Credit Risk and the Zero-Interest Rate Bound*

Fiorella De Fiore† Oreste Tristani‡

European Central Bank European Central Bank

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Abstract

We study the implications for monetary policy of the zero-interest rate bound (ZLB) when agency costs generate a spread between deposit and lending rates. We show that, when policy follows a Taylor rule, the deflationary effects of a negative preference shock are amplified relative to the case with frictionless financial markets and the likelihood of hitting the zero bound is higher. Under optimal policy, adverse financial shocks can also lead the nominal rate to the ZLB, because they cause an inefficient fall in households' consumption. The policy easing occurs in spite of the absence of deflationary pressures.

Keyworks: optimal monetary policy, financial frictions, zero-lower bound, asymmetric information

JEL codes: E44, E52, E61

1 Introduction

The literature on optimal monetary policy in the presence of a zero lower-bound on nominal interest rates has mainly used the New-Keynesian model with frictionless financial markets as a benchmark (see e.g. Eggertsson and Woodford, 2003, Adam and Billi, 2006, and Nakov, 2008). In that model, the risk-free rate, current and expected, is the only interest rate which affects aggregate demand and only adverse demand-type shocks – e.g. preference shocks –

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†Directorate General Research, European Central Bank. Email: fiorella.de_fiore@ecb.int.

‡Directorate General Research, European Central Bank. Email: oreste.tristani@ecb.int.

can drive the economy to the zero lower bound (henceforth, ZLB). One main finding in that literature is that optimal monetary policy can reduce the likelihood of hitting the ZLB and the severity of the ensuing recession through the promise of low interest rates in the future. There is no scope for additional instruments, or "unconventional" measures, which would be completely ineffective.

These features of the new-Keynesian model and its optimal policy implications are at odds with the evidence of many historical episodes in which policy rates have reached the ZLB. First, the ZLB has typically been reached during periods of financial distress, which may be more easily interpreted as the result of shocks originating in the financial sector. Second, unconventional policy measures have been used extensively in these episodes, going from quantitative easing in Japan in the nineties, to credit easing at the Federal Reserve during the financial and economic crisis of 2008-2009.

From a more general perspective, many interest rates contribute to shape aggregate demand in the real world, including bank deposit and lending rates. While the spreads between these interest rates move moderately under normal circumstances, they tend to increase dramatically in episodes of financial distress, potentially producing large effects on the real economy. In this environment, promising a low future path for the policy rate may not be all that monetary policy can do to alleviate the consequences of the ZLB constraint.

Some recent papers have analysed the optimal policy response to financial shocks in linearised models (see De Fiore and Tristani, 2009, and Curdia and Woodford, 2009). An analysis of the ZLB, however, has proven to be more challenging, because it requires the use of non-linear solution methods – methods that can be computationally prohibitive for models with several state variables. In this paper we redress this shortcoming and study the implications of the zero lower bound for optimal monetary policy in a model featuring both sticky prices and simple, but microfounded, financial market imperfections. More specifically, we assume that firms must raise nominal debt to finance production, and that asymmetric information and monitoring costs generate an endogenous spread between deposit and lending rates. Differently from Bernanke et al. (1999), variations in leverage can occur in our model even if it does not include capital. In its simplest, linearised version, our model, like the new-Keynesian (henceforth, NK) model, features only exogenous state variables and it is therefore amenable to a fully nonlinear analysis with global solution methods.

We first show that, contrary to the case of the NK model, the natural rate is not the relevant benchmark for optimal policy after demand-type shocks. In our economy, all shocks have a "cost-push" component – namely, they all affect inflation directly, and not just through their impact on aggregate demand. As a result, shocks generate inefficiencies. The *efficient* equilibrium, i.e. the equilibrium when all real and financial frictions are eliminated, is the appropriate benchmark from a welfare perspective. The real interest rate which would prevail in the efficient equilibrium (henceforth, "efficient interest rate") responds differently to different shocks and is therefore not a summary statistic for monetary policy. More specifically, the efficient interest rate falls in response to a negative demand-type shock, while it remains constant in response to a financial shock. These different shocks, therefore, warrant different responses from an optimal monetary policy perspective.

Our numerical results demonstrate that, in this environment, the zero lower bound exerts a tighter constraint on policy. In the absence of financial frictions, a preference shock that induces the economy to hit the ZLB and reduces expected inflation increases the real interest rate, thus depressing aggregate demand. In the presence of agency costs, the same shock also reduces credit spreads and lending rates. Because firms charge lower prices, expected inflation is reduced even further, exacerbating the upward pressure on the real interest rate. When monetary policy follows a Taylor rule, the likelihood of hitting the zero lower bound constraint is higher than in the standard NK model.

Contrary to preference shocks, a negative financial shock generates an increase in credit spreads and a fall in output. The increase in credit spreads also produces inflationary pressure. In equilibrium, inflation increases in spite of the fall in the output gap. As a result, the policy rate increases when set according to a Taylor rule. Consequently, the economy does not hit the ZLB.

Under the maintained assumption that policy follows a Taylor rule, therefore, we can generate a scenario akin to the recent financial crisis in our model if we assume that the crisis was the combination of an adverse financial shock and an adverse preference, or "confidence", shock. Such a combination can generate some deflationary pressure and, at the same time, a large output contraction and an increase in credit spreads. We show that the zero bound can be binding for many periods.

Results are very different under optimal policy. To develop an intuition for what a central bank ought to do in our model, we first derive in closed form the target rule which would implement the Ramsey allocation (under the timeless perspective) if the ZLB were ignored. Compared to the NK benchmark, we demonstrate that the target rule implies a stronger mean reversion of the price level. This implies that, in case of deflationary shocks, the price level eventually reaches a higher value than the initial conditon.

Following an adverse financial shock, optimal policy requires a cut in the nominal interest rate, in spite of the inflationary pressure created by the increase in spreads. The policy response aims at limiting the negative effect of this inefficient shock on households' consumption. For large enough financial shocks, therefore, the zero lower bound can be reached even in the absence of deflationary pressure.

Our model relates to papers that address the so-called cost channel, i.e. the assumption that firms raise nominal debt to finance production, and its consequences for the optimal monetary policy (see e.g. Ravenna and Walsh, 2006). In those models, however, there are no agency costs and firms pay the risk-free rate when borrowing funds. As in our previous work (De Fiore and Tristani, 2009, 2011), we nest the cost channel in a set-up with agency costs (as in Carlstrom and Fuerst, 1998, and Bernanke, Gertler and Gilchrist, 1999).

Another related paper to ours is Eggertsson and Krugman (2010), which addresses the possibility of liquidity traps and the implications for policy when some agents are debt-constrained. One major difference is that in Eggertsson and Krugman (2010), the risk-free rate remains the only relevant rate for aggregate demand, despite agents being debt constrained. A second difference is that, in that model, a share of agents pre-sets prices for a given amount of time, while the remaining share sets prices flexibly. The resulting aggregate supply relationship is backward-looking. In our model, prices are set as in Calvo (1983) so the aggregate supply relation is forward-looking. This has important implications for the propagation of shocks that induce the economy to hit the zero-lower bound and for the design of appropriate monetary and fiscal policies.

The paper proceeds as follows. In section 2, we describe the model. In section 3, we characterize the natural and efficient equilibrium and argue that the latter provides the relevant benchmark for a welfare analysis in our economy, where several distortions interact. In section 4, we derive a second-order approximation to the welfare function and the first order conditions of the Ramsey allocation. We also derive in closed form the target rule which, absent the ZLB constraint, would implement the Ramsey allocation. In section 5, we present impulse responses to preference and financial shocks under a Taylor rule and under the optimal policy. In section 6, we conclude.

2 The model

The model builds upon the setup presented in De Fiore and Tristani (2009, henceforth DT). Here, we provide a short description of the environment, focusing on the differences relative to the DT model. We then report the system of log-linear conditions that characterize a competitive equilibrium. A derivation of the system is reported in an appendix which is available from the authors upon request.

The economy is inhabited by a representative infinitely-lived household, wholesale firms owned by risk-neutral entrepreneurs, monopolistically competitive retail firms owned by the households, zero-profit financial intermediaries, a government and a central bank. We describe in turn the problem faced by each class of agents.

2.1 Households

At the beginning of period t, interest is paid on nominal financial assets acquired at time $t-1$. The households, holding an amount W_t of nominal wealth, choose to allocate it among existing nominal assets, namely money M_t, a portfolio of nominal state-contingent bonds A_{t+1} each paying a unit of currency in a particular state in period $t+1$, and one-period deposits denominated in units of currency D_t paying back $R_t^d D_t$ at the end of the period.

In the second part of the period, the goods market opens. Households' money balances are increased by the nominal amount of their revenues and decreased by the value of their expenses. Taxes are also paid or transfers received. The amount of nominal balances brought into period $t+1$ is equal to

$$M_t + P_t w_t h_t + Z_t - P_t c_t + T_t, \tag{1}$$

where h_t is hours worked, w_t is the real wage, Z_t are nominal profits transferred from retail producers to households, and T_t are lump-sum nominal transfers from the government. c_t denote a CES aggregator of a continuum $j \in (0,1)$ of differentiated consumption goods produced by retail firms, $c_t = \left[\int_0^1 c_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$, with $\varepsilon > 1$. P_t is the price of the CES aggregator.

Nominal wealth at the beginning of period $t+1$ is given by

$$W_{t+1} = A_{t+1} + R_t^d D_t + R_t^m \{M_t + P_t w_t h_t + Z_t - P_t c_t - T_t\}, \tag{2}$$

where R_t^m denotes the interest paid on money holdings.

The household's problem is to maximize preferences, defined as

$$E_o \left\{ \sum_0^\infty \beta^t \left[u(c_t; \xi_t) + \kappa(m_t) - v(h_t) \right] \right\}, \tag{3}$$

where $u_c > 0$, $u_{cc} < 0$, $\kappa_m \geq 0, \kappa_{mm} < 0$, $v_h > 0$, $v_{hh} > 0$, $m_t \equiv M_t/P_t$ denotes real balances, and ξ_t is a preference shock. The problem is subject to the budget constraint

$$M_t + D_t + E_t \left[Q_{t,t+1} A_{t+1} \right] \leq W_t, \tag{4}$$

In our model, because external finance needs to be raised before production, financial markets open at the beginning of the period and goods market at the end of the period, as in Lucas and Stokey (1987). One implication of this timing is that real balances affect the equilibrium. In order to relate to the new-Keynesian model with no financial frictions, which has been the main model used to analyse monetary policy at the zero-lower bound, we neutralize the effect of the different timing on the equilibrium. We do so by assuming that monetary policy remunerates money holdings at a rate R_t^m that is proportional to the risk-free rate R_t. Define $\Lambda_{m,t} \equiv \frac{R_t - R_t^m}{R_t}$. Under our assumption, $\Lambda_{m,t} = \Lambda_m$ for all t, and money demand satisfies $\kappa_m(m_t) = \frac{\Lambda_m}{1-\Lambda_m} u_c(c_t; \xi_t)$. The households' optimality conditions are then identical to those obtained in the standard New Keynesian model without financial frictions. They are given by $R_t = R_t^d = E_t \left[Q_{t,t+1} \right]^{-1}$ and

$$-\frac{v_h(h_t)}{u_c(c_t; \xi_t)} = w_t, \tag{5}$$

$$u_c(c_t; \xi_t) = \beta R_t E_t \left\{ \frac{u_c(c_{t+1}; \xi_{t+1})}{\pi_{t+1}} \right\},$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$. The optimal allocation of expenditure between the different types of goods is given by $c_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} c_t$, where $P_t(j)$ is the price of good j.

2.2 Wholesale firms

Wholesale firms, indexed by i, are competitive and owned by infinitely lived entrepreneurs. Each firm i produces the amount $y_{i,t}$ of an homogeneous good, using a linear technology

$$y_{i,t} = A_t \omega_{i,t} l_{i,t}. \tag{6}$$

Here A_t is an aggregate exogenous productivity shock and $\omega_{i,t}$ is an iid productivity shock with distribution function Φ and density function ϕ. Aggregate shocks are publicly observed, while idiosyncratic shocks are observed at no cost only by firms.

At the beginning of the period, each firm receives an exogenous endowment τ_t, which can be used as internal funds. Since these funds are not sufficient to finance the firm's desired level of production, firms need to raise external finance. Before observing $\omega_{i,t}$, firms sign a contract with the financial intermediary to raise a nominal amount $P_t\left(x_{i,t}-\tau_t\right)$, where

$$x_{i,t} \geq w_t l_{i,t}. \tag{7}$$

Each firm i's demand for labor is derived by maximizing the firm's expected profits, subject to the financing constraint (7).

Let $\overline{P}_t$ be the price of the wholesale homogenous good, $\frac{\overline{P}_t}{P_t} = \chi_t^{-1}$ the relative price of wholesale goods to the aggregate price of retail goods, and $(q_{i,t}-1)$ the Lagrange multiplier on the financing constraint. Optimality requires that

$$q_{i,t} = q_t = \frac{A_t}{w_t \chi_t} \tag{8}$$

$$x_{i,t} = w_t l_{i,t} \tag{9}$$

implying that

$$\mathcal{E}\left[y_{i,t}\right] = \chi_t q_t x_{i,t}. \tag{10}$$

Equation (10) states that wholesale firms must sell at a mark-up $\chi_t q_t$ over firms' production costs to cover for the presence of credit frictions and for the monopolistic distortion in the retail sector.

The assumption that firms receive an endowment from the government at the beginning of the period is made for simplicity, in order to reduce the number of state variables and to facilitate the computation of the non-linear numerical solution of the model. The absence of accumulation of firms' net worth implies that the persistence of the endogenous variables merely reflects the persistence of the exogenous shocks. Nonetheless, financial frictions provide an important transmission channel in our economy, through the credit constraint faced by firms and the endogenous spread charged by financial intermediaries. As documented in DT, up to a linear approximation, the model with and without capital accumulation delivers qualitatively similar responses to both real and financial shocks. Moreover, the characterization of optimal monetary policy is broadly similar in these two cases.

2.3 The financial contract

Lending occurs through financial intermediaries (competitive banks), which collect deposits from households and use them to finance loans to firms. Banks can monitor the realization of the idiosyncratic shock but a fraction of the firm's input is consumed in the monitoring activity. If the realization of the idiosyncratic shock is sufficiently low, the value of firm's production is not sufficient to repay the loan and the firm defaults. Despite default risk, banks are able to ensure a safe return because they lend to a continuum of firms facing idiosyncratic shocks. The informational structure corresponds to a standard costly state verification problem, whose solution takes the form of risky debt (see e.g. Gale and Hellwig (1985)). Define

$$f\left(\overline{\omega}\right) \equiv \int_{\overline{\omega}}^{\infty} \omega \Phi\left(d\omega\right) - \overline{\omega}\left[1 - \Phi\left(\overline{\omega}\right)\right]$$

$$g\left(\overline{\omega};\mu\right) \equiv \int_{0}^{\overline{\omega}} \omega \Phi\left(d\omega\right) - \mu \Phi\left(\overline{\omega}\right) + \overline{\omega}\left[1 - \Phi\left(\overline{\omega}\right)\right]$$

as the expected shares of output accruing respectively to an entrepreneur and to a lender, after stipulating a contract that sets the fixed repayment at $\overline{P}_t \chi_t q_t \overline{\omega}_{it} x_{i,t}$ units of money. In case of default, a stochastic fraction μ_t of the input costs $x_{i,t}$ is used in monitoring. At the individual firm level, total output and the government subsidy are split between the entrepreneur, the lender, and monitoring costs so that $f\left(\overline{\omega}_t\right) + g(\overline{\omega}_t;\mu_t) = 1 - \mu_t \Phi\left(\overline{\omega}_t\right)$.

The optimal contract is the pair $(x_{i,t}, \overline{\omega}_{i,t})$ that solves the following problem:

$$\max\ \overline{P}_t \chi_t q_t f(\overline{\omega}_{i,t}) x_{i,t}$$

subject to

$$\begin{aligned}
\overline{P}_t \chi_t q_t g(\overline{\omega}_{i,t}) x_{i,t} &\geq R_t^d P_t \left(x_{i,t} - \tau_t\right) && (11)\\
\overline{P}_t\left[f\left(\overline{\omega}_{i,t}\right) + g\left(\overline{\omega}_{i,t};\mu_t\right) - 1 + \mu_t \Phi\left(\overline{\omega}\right)\right] &\leq 0 && (12)\\
\overline{P}_t \chi_t q_t f(\overline{\omega}_{i,t}) x_{i,t} &\geq P_t \tau_t && (13)
\end{aligned}$$

The optimal contract maximizes the entrepreneur's expected nominal profits subject to the lender being willing to lend out funds, (11), the feasibility condition, (12), and the entrepreneur being willing to sign the contract, (13).

The optimality conditions are the same for all firms. They can be written as

$$q_t = \frac{R_t}{1 - \mu_t \Phi\left(\overline{\omega}_{i,t}\right) + \frac{\mu_t f(\overline{\omega}_{i,t}) \phi(\overline{\omega}_{i,t})}{f'(\overline{\omega}_{i,t})}} \tag{14}$$

$$x_t = \left\{ \frac{R_t}{R_t - q_t g\left(\overline{\omega}_t; \mu_t\right)} \right\} \tau_t. \quad (15)$$

Notice that the gross interest rate on loans, R_t^l, can be backed up from the debt repayment. It is given by $\overline{P}_t \overline{\omega}_t \chi_t q_t x_t = R_t^l P_t \left(x_t - \tau_t\right)$. Define the spread between the loan rate and the risk-free rate as $\Lambda_t \equiv \frac{R_t^l}{R_t^d}$. It can be written as

$$\Lambda_t = \frac{\overline{\omega}_t}{g(\overline{\omega}_t; \mu_t)}.$$

2.3.1 Entrepreneurs

Entrepreneurs die with probability γ_t. They have linear preferences over the same CES basket of differentiated consumption goods as households, with rate of time preference β^e. This latter is sufficiently high so that the return on internal funds is always larger than the rate of time preference, $\frac{1}{\beta^e} - 1$, and entrepreneurs postpone consumption until the time of death.

As in De Fiore, Teles and Tristani (2011), we assume that the government imposes a tax ν on entrepreneurial consumption. It follows that

$$(1+\nu) \int_0^1 P_t\left(j\right) e_{i,t}\left(j\right) dj = \overline{P}_t \left(\omega_{i,t} - \overline{\omega}_t\right) \chi_t q_t x_{i,t},$$

where $e_{i,t}\left(j\right)$ is firm i's consumption of good j. Notice that $\int_0^1 P_t\left(j\right) e_{i,t}\left(j\right) = P_t e_{i,t}$,where $e_{i,t}$ is the demand of the final consumption good of entrepreneur i. Aggregating across firms, we obtain

$$(1+\nu) e_t = f\left(\overline{\omega}_t\right) q_t x_t,$$

where $e_t = \int_0^1 e_{i,t} di$ is aggregate entrepreneurial consumption of the composite good.

We consider the case where ν becomes arbitrarily large. The tax revenue, $T_t^e = \frac{\nu_t f(\overline{\omega}_t) q_t x_t}{1+\nu}$, approaches the total funds of the entrepreneurs that die and the consumption of the entrepreneurs approaches zero, $e_t \rightarrow 0$. The reason for this is that, with $e_t > 0$, it would be optimal for policy to generate a redistribution of resources between households and entrepreneurs. This would enable to exploit the risk-neutrality of the latter to smooth out consumption of the former. Since risk neutrality of entrepreneurs is a simplifying assumption needed to derive debt as an optimal contract, we eliminate this type of incentives for monetary policy by completely taxing away entrepreneurial consumption. Allowing entrepreneurs to consume would also require arbitrary choices on the weight of entrepreneurs to be given in the social welfare function.

2.4 Government

Revenues from taxes on entrepreneurial consumption are used by the government to finance the transfer τ_t. Funds below (in excess of) τ_t are supplemented through (rebated to) households lump-sum taxes (transfers), T_t^h. The budget constraint of the government is

$$T_t^e = \tau_t - T_t^h.$$

2.5 Retail firms

As in Bernanke, Gertler and Gilchrist (1999), monopolistic competition occurs at the "retail" level. A continuum of monopolistically competitive retailers buy wholesale output from entrepreneurs in a competitive market and then differentiate it at no cost. Because of product differentiation, each retailer has some market power. Profits, Z_t, are distributed to the households, who own firms in the retail sector.

Output sold by retailer j, $Y_t(j)$, is used for households' consumption, $c_t(j)$. Hence, $Y_t(j) = c_t(j)$. The final good Y_t is a CES composite of individual retail goods $Y_t = \left[\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$,with $\varepsilon > 1$.

We assume that each retailer can change its price with probability $1-\theta$, following Calvo (1983). Let $P_t^*(j)$ denote the price for good j set by retailers that can change the price at time t, and $Y_t^*(j)$ the demand faced given this price. Then each retailer chooses its price to maximize expected discounted profits. The optimality conditions are given by

$$\begin{aligned} 1 &= \theta\pi_t^{\varepsilon-1} + (1-\theta)\left(\frac{\varepsilon}{\varepsilon-1}\frac{\overline{\Theta}_{1,t}}{\overline{\Theta}_{2,t}}\right)^{1-\varepsilon} \\ \overline{\Theta}_{1,t} &= \frac{1}{\chi_t}Y_t + \theta E_t\left[\pi_{t+1}^{\varepsilon}\overline{Q}_{t,t+1}\overline{\Theta}_{1,t+1}\right] \\ \overline{\Theta}_{2,t} &= Y_t + \theta E_t\left[\pi_{t+1}^{\varepsilon-1}\overline{Q}_{t,t+1}\overline{\Theta}_{2,t+1}\right], \end{aligned}$$

where $\overline{Q}_{t,t+k} = \beta^k \left[\frac{u_c(c_{t+k};\xi_{t+k})}{u_c(c_t;\xi_t)}\right]$.

Recall that the aggregate retail price level is given by $P_t = \left[\int_0^1 P_t(j)^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}}$. Define the relative price of differentiated good j as $p_t(j) \equiv \frac{P_t(j)}{P_t}$ and divide both sides by P_t to express everything in terms of relative prices, $1 = \int_0^1 (p_t(j))^{1-\varepsilon} dj$.

Now define the relative price dispersion term as

$$s_t \equiv \int_0^1 (p_t(j))^{-\varepsilon} dj.$$

This equation can be written in recursive terms as

$$s_t = (1-\theta)\left(\frac{1-\theta\pi_t^{\varepsilon-1}}{1-\theta}\right)^{-\frac{\varepsilon}{1-\varepsilon}} + \theta\pi_t^{\varepsilon}s_{t-1}.$$

2.6 Monetary policy

We characterize monetary policy as either an optimal Ramsey plan, or a simple Taylor rule. Both types of policies are subject to a non-negativity constraint on the nominal interest rate R_t. In addition, the central bank remunerates money holdings at a rate R_t^m that is proportional to the risk-free rate R_t.

We assume a simple Taylor rule of the type

$$R_t = \digamma\left(\pi_t, y_t\right) \tag{16}$$

Here $\digamma(\cdot)$ is a non-negative-valued, nondecreasing function of both arguments.

2.7 Market clearing

Market clearing conditions for money, bonds, labor, loand, wholesale goods and retail goods are given, respectively, by

$$\begin{aligned}
M_t^s &= M_t, \\
Z_t &= 0, \\
h_t &= l_t, \\
D_t &= P_t\left(x_t-\tau\right) \\
y_t &= \int_0^1 Y_t(j)\,dj \\
Y_t(j) &= c_t(j)+e_t(j)\,, \text{ for all } j.
\end{aligned}$$

2.8 Equilibrium

We log-linearize the system of equilibrium conditions around a generic steady state where $p_t(j) = s_t = 1$ and $A = 1$. Define $\widehat{\pi}_{t+1} \equiv \log \pi_{t+1}$, $\widehat{p}_t(j) = \log p_t(j)\,,$ $a_t = \log A_t$, $\widehat{\tau}_t = \log(\tau_t/\tau)\,.$ Under the functional form $u(c_t;\xi_t) - v(h_t) = \xi_t\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{h_t^{1+\varphi}}{1+\varphi}$, the system can be reduced to

$$
\begin{aligned}
(\alpha_3-\alpha_1)\,\widehat{\Lambda}_t &= (1+\sigma+\varphi)\,\widehat{y}_t-(1+\varphi)\,a_t-\widehat{\xi}_t-\widehat{\tau}_t-(\alpha_2+\alpha_4)\,\widehat{\mu}_t && (17)\\
\sigma\left(E_t\widehat{y}_{t+1}-\widehat{y}_t\right) &= \widehat{R}_t-E_t\widehat{\pi}_{t+1}+\widehat{\xi}_{t+1}-\widehat{\xi}_t && (18)\\
\widehat{\pi}_t &= -\lambda\left[(1+\varphi)\,a_t+\widehat{\xi}_t+\alpha_2\widehat{\mu}_t-(\sigma+\varphi)\,\widehat{y}_t-\widehat{R}_t-\alpha_1\widehat{\Lambda}_t\right]+\beta E_t\widehat{\pi}_{t+1} && (19)\\
\widehat{R}_t &\geq 0, && (20)
\end{aligned}
$$

for coefficients α_1, α_2, α_3 and α_4 defined in appendix A, and $\lambda \equiv (1-\theta)(1-\beta\theta)/\theta$. Notice that $\alpha_1 > 0$ and $\alpha_3 > 0$. Under our calibration, α_2, α_4 and $(\alpha_3-\alpha_1)$ are also strictly positive.

The system needs to be complemented with a specification of the path for the nominal interst rate $\widehat{R}_t$ that depends on the policy considered.

3 Natural and efficient equilibrium

It is common in the literature on the zero lower-bound to analyze a situation of liquidity trap when the economy is hit by shocks that reduce the natural rate of interest (see Eggertsson and Woodford (2003), and Adam and Billi (2006)). In a model without financial frictions, preference shocks are the most prominent candidates to affect the natural rate of interest. It is often claimed that these shocks are proxies for disturbances that arise in the financial sector of the economy, which is not modeled in the benchmark New-Keynesian (henceforth NK) model. Our model provides a setup that enables to confirm this conjecture.

To realize this point, consider the standard NK model with frictionless financial markets. It can be obtained by shutting down all financial market frictions in the model of section 2. In this case, $\widehat{\mu}_t = \widehat{\tau}_t = \widehat{\Lambda}_t = 0$, and the nominal interest rate disappears from equation (19), because the cost channel is absent. The equilibrium of the model can be characterized by restriction (20) and

$$
\begin{aligned}
\sigma\left(E_t\widehat{y}_{t+1}-\widehat{y}_t\right) &= \widehat{R}_t-E_t\widehat{\pi}_{t+1}+\widehat{\xi}_{t+1}-\widehat{\xi}_t\\
\widehat{\pi}_t &= -\lambda\left[(1+\varphi)\,a_t+\widehat{\xi}_t-(\sigma+\varphi)\,\widehat{y}_t\right]+\beta E_t\widehat{\pi}_{t+1}.
\end{aligned}
$$

The natural equilibrium, defined as one where prices are flexible and denoted by the superscript n, is described by

$$
\begin{aligned}
\sigma\left(E_t\widehat{y}^n_{t+1}-\widehat{y}^n_t\right) &= \widehat{r}^n_t+E_t\widehat{\xi}_{t+1}-\widehat{\xi}_t,\\
\widehat{y}^n_t &= \frac{1}{(\sigma+\varphi)}\left[(1+\varphi)\,a_t+\widehat{\xi}_t\right],
\end{aligned}
$$

so that the natural rate of interest can be written as

$$\widehat{r}_t^n = \frac{\sigma(1+\varphi)}{(\sigma+\varphi)} E_t (a_{t+1} - a_t) - \frac{\varphi}{(\sigma+\varphi)} E_t \left(\widehat{\xi}_{t+1} - \widehat{\xi}_t\right).$$

Now, define $\widetilde{x}_t = \widehat{y}_t - \widehat{y}_t^n$ and rewrite the system in deviations from the natural equilibrium:

$$\begin{aligned} \widehat{R}_t - E_t \widehat{\pi}_{t+1} - \widehat{r}_t^n &= \sigma (E_t \widetilde{x}_{t+1} - \widetilde{x}_t) \\ \widehat{\pi}_t &= \lambda (\sigma + \varphi) \widetilde{x}_t + \beta E_t \widehat{\pi}_{t+1}. \end{aligned}$$

It is clear that the natural rate of interest acts as a summary statistics in the benchmark NK model. Any shock that moves $\widehat{r}_t^n$ affects the output gap through the IS relation and inflation through the AS relation.

Consider now the natural equilibrium in our model. As in DT, we define it as an equilibrium where all nominal distortions are eliminated. Namely, prices are flexible, and the nominal interest rate reacts to shocks in such a way as to maintain inflation to zero. In such equilibrium, where the nominal and real interest rate coincide, the price stickiness and cost channel distortions are shut down. Our definition of natural equilibrium as one where the cost channel is not active avoids dependence of the natural equilibrium upon monetary policy itself. This is important if this latter is seen as a benchmark that the social planner may wish to replicate by setting the appropriate path of the policy instruments.

According to the definition above, the natural equilibrium is characterized by the conditions

$$\begin{aligned} (\alpha_3 - \alpha_1) \widehat{\Lambda}_t^n &= (1 + \sigma + \varphi) \widehat{y}_t^n - (1+\varphi) a_t - \widehat{\xi}_t - \widehat{\tau}_t - (\alpha_2 + \alpha_4) \widehat{\mu}_t \\ \sigma \left(E_t \widehat{y}_{t+1}^n - \widehat{y}_t^n\right) &= \widehat{r}_t^n + \widehat{\xi}_{t+1} - \widehat{\xi}_t \\ (\sigma + \varphi) \widehat{y}_t^n + \widehat{r}_t^n + \alpha_1 \widehat{\Lambda}_t^n &= (1+\varphi) a_t + \widehat{\xi}_t + \alpha_2 \widehat{\mu}_t, \end{aligned}$$

together with the restriction on the zero-lower bound, (20).

The first and last equations can be combined to obtain

$$[(\sigma+\varphi)\alpha_3 + \alpha_1] \widehat{y}_t^n = (\alpha_1 - \alpha_3) \widehat{r}_t^n + \alpha_3 (1+\varphi) a_t + \alpha_3 \widehat{\xi}_t + \alpha_1 \widehat{\tau}_t + (\alpha_2 \alpha_3 + \alpha_1 \alpha_4) \widehat{\mu}_t.$$

Assuming that all shocks have persistence ρ, using the expression for $\widehat{y}_t^n$ in the Euler equation, and solving forward for $\widehat{r}_t^n$, we can obtain an expression for the natural rate as a function of current shocks

$$\widehat{r}_t^n = \left[\frac{\sigma}{\alpha_1(1+\sigma) + \varphi\alpha_3}\right] \left\{\sigma^{-1} (\alpha_1 + \varphi\alpha_3) \widehat{\xi}_t - \alpha_3 (1+\varphi) a_t - \alpha_1 \widehat{\tau}_t - (\alpha_2 \alpha_3 + \alpha_1 \alpha_4) \widehat{\mu}_t\right\}. \tag{21}$$

The expression shows that, similar to preference shocks, financial shocks can indeed reduce the natural rate and lead the economy to hit the zero-lower bound.

Express now the equilibrium of the model in terms of gaps from the natural equilibrium:

$$(\alpha_3 - \alpha_1)\,\widetilde{\Lambda}_t = (1 + \sigma + \varphi)\,\widetilde{x}_t,$$

$$\sigma\left(E_t \widetilde{x}_{t+1} - \widetilde{x}_t\right) = \widehat{R}_t - E_t \widehat{\pi}_{t+1} - \widehat{r}_t^n,$$

$$\widehat{\pi}_t = \lambda\left[(\sigma + \varphi)\,\widetilde{x}_t + \widehat{R}_t - \widehat{r}_t^n + \alpha_1 \widetilde{\Lambda}_t\right] + \beta E_t \widehat{\pi}_{t+1},$$

where $\widetilde{\Lambda}_t = \widehat{\Lambda}_t - \widehat{\Lambda}_t^n$. The system can be written as

$$\sigma\left(E_t \widetilde{x}_{t+1} - \widetilde{x}_t\right) = \widehat{R}_t - E_t \widehat{\pi}_{t+1} - \widehat{r}_t^n, \tag{22}$$

$$\widehat{\pi}_t = \lambda\left[\frac{\alpha_1 + (\sigma + \varphi)\,\alpha_3}{(\alpha_3 - \alpha_1)}\widetilde{x}_t + \widehat{R}_t - \widehat{r}_t^n\right] + \beta E_t \widehat{\pi}_{t+1}. \tag{23}$$

The expressions above show that, as in the NK model, the natural rate of interest acts as a summary statistics. A decrease in the natural rate of interest (irrespective of what shock drives such decrease) reduces the output gap and, through the effect of this on marginal costs, lowers inflation.

However, in our economy, the natural equilibrium does not describe the allocation that the social planner would want to achieve. One reason is that the economy is characterized by several distortions - sticky prices, monopolistic competition, credit constraints, imperfect information. In this second-best environment, where policy faces several trade-offs, the social planner can possibly achieve the highest welfare by implementing an allocation that does not deliver zero inflation at all times. Below, we derive a second-order approximation of welfare and show that the social planner aims at closing the gaps, when defined as deviations from the efficient equilibrium, i.e. an equilibrium where all financial frictions (nominal and real), as well as nominal price stickiness, are removed.

Here, we show that the real rate of interest arising in the efficient equilibrium does not have the property of acting as a summary statistics. Financial shocks that reduce the efficient rate of interest also affect autonomously inflation.

Define $\widehat{r}_t^e = \widehat{R}_t^e - E_t \widehat{\pi}_{t+1}^e$as the real interest rate arising in the efficient equilibrium. This latter is characterized by

$$\sigma\left(E_t \widehat{y}_{t+1}^e - \widehat{y}_t^e\right) = \widehat{r}_t^e + \widehat{\xi}_{t+1} - \widehat{\xi}_t$$

$$\widehat{y}_t^e = \frac{1}{(\sigma + \varphi)}\left[(1 + \varphi)\,a_t + \widehat{\xi}\right],$$

implying that

$$\widehat{r}_t^e = \frac{\sigma(1+\varphi)}{(\sigma+\varphi)} E_t(a_{t+1}-a_t) - \frac{\varphi}{(\sigma+\varphi)} E_t\left(\widehat{\xi}_{t+1}-\widehat{\xi}_t\right). \tag{24}$$

Define $x_t = \widehat{y}_t - \widehat{y}_t^e$. The system of log-linear equilibrium conditions, in deviation from the efficient equilibrium, is given by (24), together with

$$(\alpha_3-\alpha_1)\widehat{\Lambda}_t = (1+\sigma+\varphi)x_t + \frac{1}{(\sigma+\varphi)}\left[(1+\varphi)a_t+\widehat{\xi}_t\right] - \widehat{\tau}_t - (\alpha_2+\alpha_4)\widehat{\mu}_t, \tag{25}$$

$$\sigma(E_t x_{t+1} - x_t) = \widehat{R}_t - E_t\widehat{\pi}_{t+1} - \widehat{r}_t^e, \tag{26}$$

$$\widehat{\pi}_t = \lambda\left[(\sigma+\varphi)x_t + \widehat{R}_t + \alpha_1\widehat{\Lambda}_t\right] + \beta E_t\widehat{\pi}_{t+1} - \lambda\alpha_2\widehat{\mu}_t, \tag{27}$$

$$\widehat{R}_t \geq 0, \tag{28}$$

and the policy rule.

Equation (25) shows that the spread between the loan rate and the policy rate increases with excess aggregate demand. A higher demand for retail (and thus also for wholesale) goods implies an implicit tightening of the credit constraint, because the exogenous amount of internal funds must be used to finance a higher level of debt. The increased default risk generates a larger spread. The spread is also increasing in the technology shock and in the preference shock, and (under our calibration) decreasing in the shock to net worth and in the shock to monitoring costs. An increase in aggregate productivity raises the demand for external finance, because firms wish to take advantage of the higher return from production. Since firms' internal funds are given, this raises the proability of default and the spread charged by banks. An increase in $\widehat{\xi}_t$ raises the marginal utility of consumption and households' willingness to supply labor. Firms' demand for external finance and leverage increases, together with the spread. A fall in firms' net worth, $\widehat{\tau}_t$, directly raises leverage, leading to a higher spread. Finally, an exogenous increase in monitoring costs, as reflected by $\widehat{\mu}_t$, leads to a higher financial markup $\widehat{\chi}_t$, which reduces real wages and the firms' need to raise external finance. Leverage falls, together with the credit spread.

Equation (26) is a standard forward-looking IS-curve describing the evolution of the gap between actual output and its efficient level. As in the standard NK model, the output gap is affected by its expected future value and by the real interest rate gap.

Equation (27) represents an extended Phillips curve. The first determinant of inflation in this equation is a standard output gap term. A higher demand for retail goods, and correspondingly for intermediate goods to be used as production inputs, implies that wholesale firms need to pay a higher real wage to induce workers to supply the required labor services. Equation (27) also includes a nominal interest rate term reflecting the cost channel, and a credit spread reflecting the agency costs, whose increase also pushes up marginal costs. Finally, the equation shows that an increase in the monitoring parameter $\widehat{\mu}_t$ lowers inflationary pressures. The reason is that the corresponding increase in the financial markup $\widehat{\chi}_t$ lowers real wages and firms' marginal costs.

From equations (25)-(27), it is clear that the efficient rate of interest, $\widehat{r}_t^e$, does not act as a summary statistic for inflationary pressures. For instance, while preference shocks affect $\widehat{r}_t^e$ and, through the output gap, inflation, they also affect the spread. This relation between the spread and $\widehat{\xi}_t$ adds autonomous inflationary pressures. Also, financial shocks (to the monitoring technology) affect inflation through their effect in equation (27), although they do not affect $\widehat{r}_t^e$.

As discussed in DT, the credit spread and the nominal interest rate act as endogenous "cost-push" terms in the economy. They push up marginal costs and inflation, while exerting downward pressure on economic activity. For the nominal interest rate, this happens because it induces an increase in the real interest rate, which make households postpone their consumption to the future. For the credit spread, this happens because the increase in the financial markup needed for firms to cover the larger cost of external finance reduces the real wage, depressing current demand. All shocks - both real and financial - are exogenous "cost-push" shocks, because they create inflationary pressure independently from the evolution of the output gap.

4 Welfare analysis

We derive a policy objective function by taking a second order approximation to the utility of the economy's representative agents. Welfare is

$$W_{t_0} = E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^t U_t \right\},$$

where households' temporary utility is given by $U_t = \xi_t \frac{c_t^{1-\sigma}}{1-\sigma} - \psi \frac{h_t^{1+\phi}}{1+\phi}$.

Under the functional form for household's utility defined above, appendix B shows that the present discounted value of social welfare can be approximated by

$$W_{t_0} \simeq c^{1-\sigma} \left[\varkappa - \frac{1}{2} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} L_t \right] + t.i.p. \tag{29}$$

where $t.i.p.$ denotes terms independent of policy and

$$L_t \equiv \kappa_\pi \pi_t^2 + (\sigma + \varphi) x_t. \tag{30}$$

Here x_t is our measure of output gap (in deviations from the efficient equilibrium), $\kappa_\pi = \frac{\varepsilon\theta}{(1-\theta)(1-\beta\theta)}$ and $\varkappa = \left(\frac{1}{1-\sigma} - \frac{1}{1+\phi}\right)$.

The planner maximizes (30) subject to the equilibrium conditions

$$\begin{aligned}
x_t &= E_t x_{t+1} - \sigma^{-1}\left(\widehat{R}_t - E_t\widehat{\pi}_{t+1} - \widehat{r}_t^e\right) \\
\widehat{\pi}_t &= \beta E_t\widehat{\pi}_{t+1} + \lambda\left[\sigma + \varphi + \frac{\alpha_1}{\alpha_3 - \alpha_1}(1 + \sigma + \varphi)\right] x_t + \lambda\widehat{R}_t + \upsilon_t \\
\upsilon_t &\equiv \lambda\frac{\alpha_1}{\alpha_3 - \alpha_1}\left(\frac{1+\varphi}{\sigma+\varphi}a_t + \frac{1}{\sigma+\varphi}\widehat{\xi}_t - \widehat{\tau}_t\left(\alpha_4 + \frac{\alpha_2}{\alpha_1}\alpha_3\right)\widehat{\mu}_t\right) \\
\widehat{R}_t &\geq \ln\beta
\end{aligned}$$

where $\lambda \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}$,

$$\widehat{r}_t^e = \sigma\frac{1+\varphi}{\sigma+\varphi}(\rho_a - 1) a_t - \frac{\varphi}{\sigma+\varphi}(\rho_\xi - 1)\widehat{\xi}_t$$

and the last equation corresponds to the ZLB constraint.

The first-order conditions can be written as

$$\begin{aligned}
\mu_t &= \frac{\sigma(\sigma+\varphi)x_t - \beta^{-1}\sigma\mu_{t-1} - \sigma\left[\sigma + \varphi + \frac{\alpha_1}{\alpha_3-\alpha_1}(1+\sigma+\varphi)\right]\phi_{t-1}}{\varphi + \frac{\alpha_1}{\alpha_3-\alpha_1}(1+\sigma+\varphi)} \\
\phi_t &= \phi_{t-1} + \frac{\beta+\lambda}{\beta\sigma}\mu_{t-1} + \frac{\left[\sigma+\varphi+\frac{\alpha_1}{\alpha_3-\alpha_1}(1+\sigma+\varphi)\right]\phi_{t-1} + \beta^{-1}\mu_{t-1} - (\sigma+\varphi)x_t}{\varphi + \frac{\alpha_1}{\alpha_3-\alpha_1}(1+\sigma+\varphi)} - \varepsilon\widehat{\pi}_t \\
0 &= \left(\widehat{R}_t - \ln\beta\right)\phi_t
\end{aligned}$$

where μ_t and ϕ_t are the lagrangean multipliers on the Euler equation and the ZLB constraint, respectively (the New-Phillips curve multiplier, ν_t, has been substituted out).

Under the assumption that the ZLB constraint can be ignored, these conditions can be rewritten in terms of the following target rule

$$\Delta x_t = -\varepsilon\left[1 + \frac{\alpha_1}{\alpha_3-\alpha_1}\left(1 + \frac{1}{\sigma+\varphi}\right)\right]\widehat{\pi}_t + \frac{\sigma}{\sigma+\varphi}\varepsilon\left(\widehat{\pi}_t - \frac{\widehat{\pi}_{t-1}}{\beta}\right) + \frac{\lambda}{\beta}x_{t-1} \tag{31}$$

Equation (31) nests the target rule which implements optimal policy in the New Keynesian model, given by $\Delta x_t = -\varepsilon\widehat{\pi}_t$ (see eg Woodford, 2003). In that model, the target rule can be interpreted as the simple prescription to keep contracting the output gap as long as inflation is positive (and viceversa for negative inflation).

The introduction of the cost channel in the model is responsible for the last two terms in equation (31). In response to a certain inflation rate, these terms suggest that the contraction in the output gap should be smaller than in the NK model, if lagged output gap remains positive and if the quasi-difference in the inflation rate $\widehat{\pi}_t - \widehat{\pi}_{t-1}/\beta$ is positive. Intuitively, these terms take into account the cost-push inflationary effects of the increase in the nominal interest rate, which will have to be implemented to induce a contraction of the output gap.

Finally, the existence of asymmetric information and credit spreads calls for a more aggressive policy response to current inflation – the coefficient is higher than in the NK case by the positive amount $\alpha_1/(\alpha_3 - \alpha_1)(1 + 1/(\sigma + \varphi))$. This is necessary to take check any additional inflationary pressures coming from credit spreads.

Equation (31) can also be written differently to highlight its implications on the price level. We then have

$$p_t = p_{t-1} - \frac{1}{\widetilde{\varepsilon}}\left[\beta\frac{\varphi + \sigma}{\sigma}\Delta x_t + \lambda\frac{\varphi + \sigma}{\sigma}(\varepsilon\sigma - 1)x_{t-1} + \lambda\widehat{R}_{t-1} + \lambda\alpha_1\widehat{\Lambda}_{t-1} + \beta \mathrm{E}_{t-1}\widehat{\pi}_t\right] \qquad (32)$$

where $\widetilde{\varepsilon}$ is a positive reaction coefficient given by $\widetilde{\varepsilon} \equiv \varepsilon\beta\sigma^{-1}\left[\varphi + \frac{\alpha_1}{\alpha_3 - \alpha_1}(1 + \sigma + \varphi)\right]$.

Note that the NK model would require $p_t = p_{t-1} - (1/\varepsilon)\Delta x_t$. Assuming to start the economy from an initial price level $p_0 = \overline{p}$, this equation says that the economy should always return to that $\overline{p}$ once the output gap is stabilised and $\Delta x_t = 0$. This implies history dependence, in the sense that an inflationary period should be induced after a deflationay shock, so as to ensure a return to the original price level.

In the case of our model, a return to the original price level is not sufficient. Note that all terms inside the square brackets on the right-hand side of equation (32) are positive. This implies that, following again a deflationary shock, some additional upward pressure on the price level must be engineered even after hen the output gap is stabilised and $\Delta x_t = 0$. As a result, prices will remain, as in the NK model, trend stationary, but they will return to a higher price level than the one from when the economy started.

5 Numerical results

We use the collocation method (see Judd, 1998, or Miranda and Fackler, 2002) to solve the system (25)-(28) closed with either a simple Taylor-type rule or the first order conditions of the Ramsey planner. The collocation method can be applied using polinomial approximants or spline functions. Our results are based on cubic splines. Given that there are no endogenous state variables in our model, the only state variables are the exogenous shocks when policy follows a Taylor rule. Under optimal policy, the state spate must be expanded to include the Lagrange multipliers of the Ramsey problem.

Parameter values are in line with Woodford (2003). More specifically, we set $\beta = 0.99$, $\sigma = 0.16$, $\varepsilon = 11, \phi = 0.11$, and $\theta = .66$. For the steady state parameters τ and σ_ω, we use the values implied in the parametrization used in DT, which matches US data on the average annual spread between lending and deposit rates (approximately 2%) and on the quarterly bankruptcy rate (around 1%). They imply that $\alpha_1 = 4.7$ and $(\alpha_3 - \alpha_1)^{-1} = 0.008$. As in Adam and Billi (2006), we assume that shocks have a serial correlation coefficient equal to 0.8.

5.1 Simple rule

Our results are based on a standard Taylor type rule of the form

$$\widehat{R}_t \equiv \phi_\pi \pi_t + \phi_x \widehat{x}_t$$

where $\phi_\pi = 1.5$ and $\phi_x = 0.5$.

Figure 1 compares the impulse responses to an adverse demand shock in our model and in the NK model. All variables are in annualised log-levels – note that all interest rates and the credit spread return to their steady state values of 4% and 2%, respectively. The size of the shock is calibrate to induce a fall in the policy rate to just zero in our model, which amounts to a fall of the natural rate from 4% to 2.8%.

This shock has relatively mild implications in the NK model. Inflation and the output gap fall by approximately 1 percentage point. The policy interest rate is cut by almost 2 percentage points, but it remains safely above the zero bound.

In the model with credit frictions, however, a fall of the efficient interest rate to just under 2.5% is sufficient for the policy rate to reach the ZLB. The difference from the NK model has to do with the larger fall in inflation in the model with credit frictions, due to the impact of the nominal interest rate and the credit spread on the Phillips curve. The faster fall to zero of the policy interest rate is so expansionary that the output gap increases slightly in the model with credit frictions.

Note that the cost channel effect and the impact of financial frictions through spreads reinforce each other. Credit spread fall, because firms find themselves less leveraged due to lower aggregate demand and hence lower output.

All in all, these results illustrate how, for given size of the shocks to the efficient interest rate, the model with financial frictions leads to a higher incidence of the ZLB and more dispersed inflation outcomes. These results echo the findings in Chung, Laforte, Reifschneider and Williams (2011), who argue that the probability of hitting the zero bound may have been underestimated in previous research.

Figure 2 and 3 show the impact of increasingly larger, negative shocks to the efficient interest rate. The more binding the zero bound constraint, the more nonlinear the policy rules become in our model. The results are striking. Figure 2 shows that, already when the zero bound binds for one period, the liquidity trap starts biasing downwards expectations of future output gaps. Eventually, in the stochastic steady state the output gap turns negative, while inflation becomes positive. In figure 3, where the shock is large enough to make the zero bound binding for 3 periods, these effects are extremely large.

We should emphasise that the precise quantitative results in figures 2 and 3 are sensitive to the parameters of the Taylor rule and to the calibration of σ and ϕ. For example, the nonlinear effects on the steady state would be much smaller if we used an RBC-style calibration with $\sigma = 1$ and $\phi = 0$. Nevertheless, figures 2 and 3 are illustrative of an additional property of our model: the zero bound is much more harmful than in the NK model. The combination of deflationary pressures which follows a negative demand shock can much more easily "trap" expectations in an adverse equilibrium.

All figures presented so far show that a demand type shock cannot be seen as a source of the recent zero bound episodes in the United States following the financial crisis of 2007-2008. While output and inflation fall, the credit spread also falls in thes simulations – something clearly counterfactual.

Figure 4 presents the impulse responses to a destruction of net worth – a negative shock to τ_t. The shock is calibrated to generate a 1% increase in the credit spread. The increase cost of financing and the higher bankruptcy rate also produce a fall in output. Through the impact of the higher spread on marginal costs, however, the shock is inflationary. If the central bank follows a Taylor rule, interest rates increase to meet the rise in inflation. This shock, therefore, cannot lead to the zero bound under a Taylor rule.

We can obtain a more realistic scenario in our model through the combination of adverse financial and demand shocks. Figure 5 shows impulse responses to a combined shock of this type, which is calibrated to generate a recession accompanied by an increase in credit spreads and a fall in inflation. The shocks are large enough to make the ZLB binding for two periods: this implies almost a 3 percentage points fall in the efficient rate, and a 60% fall in firms' net worth. As in figure 3, the ZLB produces a strong nonlinearity in the policy rules. In the stochastic steady state inflation is positive and the output gap negative. The cost push nature of the net worth shock implies that inflation returns towards the stochastic steady state faster than output. Consequently, the policy rate also rises fast after being at the ZLB.

5.2 Ramsey policy

In this section we compare the impulse responses under the Taylor rule with those which would be obtained if policy followed the target rule which implements optimal policy under commitment.

To develop an intuition for our more general results, Figure 6 shows the impulse responses to the negative, persistent shock to τ_t. The shock is of the same size as in Figure 4. As already discussed above, this shock acts like a cost-push shock. On the one hand, it generates an immediate increase in the loan-deposit rate spread, which pushes up firms' marginal costs and thus generates inflationary pressure. On the other hand, the increase in marginal costs generates a persistent increase in the mark-up q_t and persistent downward pressure on wages, hence a reduction in both labour supply and the demand for consumption goods. Hence, the spread moves anti-cyclically in response to a financial shock.

Compared to the Taylor rule case – the dashed line in green in this figure – optimal policy requires a cut in interest rates, in spite of the inflationary pressure created by the increase in spreads. The main reason for this policy response is that the financial shock is inefficient, hence the fall in households' consumption is entirely undesirable. The expansion in the monetary policy stance helps smooth the adjustment of households' consumption after the shock, at the cost of producing a short inflationary episode.

The most striking difference is in the response of the price level, which is well-known to be an integrated variable under the Taylor rule. Under the target rule, the price level reverts back to the original level and then crosses it after 1 and $\frac{1}{2}$ years, ending up *below* the starting value. The promise of a future fall in the price level keeps expectations of future inflation down and fosters a short inflationary episode in spite of the upward pressure from the increased credit spread.

Figure 7 shows impulse responses under the optimal target rule to the same combination of shocks displayed in Figure 5. Compared to the Taylor rule case, the interest rate falls less on impact under optimal policy, so that the ZLB is never binding. Through the cost channel, together with the increase in credit spreads, this implies that inflation increases on impact, instead of falling as in the Taylor rule case. At the same time, however, the central bank promises to create a mild deflation in the future, when the economy starts recovering. As a result, the initial increase in inflation is modest. The real rate falls less than under the Taylor rule, but more, and more persistently, than the efficient rate, so as to cushion the fall in the output gap. Once the policy rate starts increasing, it does so less abruptly than under the Taylor rule. All in all, both inflation and output are better stabilised under optimal policy.

6 Conclusions

[To be written]

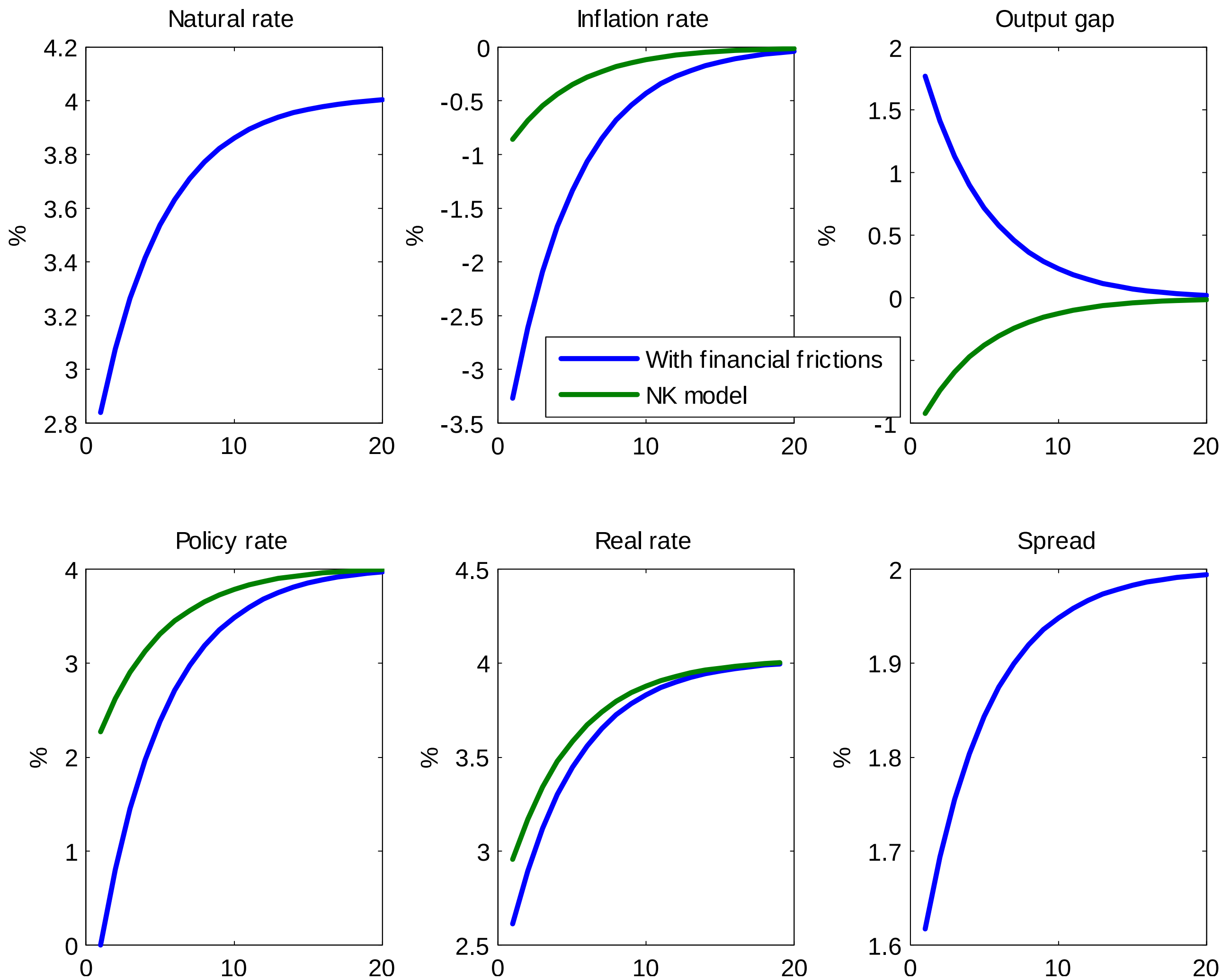

Figure 1: Preference shock, Taylor rule

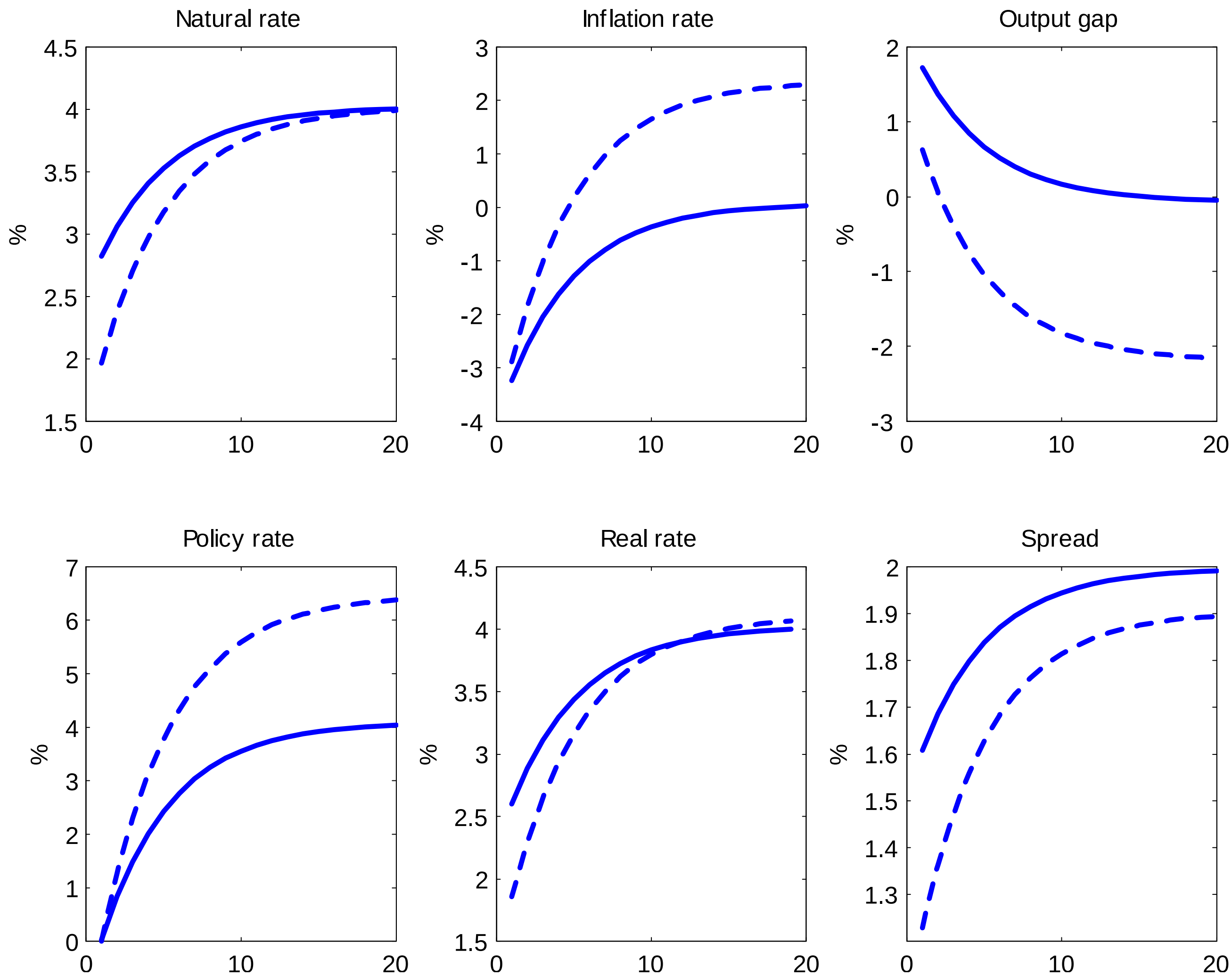

Figure 2: Larger preference shock, Taylor rule

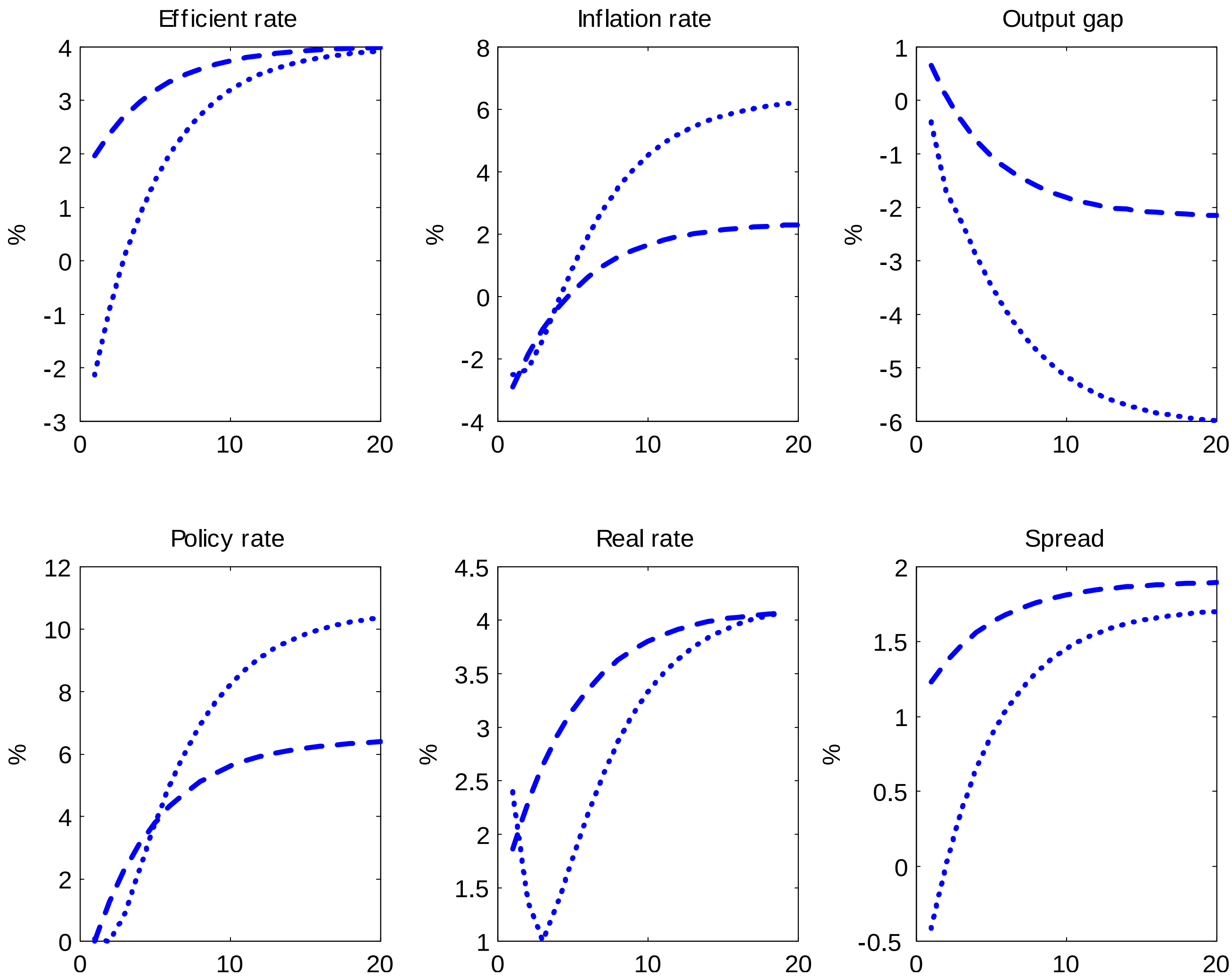

Figure 3: Even larger shock, Taylor rule

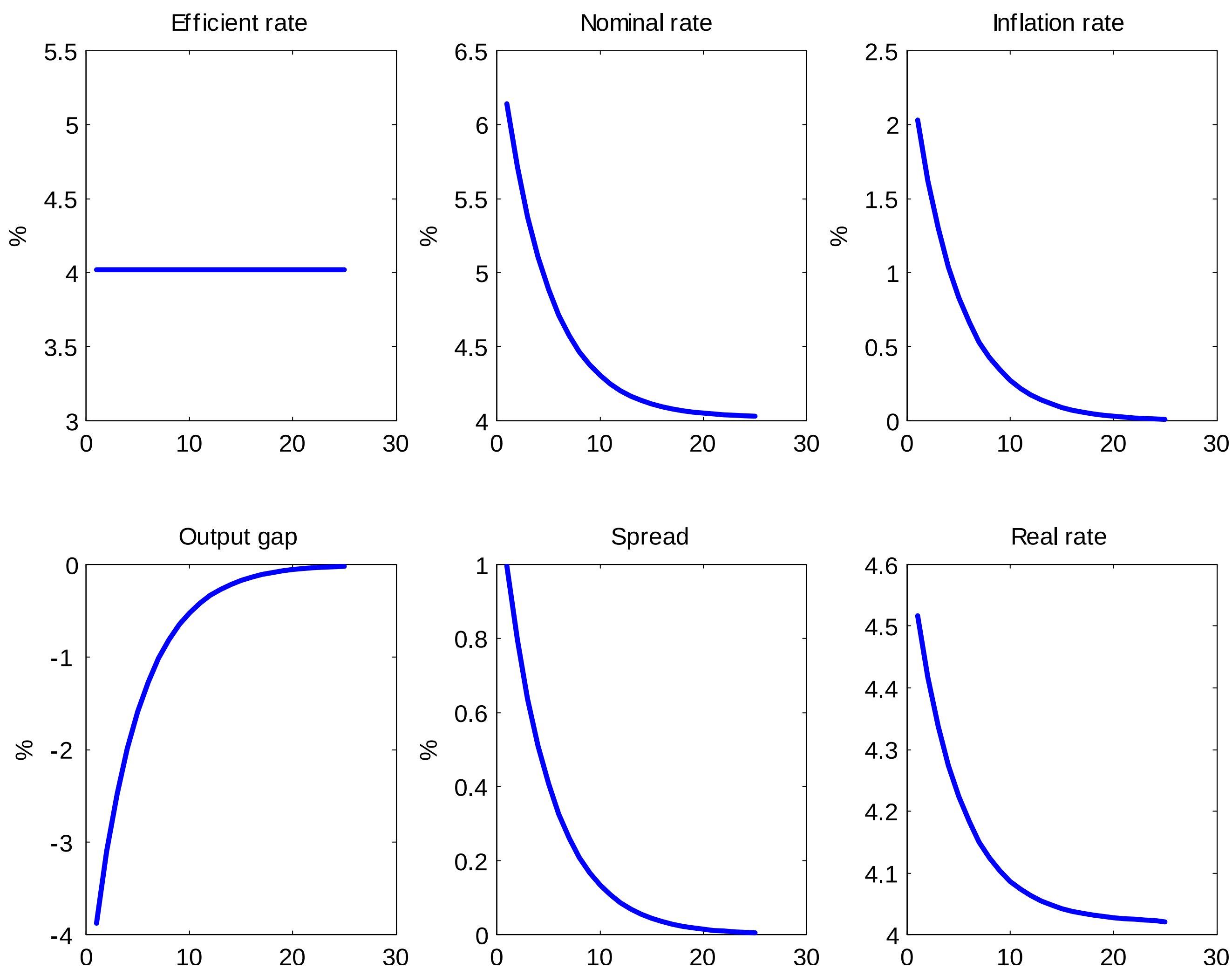

Figure 4: Net worth shock, Taylor rule

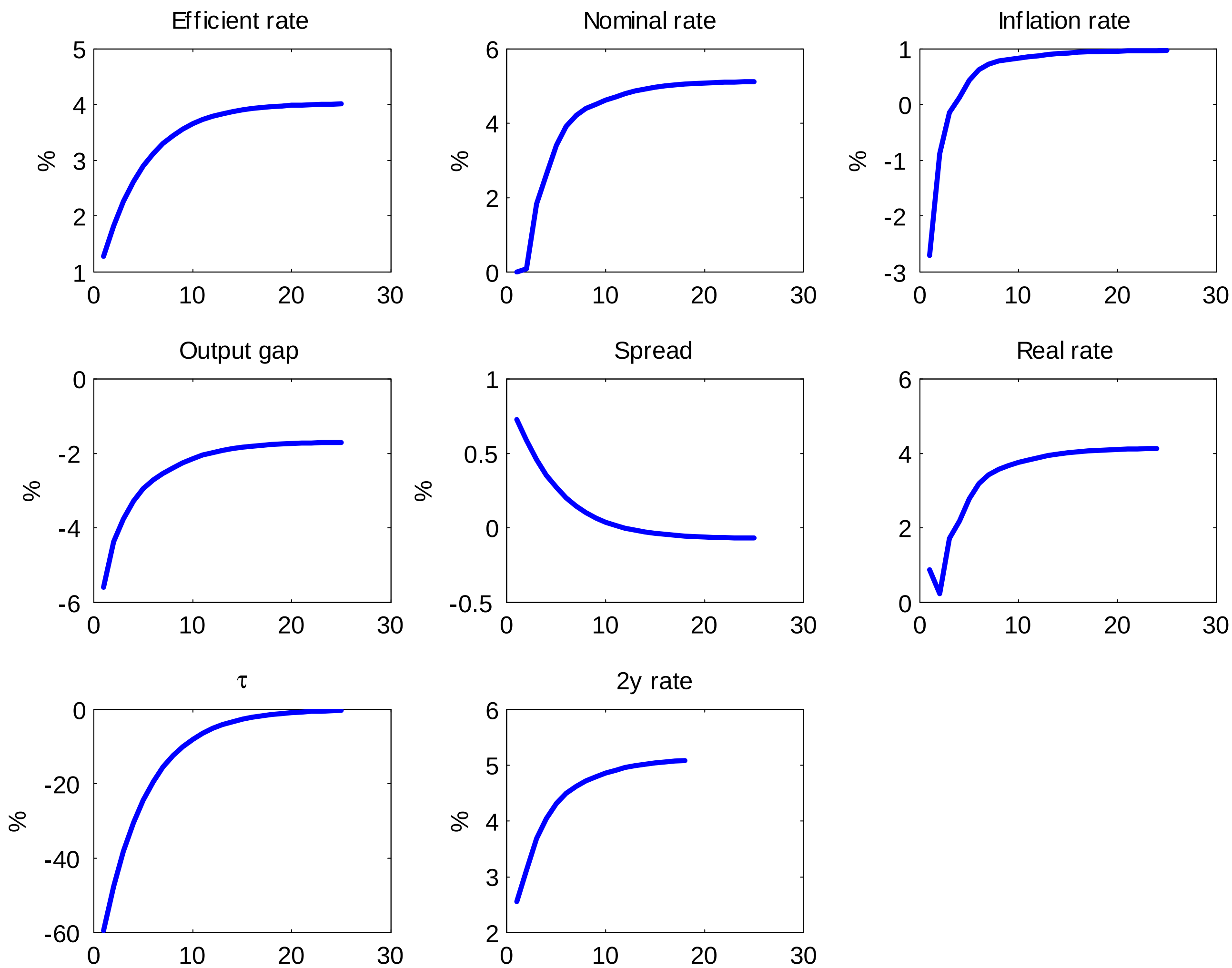

Figure 5: Combined shocks, Taylor rule

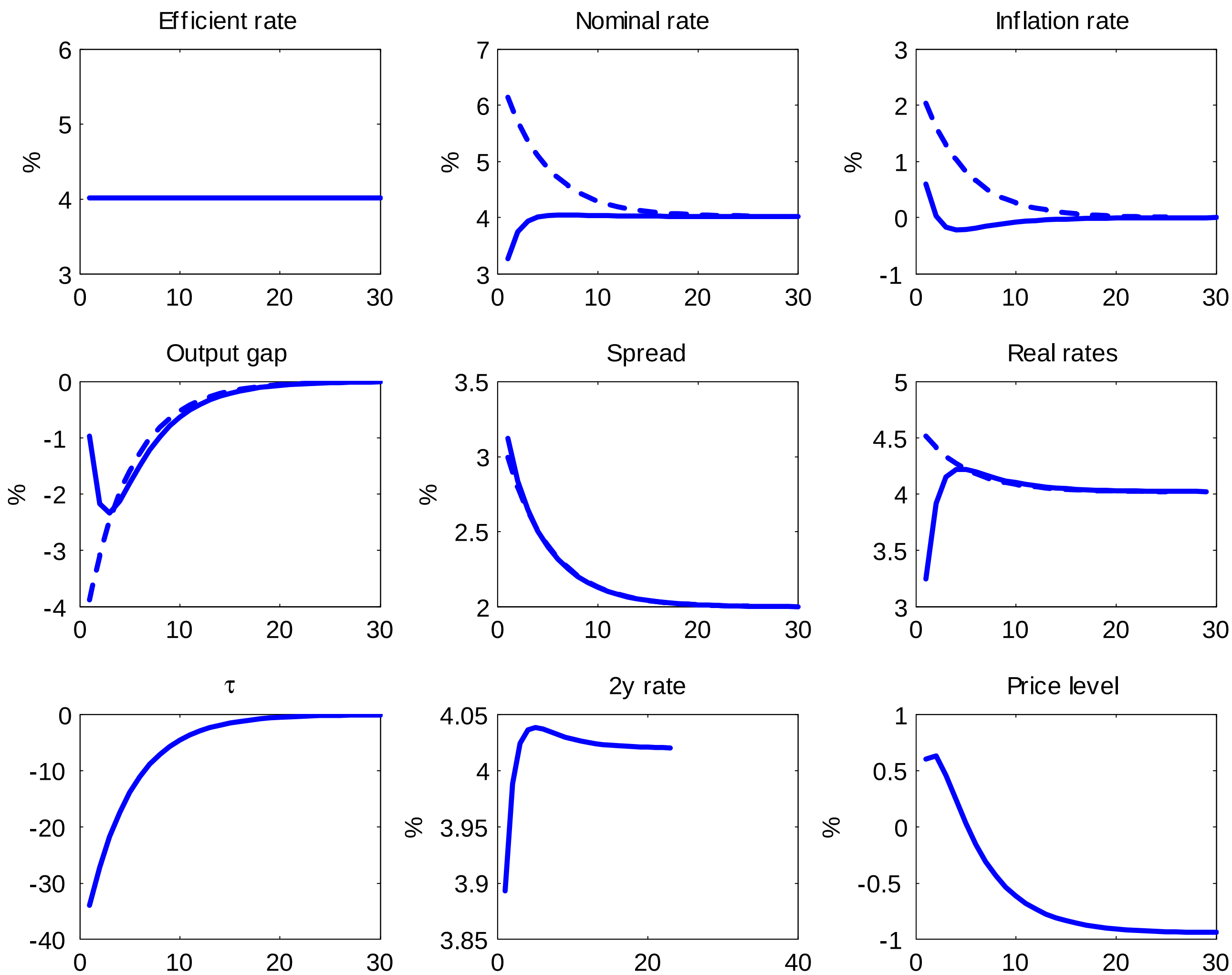

Figure 6: Net worth shock, Taylor rule vs. optimal policy

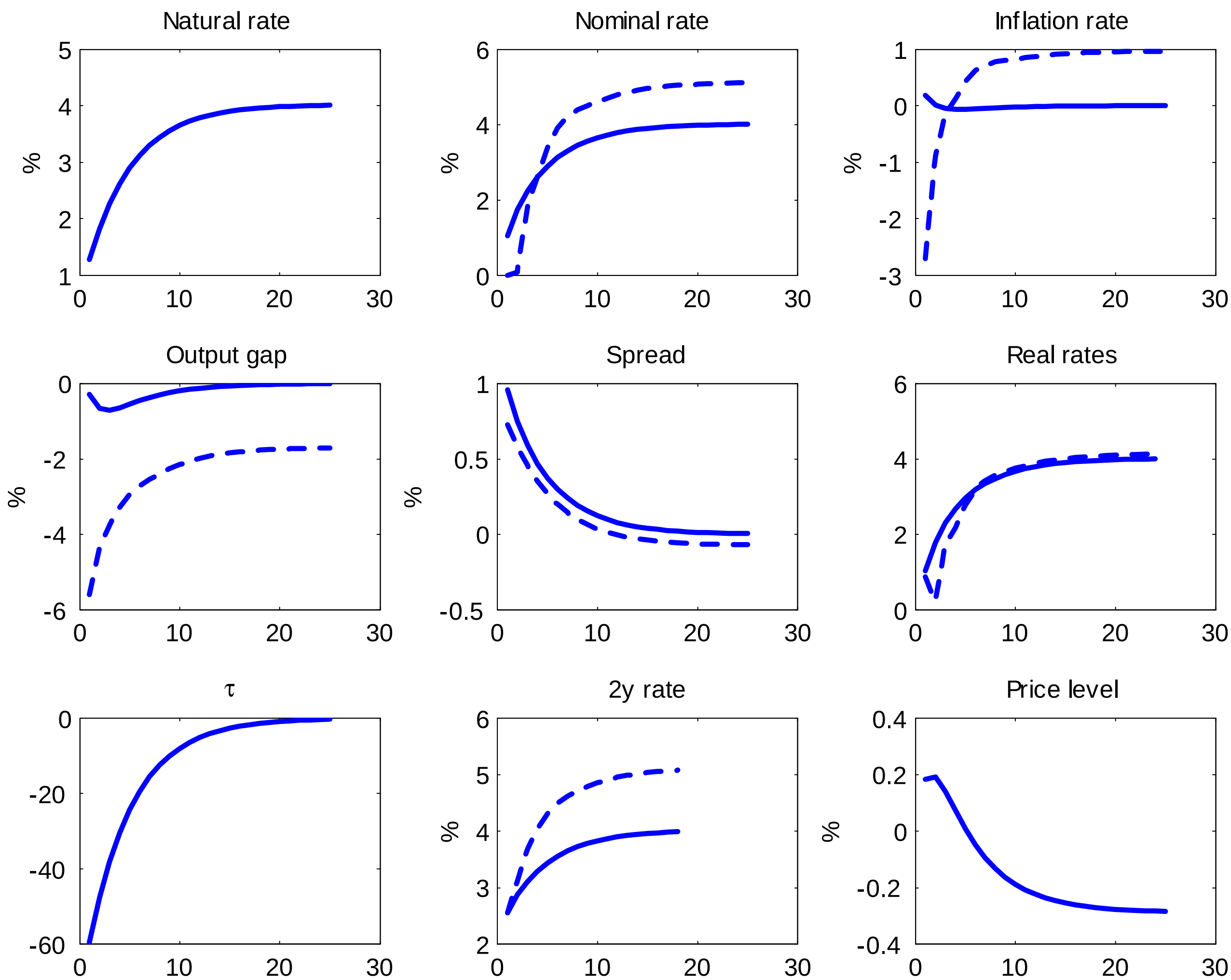

Figure 7: Combined shocks, optimal policy vs. Taylor rule

7 Appendix

A. Coefficients

The coefficients of the system of log-linearized equilibrium conditions are given by

$$\begin{aligned}
\alpha_1 &= -\frac{q}{R}\frac{\mu\frac{f\overline{\omega}}{f_{\overline{\omega}}}\left(\phi_{\overline{\omega}}-\frac{\phi^2}{f_{\overline{\omega}}}\right)}{(1-g_{\overline{\omega}}\Lambda)} \\
\alpha_2 &= \mu\frac{q}{R}\left[-\frac{f\overline{\omega}}{f_{\overline{\omega}}}\left(\phi_{\overline{\omega}}-\frac{\phi^2}{f_{\overline{\omega}}}\right)\frac{\frac{\mu\Phi}{g}}{(1-g_{\overline{\omega}}\Lambda)}+\frac{f\phi}{f_{\overline{\omega}}}-\Phi\right] \\
\alpha_3 &= -\left(\frac{\mu\frac{f}{f_{\overline{\omega}}}\left(\phi_{\overline{\omega}}-\frac{\phi^2}{f_{\overline{\omega}}}\right)+(f_{\overline{\omega}}+\mu\phi)}{f+\frac{\mu f\phi}{f_{\overline{\omega}}}}\right)\frac{\overline{\omega}}{(1-g_{\overline{\omega}}\Lambda)} \\
\alpha_4 &= -\frac{\mu\Phi}{g}\alpha_3-\frac{\mu\frac{f\phi}{f_{\overline{\omega}}}}{f+\frac{\mu f\phi}{f_{\overline{\omega}}}}.
\end{aligned}$$

B. Welfare approximation

Welfare is

$$W_{t_0}=E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^t U_t\right\},$$

where households' temporary utility is given by $U_t=u\left(c_t;\xi_t\right)-v\left(h_t\right)$. This latter can then be approximated as

$$\begin{aligned}
U_t \simeq\ & U+u_c c\left(\widehat{c}_t+\frac{1}{2}\left(1+\frac{u_{cc}c}{u_c}\right)\widehat{c}_t^2\right)-v_h h\left(\widehat{h}_t+\frac{1}{2}\left(1+\frac{v_{hh}h}{v_h}\right)\widehat{h}_t^2\right)+u_{c\xi}c\widehat{c}_t\widehat{\xi}_t \\
&+u_\xi\left(\widehat{\xi}_t+\frac{1}{2}\left(1+\frac{u_{\xi\xi}}{u_\xi}\right)\widehat{\xi}_t^2\right)
\end{aligned}$$

where hats denote log-deviations from the deterministic steady state and c and h denote steady state levels.

Under the functional form $U_t=\xi_t\frac{c_t^{1-\sigma}}{1-\sigma}-\psi\frac{h_t^{1+\phi}}{1+\phi}$, and assuming that in steady state $\xi=1$, households' temporary utility can be rewritten as

$$\begin{aligned}
U_t \simeq\ & \frac{c_t^{1-\sigma}}{1-\sigma}-\psi\frac{h_t^{1+\phi}}{1+\phi}+c^{1-\sigma}\widehat{c}_t-\psi h^{1+\phi}\widehat{h}_t+\frac{1}{2}c^{1-\sigma}\left(1-\sigma\right)\widehat{c}_t^2-\frac{1}{2}\psi h^{1+\phi}\left(1+\phi\right)\widehat{h}_t^2 \\
&+c^{1-\sigma}\widehat{c}_t\widehat{\xi}_t+\frac{c^{1-\sigma}}{1-\sigma}\left(\widehat{\xi}_t+\frac{1}{2}\widehat{\xi}_t^2\right).
\end{aligned}$$

We can now express hours and households' consumption as $h_t = \frac{s_t y_t}{A_t}$ so that $\widehat{h}_t = \widehat{s}_t + \widehat{y}_t - \widehat{a}_t$. Using this expression together with $c_t = y_t$, we can write utility as

$$\begin{aligned}\frac{U_t}{c^{1-\sigma}} \simeq\ & \frac{1}{1-\sigma} - \frac{\psi}{1+\phi}\frac{h_t^{1+\phi}}{c_t^{1-\sigma}} + \left(1 - \frac{\psi h^{1+\phi}}{c^{1-\sigma}}\right)\widehat{y}_t - \psi\frac{h^{1+\phi}}{c^{1-\sigma}}\widehat{s}_t - \frac{1}{2}\left(\frac{\psi h^{1+\phi}}{c^{1-\sigma}}(1+\varphi) - (1-\sigma)\right)\widehat{y}_t^2 \\ & + \frac{\psi h^{1+\phi}}{c^{1-\sigma}}(1+\varphi)\widehat{y}_t\widehat{a}_t + \widehat{\xi}_t\widehat{y}_t - \frac{\psi h^{1+\phi}}{c^{1-\sigma}}(1+\varphi)\widehat{s}_t\widehat{y}_t + \frac{\psi h^{1+\phi}}{c^{1-\sigma}}(1+\varphi)\widehat{s}_t\widehat{a}_t - \frac{1}{2}\frac{\psi h^{1+\phi}}{c^{1-\sigma}}(1+\varphi)\widehat{s}_t^2 \\ & + \frac{1}{1-\sigma}\left(\widehat{\xi}_t + \frac{1}{2}\widehat{\xi}_t^2\right) + \frac{\psi h^{1+\phi}}{c^{1-\sigma}}\widehat{a}_t - \frac{1}{2}\frac{\psi h^{1+\phi}}{c^{1-\sigma}}(1+\psi)\widehat{a}_t^2\end{aligned}$$

or, given that s_t is of second order, as

$$\begin{aligned}\frac{U_t}{c^{1-\sigma}} \simeq \frac{1}{1-\sigma} - \frac{\psi}{1+\phi}\frac{h_t^{1+\phi}}{c_t^{1-\sigma}} & + \left(1 - \frac{\psi h^{1+\phi}}{c^{1-\sigma}}\right)\widehat{y}_t - \psi\frac{h^{1+\phi}}{c^{1-\sigma}}\widehat{s}_t \\ -\frac{1}{2}\left(\frac{\psi h^{1+\phi}}{c^{1-\sigma}}(1+\varphi) - (1-\sigma)\right)\widehat{y}_t^2 & + \frac{\psi h^{1+\phi}}{c^{1-\sigma}}(1+\varphi)\widehat{y}_t\widehat{a}_t + \widehat{\xi}_t\widehat{y}_t + t.i.p.s\end{aligned}$$

Assume a subsidy such that $\frac{\psi h^{1+\phi}}{c^{1-\sigma}} = 1$. Then

$$\frac{U_t}{c^{1-\sigma}} \simeq \frac{1}{1-\sigma} - \frac{1}{1+\phi} - \widehat{s}_t - \frac{1}{2}(\varphi+\sigma)\widehat{y}_t^2 + \left[(1+\varphi)\widehat{a}_t + \widehat{\xi}_t\right]\widehat{y}_t + t.i.p.s.$$

Now recall that $\widehat{y}_t^e = \frac{1}{(\sigma+\varphi)}\left[(1+\varphi)a_t + \widehat{\xi}\right]$. Then

$$\frac{U_t}{c^{1-\sigma}} \simeq \frac{1}{1-\sigma} - \frac{1}{1+\phi} - \widehat{s}_t - \frac{1}{2}(\sigma+\varphi)\widehat{y}_t^2 + (\sigma+\varphi)\widehat{y}_t^e\widehat{y}_t + t.i.p.s$$

This can be rewritten as

$$\frac{U_t}{c^{1-\sigma}} - \left(\frac{1}{1-\sigma} - \frac{1}{1+\phi}\right) \simeq -\frac{1}{2}\frac{\varepsilon\theta}{(1-\theta)(1-\beta\theta)}\widehat{\pi}_t^2 - \frac{1}{2}(\sigma+\varphi)x_t^2 + t.i.p.s.$$

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