

$$\begin{aligned} Q_{p,t} &= W_{XX,p} + W_{Xx,p}\overline{A}_{p,t} + \overline{A}'_{p,t}W'_{Xx,p} + \overline{A}'_{p,t}W_{xx,p}\overline{A}_{p,t}, \\ N_{p,t} &= \ W_{Xx,p}\overline{B}_{p,t} + \overline{A}'_{t}W_{xx,p}\overline{B}_{p,t} + W_{Xi,p} + \overline{A}'_{p,t}W_{xi,p} \end{aligned}$$

and

$$R_{p,t} = W_{ii,p} + \overline{B}'_{p,t}W_{xx,p}\overline{B}_{p,t} + \overline{B}'_{p,t}W_{xi,p} + W'_{xi,p}\overline{B}_{p,t}.$$

The Bellman equation can be written as

$$\frac{1}{2}[(1-\delta)X'_tV_{p,t}X_t + \delta\omega_{p,t}] = (1-\delta)\min_{i_t}\left[L_{p,t} + \delta E_t \frac{1}{2}\left(X'_{t+1}V_{p,t+1}X_{t+1} + \frac{\delta}{1-\delta}\omega_{p,t}\right)\right].$$

From the first order condition, we obtain

$$F_{p,t} = -(R_{p,t} + \delta\widehat{B}'_{p,t}V_{p,t+1}\widehat{B}_{p,t})^{-1}(N_{p,t} + \delta\widehat{B}'_{p,t}V_{p,t+1}\widehat{A}_{p,t})$$

and

$$G_{p,t} = \overline{A}_{p,t} + \overline{B}_{p,t} F_{p,t},$$

and we denote the corresponding equilibrium functions by F_p and G_p. Forecasts of X_t are based on

$$X_{t+1} = M_p X_t + C e_{t+1}$$

with

$$M_p = \widehat{A}_p + \widehat{B}_p F_p,$$

where $\widehat{A}_p$ and $\widehat{B}_p$ are the fixed points of the mapping from $(\widehat{A}_{p,t+1},\, \widehat{B}_{p,t+1})$ to $(\widehat{A}_{p,t},\, \widehat{B}_{p,t})$.

D Longer-term rate in the IS curve

If the IS curve (3) is changed to

$$y_t = b_y E_t y_{t+1} + (1 - b_y) y_{t-1} - b_r (i_{3,t} - E_t \pi_{3,t+1} - \mu_{3,r}) + u_{y,t},$$

with $E_t \pi_{3,t+1} = \frac{1}{3} E_t(\pi_{t+1} + \pi_{t+2} + \pi_{t+3})$ and $\mu_{3,r} = \theta_3/(1 - \rho_3) + \tau_3$. Matrices H, $\widetilde{A}_{21}$ and B_2 need to be adjusted to

$$H = \begin{bmatrix} a_\pi & 0 \\ \frac{1}{3} b_r & b_y \end{bmatrix},$$

$$\begin{aligned}\widetilde{A}_{21} = & \begin{bmatrix} 0 & -(1-a_\pi) & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -b_r \mu_{3,r} & 0 & -(1-b_y) & 0 & 0 & -1 & 0 & b_r & 0 & 0 & 0 & 0 \end{bmatrix} \\ & + \begin{bmatrix} 0 \\ \frac{1}{3} b_r (f_{i,p} M + f_{i,p} M^2 - f_{\pi,p} M^2 - f_{\pi,p} M^3) \end{bmatrix},\end{aligned}$$

where the last line captures $E_t i_{t+1}$, $E_t i_{t+2}$, $E_t \pi_{t+2}$ and $E_t \pi_{t+3}$, and

$$B_2 = \begin{bmatrix} 0 \\ \frac{1}{3} b_r \end{bmatrix}.$$

When solving this model for the LMR procedure, indeterminacy problems arise. While under the RR and SMR procedures, the smoothing objective of the policy rate i_t and $i_{1,t}$, respectively, constrains the path of these variables, the repo rate is not pinned down under the LMR procedure.[22] To overcome this problem in the simplest way, we attach in the simulations for the LMR procedure a small positive weight of 0.05 to smoothing the repo rate and reduce the weight attached to the three-month money market rate to 0.95.

[22] On determinacy conditions for monetary policy rules, see Evans and McGough [16] and [17], McGough, Rudebusch and Williams [30] and Woodford [47].

References

[1] Amisano, Gianni and Oreste Tristani (2008), A DSGE model of the term structure with regime shifts, mimeo.

[2] Atkeson, Andrew and Patrick J. Kehoe (2008), On the need for a new approach to analyzing monetary policy, Federal Reserve Bank of Minneapolis Staff Report 412.

[3] Bernanke, Ben S. (2009), The crisis and the policy response, speech available at http://www.federalreserve.gov/newsevents/speech/bernanke20090113a.htm

[4] Bernanke, Ben S. (2005), Implementing monetary policy, speech available at http://www.federalreserve.gov/boarddocs/speeches/2005/20050330/default.htm

[5] Bindseil, Ulrich (2004), Monetary policy implementation: Theory, past and present, Oxford University Press, Oxford.

[6] Blinder, Alan S. (2006), Monetary policy today: sixteen questions and about twelve answers, in: S. Fernandez de Lis and F. Restoy, eds., Central Banks in the 21st Century, Banco de España, 31-72.

[7] Borio, Claudio (1997), Monetary policy operating procedures in industrial countries, BIS working paper 40.

[8] Borio, Claudio and William Nelson (2008), Monetary operation and the financial turmoil, BIS Quarterly Review, March, 31-46.

[9] Buiter, Willem H. (2008), Central banks and financial crises, mimeo.

[10] Committee on the Global Financial System (2008), Central bank operations in response to the financial turmoil, CGFS Paper 31.

[11] Cukierman, Alex (1991), Why does the Fed smooth interest rates? in: Michael T. Belongia (ed), Monetary policy on the 75th anniversary of the Federal Reserve System, Federal Reserve Bank of St. Louis, Kluwer Academic Publishers, Boston/Dordrecht/London, 111-144.

[12] Cúrdia, Vasco and Michael Woodford (2008), Credit frictions and monetary policy, mimeo.

[13] Dennis, Richard and Ulf Söderström (2006), How important is precommitment for monetary policy? Journal of Money, Credit, and Banking 38(4), 847-872.

[14] Eijffinger, Sylvester, Eric Schaling and Willem Verhagen (2000), The term structure of interest rates and inflation forecast targeting, CEPR Discussion Paper 2375.

[15] English, William B., William R. Nelson and Brian P. Sack (2002), Interpreting the significance of the lagged interest rate in estimated monetary policy rules, Contributions to Macroeconomics 3(1), Article 5.

[16] Evans, George W. and Bruce McGough (2005), Stable sunspot solutions in models with predetermined variables, Journal of Economic Dynamics and Control 29, 601-625.

[17] Evans, George W. and Bruce McGough (2005), Monetary policy and stable indeterminacy with inertia, Economics Letters 87, 1-7.

[18] Fendel, Ralf (forthcoming), Note on Taylor rules and the term structure, Applied Economics Letters.

[19] Flemming, John S. (1990), Three points on the yield curve, Greek Economic Review 12 (supplement), 119-132.

[20] Gerlach-Kristen, Petra (2004), Interest rate smoothing: Monetary policy inertia or unobserved variables? B.E. Journals in Macroeconomics, Contributions to Macroeconomics 4(1), Article 3.

[21] Goodfriend, Marvin (1987), Interest-rate smoothing and price level trend-stationarity, Journal of Monetary Economics 19, 335-348.

[22] Goodfriend, Marvin and Bennett T. McCallum (2007), Banking and interest rates in monetary policy analysis: A quantitative explorations, Journal of Monetary Economics 54, 1480-1507.

[23] Goodhart, Charles (1999), Central bankers and uncertainty, Bank of England Quarterly Bulletin, February. Reprint of a speech at the annual Keynes lecture, British Academy, London, October 1998, 102-114.

[24] Jordan, Thomas J. and Peter Kugler (2004), Implementing Swiss monetary policy: Steering the 3M-Libor with repo transactions, Swiss Journal of Economics and Statistics 140(3), 381-393.

[25] Jordan, Thomas J., Angelo Ranaldo and Paul Söderlind (2009), The implementation of SNB monetary policy, mimeo.

[26] Kulish, Mariano (2007), Should monetary policy use long-term rates? B.E. Journal of Macroeconomics, Advances 7(1), Article 15.

[27] Lansing, Kevin J. and Bharat Trehan (2003), Forward-looking behavior and optimal discretionary monetary policy, Economics Letters 81(2), 249-256.

[28] Marcet, Albert and Ramon Marimon (1998), Recursive contracts, mimeo.

[29] Martin, Christopher and Costas Milas (2008), The sub-prime crisis and UK monetary policy, mimeo.

[30] McGough, Bruce, Glenn Rudebusch and John C. Williams (2005), Using a long-term interest rates as the monetary policy instrument, Journal of Monetary Economics 52, 855 – 879.

[31] Michaud, François-Louis and Christian Upper (2008), What drives interbank rates? Evidence from the libor panel, BIS Quarterly Review, March, 47-58.

[32] Orphanides, Athanasios (1998), Monetary policy evaluation with noisy information, Federal Reserve Board Finance and Economics Discussion Series Paper No 50, November.

[33] Rudebusch, Glenn D. (2001), Is the Fed too timid? Monetary policy in an uncertain world, Review of Economics and Statistics 83, 203-217.

[34] Rudebusch, Glenn D. (2002), Term structure evidence on interest-rate smoothing and monetary policy inertia, Journal of Monetary Economics 49(6), 1161-1187.

[35] Rudebusch, Glenn D., Brian P. Sack and Eric T. Swanson (2007), Macroeconomic implications of changes in the term premium, Federal Reserve Bank of St. Louis Review 89(4), 241-269.

[36] Sack, Brian (1998b), Uncertainty, learning, and gradual monetary policy, Federal Reserve Board Finance and Economics Discussion Series Paper No 34, August.

[37] Sack, Brian and Volker Wieland (2000), Interest-rate smoothing and optimal monetary policy: A review of recent empirical evidence, Journal of Economics and Business 52, 205-228.

[38] Smets, Frank (1998), Output gap uncertainty: Does it matter for the Taylor rule? in: Monetary policy under uncertainty, Reserve Bank of New Zealand.

[39] Svensson, Lars E.O. (2000), Open-economy inflation targeting, Journal of International Economics 50, 155-183.

[40] Svensson, Lars E.O. (2007), Optimization under commitment and discretion, the recursive saddlepoint method, and targeting rules and instrument rules: Lecture notes.

[41] Svensson, Lars E.O. and Noah Williams (2007), Monetary policy with model uncertainty: Distribution forecast targeting, mimeo.

[42] Svensson, Lars E.O. and Michael Woodford (2005), Implementing optimal policy through inflation-forecast targeting, in: Bernanke, Ben S. and Michael Woodford (eds), Inflation targeting, University of Chicago Press.

[43] Taylor, John B. (2008), The costs and benefits of deviating from the systematic component of monetary policy, speech available at frbsf.org/economics/conferences/0802/taylor.pdf

[44] Taylor, John B. and John C. Williams (forthcoming), A black swan in the money market, American Economic Journal: Macroeconomics.

[45] Woodford, Michael (1999), Optimal monetary policy inertia, NBER Working Paper No 7261.

[46] Woodford, Michael (2003), Interest and prices: Foundations of a theory of monetary policy, Princeton University Press.

[47] Woodford, Michael (2005), Comment on "Using a long-term interest rate as the monetary policy instrument", Journal of Monetary Economics 52(5), 855-879.