Value-Added Exchange Rates: measuring competitiveness with vertical specialization in trade*

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Abstract

This paper updates the conceptual foundations for measuring real effective exchange rates (REERs) to take account of the pervasive use of imported intermediate inputs to produce exports. We derive a value-added REER describing how demand for the value added that a country produces changes as the price of its value added changes relative to competitors. We then compute this index using trade measured in value added terms and prices for value added. We identify substantial differences between value-added and conventional REERs. These are driven by the theory-motivated shift in prices used to construct the value-added REER, not changes in bilateral weights.

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Real effective exchange rates (REERs) are a core piece of macroeconomic data. Designed to gauge the effect of price changes on demand for output originating from each source country, they have wide application. For example, they are commonly used for assessing export competitiveness, judging the size of price adjustments necessary to close external imbalances, and gauging misalignment of nominal exchange rates.[1]

Despite their wide application, conventional REERs are not well suited to analyzing competitiveness when imports are used to produce exports – i.e., with vertical specialization in trade.[2] The problem lies in an outdated interpretation of how countries compete with one another. The conventional REER rests on theoretical foundations provided by the Armington (1969) demand system. In that framework, each country's differentiated 'product' competes against 'products' from other countries in destination markets. Conventional REER formulas define each country's gross output and exports to be that country's 'product,' implicitly assuming that these 'products' are entirely domestically produced. Given the pervasive use of imports to produce exports in the modern international economy, this is problematic.

To fix ideas, consider the production of an iPhone. The conventional Armington approach classifies the iPhone as China's 'product,' and supposes that China competes against other suppliers of digital music players in foreign markets. Given this definition, a rise in the price of an iPhone would imply a loss of competitiveness for China. In reality, China is the final assembly point for the iPhone, one link in a production chain spread over many countries. Therefore, China competes directly against other possible assemblers of iPhones, not suppliers of digitial music players per se. This suggests that what we should be interested in measuring is how demand for assembly services (Chinese value added) changes following changes in the price of those services (Chinese wages). Put differently, we should re-define China's 'product' to be the fragment of iPhone value added produced in China.

This example points to a general idea: with the spread of global supply chains, countries increasingly specialize in adding value at a particular stage of production rather than producing entire finished products.[3] As such, countries compete over supplying value added to foreign markets, not final goods or even gross exports per se. The time is therefore ripe to update the theoretical foundations of the REER to reflect this new reality.

In this paper, we extend the benchmark Armington framework to include cross-border input linkages on the supply side.[4] We assume that gross output is produced by combining

[1]For an overview of applications, see Chinn (2006).

[2]See Hummels, Ishii, and Yi (2001) or Yi (2003) for discussion of vertical specialization.

[3]For many OECD countries and emerging markets, the ratio of value added to gross trade (or domestic content of exports) is now on the order of 60-70% and falling. See Hummels, Ishii, and Yi (2001) or Johnson and Noguera (2012a, 2012b).

[4]In developing this new framework, we draw on a rapidly growing body of work on the construction and use of global bilateral input-output accounting frameworks, including our own previous work in Bems,

domestic value added with both domestic and imported inputs. Further, gross output from each source country is allocated to final and intermediate use in all countries, so gross trade includes both final goods and intermediate inputs. On the demand side, consumer preferences are defined over final goods.

We use this extended framework to derive a formula that links changes in prices of real value added to changes in demand for real value added originating in a given source country. Using this formula, we define a real effective exchange rate for trade in value added. The value-added REER differs from the conventional REER both in the data used to construct weights to aggregate bilateral price changes and in the measure of price changes themselves. Our formula uses trade measured in value added terms to construct bilateral weights, whereas the conventional formula uses gross trade flows. To construct these weights, we draw global input-output tables assembled by Johnson and Noguera (2012b). Further, we use prices of real value added (GDP deflators) to measure price changes, whereas the conventional formula uses changes in consumer prices as a proxy gross output prices.

Despite these differences, the formula we derive 'looks like' conventional Armington-based REER formulas. This may seem initially surprising given that we derive the formula from a model with both supply and demand linkages across borders. The key insight is that, under several parametric assumptions, the gross model collapses to a model in which consumers have preferences defined directly over value added.[5] That is, we can interpret our value-added REER as if it were derived from an Armington-style framework in which countries produce differentiated value added and consumers purchase value added directly from each source country. We emphasize that this result depends on several key assumptions, including having equal elasticities of substitution in production and demand. After presenting the main results with these maintained assumptions, we discuss how results change as we relax them.[6]

Our empirical analysis compares these new value-added REERs indexes to conventional REERs. We find significant differences between alternative indexes, and these differences appear informative about external adjustment (or lack thereof) in salient examples. For example, we find that the US value-added REER has depreciated more than the conventional REER since 1995, whereas Chinese value-added REER has appreciated more than

Johnson, and Yi (2010, 2011), Johnson (2012), Johnson and Noguera (2012a, 2012b).

[5] Two assumptions are important in deriving this result. First, the elasticity of substitution in consumption must be the same as the elasticity of substitution in production. Second, technological progress must be factor augmenting in production. We discuss these important assumptions further and the implications of relaxing them below. We view other assumptions needed to generate the result, such as the CES form for preferences and technologies themselves, as more innocuous given their widespread use in the international literature.

[6] As we relax assumptions, we generate alternative formulas for constructing REERs that are valid when inputs are used to produce exports. These formulas do not use value added trade flows directly, but do make use of data on global input-output linkages.

the conventional REER. Within Europe, we find that the German value-added REER has appreciated more than the conventional REER since 1995, while the opposite is true in Portugal, Ireland, Italy, Greece, and Spain.

To get insight into these differences, we decompose the gaps into components due to changing from gross to value-added weights versus changing from CPIs to value-added prices. We show that gaps are driven mostly by our theory-motivated shift from CPIs to value-added prices. Changes in weights, while sizable and intuitively consistent with anecdotes about the expansion of global supply chains, do not play a large role.[7] The reason is that changes in weights are weakly correlated with price changes vis-à-vis bilateral partners. To make this concrete, note that if price changes were uniform across bilateral partners, then even large changes in weights would have no effect on aggregate REER indexes. The corollary is that even substantial deepening of global supply chains may have small additional effects on gaps between conventional and value-added REERs if future changes are proportional to past changes across partners.

The paper proceeds as follows. In Section 1 we outline the basic framework underlying construction of value added REERs. In Section 2, we discuss the intuition lying behind the value added formulas and link our indexes to existing practice. Section 3 then describes the data, and Section 4 presents results on similarities and differences between our indexes and conventional REERs. We present extensions of our approach under relaxed parametric assumptions in Section 5, and Section 6 concludes the paper.

1 Deriving the Real Effective Exchange Rate

This section presents a framework for computing real effective exchange rates with traded intermediate inputs. The framework includes many countries, each of which produces an aggregate Armington differentiated good that is used as a final good and an intermediate input in production.[8] This Armington framework is chosen explicitly to facilitate comparisons to existing theory and practice for constructing REERs.[9]

In Section 1.1, we describe the basic economic environment and construct linear approximations of the key equations needed to derive real exchange rate formulas. In Section 1.2, we focus a restricted case of the framework in which elasticities of substitution are equal

[7]For example, weights attached to Canada and Mexico for the United States value-added REER fall relative to gross weights. More generally, declines in weights are larger for nearby countries and countries that have signed regional trade agreements.

[8]The framework can be extended to allow for multiple sectors, an extension we discuss in Section 5.

[9]We discuss the relationship between our approach and existing practice further in Section 2.3.

in preferences and production functions.[10] With this restriction, we derive a formula that links demand for value added to prices of value added through a system of value-added trade weights. In doing so, we rely heavily on methods for working with global input-output frameworks developed in Bems, Johnson, and Yi (2010, 2011), Johnson (2012), Johnson and Noguera (2012a, 2012b). We translate these results into a formula for the value-added REER in Section 1.3.

1.1 Framework

We consider a partial equilibrium environment. Specifically, we take changes in the price of value added from each source country as given, and then take these values from data.[11] This approach requires us to specify only three basic components of the economic environment: (1) preferences over final goods, (2) production functions for gross output, and (3) market clearing conditions for gross output. We linearize these three components, along with corresponding price indexes, to form the building blocks for the real exchange rate formula.

1.1.1 Economic Environment

Suppose there are many countries indexed by $i, j, k \in \{1, \ldots, N\}$. Each country is endowed with a production function for an aggregate Armington differentiated good, which is used both as a final good and intermediate input. Gross output in country i, denoted Q_i, is produced by combining domestic real value added, denoted V_i, with a composite intermediate input, denoted X_i. This composite input is a bundle of domestic and imported inputs, where inputs purchased by country i from country j are denoted X_{ji}.

We assume that the production structure takes the nested constant elasticity of subsitution (CES) form:

$$Q_i = \left((\omega_i^v)^{1/\gamma} V_i^{(\gamma-1)/\gamma} + (\omega_i^x)^{1/\gamma} X_i^{(\gamma-1)/\gamma}\right)^{\gamma/(\gamma-1)} \tag{1}$$

$$\text{with} \quad X_i = \left(\sum_j \left(\frac{\omega_{ji}^x}{\omega_i^x}\right)^{1/\rho} X_{ji}^{(\rho-1)/\rho}\right)^{\rho/(\rho-1)}, \tag{2}$$

where the ω's are aggregation weights, γ is the elasticity of substitution between real value added and the composite input, and ρ is the elasticity of substitution among inputs.

We assume that agents in each country have CES preferences defined of over final goods.[12]

[10] We discuss the consequences of relaxing these elasticity restrictions in Section 5.

[11] This partial equilibrium approach follows McGuirk (1987).

[12] Final goods are defined as in the national accounts, including consumption, investment, and government spending. Therefore, though we call the final goods aggregator 'preferences' throughout the paper, it

Denoting the quantity of final goods purchased by country i from country j as F_{ji}, preferences take the form:

$$F_i = \left(\sum_j (\omega^f_{ji})^{1/\sigma} F_{ji}^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}, \tag{3}$$

where ω's here denote preference weights and σ is the elasticity of substitution among final goods.

Two assumptions embedded Equations (1)-(3) are worthy of further comment. First, the choice of CES preferences and technologies here follows the vast majority of papers in international macroeconomics. As written above, we allow the elasticity of substitution to differ between preferences and production, and within nests in the production structure. We impose additional restrictions on these elasticities in Section 1.2, and analyze this general case in Section 5. Second, we have not explicitly included a productivity term in the production function. Our derivation proceeds under the assumption that productivity raises output of real value added, but does not directly increase the efficiency with which real value added and inputs are combined. For example, if we assume that real value added is the product of productivity and a composite factor input – e.g., $V_i = Z_i L_i$ where Z_i is productivity and L_i is a composite factor – then we are implicitly assuming that technological change is factor augmenting. This assumption allows us to derive a formula for the real exchange rate that depends only on observed prices of real value added.[13]

Given these preferences and technology, the standard first order conditions for consumers and competitive firms are:

$$F_{ji} = \omega^f_{ji} \left(\frac{p_j}{P_i} \right)^{-\sigma} F_i \tag{4}$$

$$V_i = \omega^v_i \left(\frac{p^v_i}{p_i} \right)^{-\gamma} Q_i \tag{5}$$

$$X_i = \omega^x_i \left(\frac{p^x_i}{p_i} \right)^{-\gamma} Q_i \tag{6}$$

$$X_{ji} = \omega^x_{ji} \left(\frac{p_j}{p^x_i} \right)^{-\rho} X_i, \tag{7}$$

where p_j is the price of gross output from j, p^v_i is the price of the composite factor, $p^x_i = \left(\sum_j \omega^x_{ji} p_j^{1-\rho} \right)^{1/(1-\rho)}$ is the price of the composite input, and $P_i = \left(\sum_j \omega^f_{ji} p_j^{1-\sigma} \right)^{1/(1-\sigma)}$ is the

might more accurately be described an aggregator that forms a composite final good that can be used for consumption, investment, and government spending.

[13] Neutral technological change implies that competitiveness depends on price changes and technology shocks separately. Setting changes in technology to zero, we recover the main formula with factor-biased technology. We do not know of strong evidence that technological changes takes one form or the other.

final goods price level. In the background, we implicitly assume that all prices are converted into a common currency so that we do not have to carry around the nominal exchange rate.

Recalling that gross output can be used as both a final good and intermediate input, the market clearing condition for gross output is:

$$Q_j = \sum_{k=1}^{N} F_{jk} + X_{jk}. \tag{8}$$

Finally, using the gross production function and prices defined above, we can write the prices of gross output as: $p_j = \left(w_j^v (p_j^v)^{1-\gamma} + w_j^x (p_j^x)^{1-\gamma}\right)^{1/(1-\gamma)}$.

1.1.2 Linear Approximation

To derive the real exchange rate formula, we linearize the first order conditions, price indexes, production functions, and market clearing conditions. We present the linearization here in 'stacked' form that facilitates manipulation of the many country system.[14]

The final goods first order condition and final goods price index can be linearized as: $\hat{F}_{ji} = -\sigma(\hat{p}_j - \hat{P}_i) + \hat{F}_i$, with $\hat{P}_i = \sum_j \left(\frac{p_j F_{ji}}{P_i F_i}\right) \hat{p}_j$. We then define a vector $\mathbb{F}$ to be a N^2 dimensional vector that records final goods shipments: $\hat{\mathbb{F}} = [\hat{F}_{11}, \hat{F}_{12}, \ldots, \hat{F}_{1N}, \hat{F}_{21}, \hat{F}_{22}, \ldots]'$. This allows us to rewrite the first order conditions and price index as:

$$\hat{\mathbb{F}} = -\sigma M_1 \hat{p} + \sigma M_2 \hat{P} + M_2 \hat{F} \tag{9}$$

$$\text{with} \quad \hat{P} = W_F \hat{p}, \tag{10}$$

where $M_1 \equiv I_{N\times N} \otimes \mathbf{1}_{N\times 1}$ and $M_2 \equiv \mathbf{1}_{N\times 1} \otimes I_{N\times N}$. The weighting matrix W_F is an $N \times N$ matrix with ij elements $\frac{p_j F_{ji}}{P_i F_i}$ equal to country i's expenditure on final goods from country j as a share of total final goods expenditure in country i.

Turning to production, the first order conditions for intermediates linearize as: $\hat{X}_i = -\gamma(\hat{p}_i^x - \hat{p}_i) + \hat{Q}_i$ and $\hat{X}_{ji} = -\rho(\hat{p}_j - \hat{p}_i^x) + \hat{X}_i$.[15] These can be stacked in a similar way:

$$\hat{X} = -\gamma \hat{p}^x + \gamma \hat{p} + \hat{Q} \tag{11}$$

$$\hat{\mathbb{X}} = -\rho M_1 \hat{p} + \rho M_2 \hat{p}^x + M_2 \hat{X} \tag{12}$$

$$\text{with} \quad \hat{p}^x = W_x \hat{p}, \tag{13}$$

where $\hat{\mathbb{X}} = [\hat{X}_{11}, \hat{X}_{12}, \ldots, \hat{X}_{1N}, \hat{X}_{21}, \hat{X}_{22}, \ldots]'$ is the N^2 dimensional vector of intermediate

[14] Johnson (2012) uses a similar stacked notation in the analysis of a many country IRBC model.

[15] We do not explicitly linearize the first order condition for real value added (V_i) here because we do not use it in the derivation.

goods shipments.

These first order conditions describe how demand for final and intermediate goods shipped from country i depends on the prices of gross output ($\hat{p}$) from each source, as well as the 'level of demand' in the destination. For intermediate goods, the level of demand depends on total gross output produced in the destination ($\hat{Q}$), while real final goods absorption ($\hat{F}$) influences the level of demand for final goods.

The market clearing conditions can be linearized as:

$$\hat{Q} = S_F \hat{\mathbb{F}} + S_X \hat{\mathbb{X}}. \tag{14}$$

The S_F and S_X matrices collect shares of final and intermediate goods sold to each destination as a share of total gross output in the source country:

$$S_F \equiv \begin{pmatrix} s_1^f & \mathbf{0} & \cdots \\ \mathbf{0} & s_2^f & \cdots \\ \vdots & \cdots & \ddots \end{pmatrix} \quad \text{and} \quad S_X \equiv \begin{pmatrix} s_1^x & \mathbf{0} & \cdots \\ \mathbf{0} & s_2^x & \cdots \\ \vdots & \cdots & \ddots \end{pmatrix}$$
$$\text{with} \quad s_i^f = [s_{i1}^f, \cdots, s_{iN}^f], \quad s_{ij}^f = \frac{p_i F_{ij}}{p_i Q_i},$$
$$s_i^x = [s_{i1}^x, \cdots, s_{iN}^x], \quad s_{ij}^x = \frac{p_i X_{ij}}{p_i Q_i}.$$

Finally, we linearize components of the production function and the gross output price index as:

$$\hat{Q} = [diag(s_i^v)]\hat{V} + W_{QX}\hat{X} \tag{15}$$
$$\hat{X} = W_X \hat{\mathbb{X}} \tag{16}$$
$$\hat{p}_i = [diag(s_i^v)]\hat{p}^v + [diag(s_i^x)]\hat{p}^x, \tag{17}$$

where $s_i^v \equiv \frac{p_i^v V_i}{p_i Q_i}$ and $s_i^x \equiv \frac{p_i^x X_i}{p_i Q_i}$ are the cost shares of real value added and the composite input in gross output. And $W_X = [diag(w_1^x), diag(w_2^x), \ldots]$ with $w_i^x = [w_{i1}^x, \cdots, w_{iN}^x]$ and $w_{ij}^x \equiv \frac{p_i X_{ij}}{p_j^x X_j}$ are shares of individual intermediates in the composite intermediate.

1.2 Demand for Real Value Added

Equations (9)-(17) are nine equations that describe how demand for value added produced by each country depends on prices of value added $\hat{p}^v$ and final demand $\hat{F}$ in all countries. To derive an intuitive formula for demand, we impose one additional restriction here. We assume that elasticities are equal in preferences and production functions: $\sigma = \gamma = \rho$ and we therefore define a new common elasticity parameter η.[16]

The derivation then proceeds in two steps. First, we use the first order conditions and price index to write the change in demand for gross output from country i as a function of price changes of gross output. Second, we convert demand for gross output as a function of gross output prices into the corresponding demand for real value added as a function of prices of real value added.

In Appendix A, we show that Equations (9)-(17) plus the common elasticity assumption imply that demand for real value added is given by:

$$\begin{aligned} \hat{V} = & -\eta \hat{p}^v + [diag(p_i Q_i)]^{-1}[I-\Omega]^{-1}[diag(p_i Q_i)] S_F M_2 W_F [I-\Omega^{'}]^{-1}[diag(s_i^v)]\hat{p}^v \\ & + [diag(p_i Q_i)]^{-1}[I-\Omega]^{-1}[diag(p_i Q_i)] S_F M_2 \hat{F}, \end{aligned} \tag{18}$$

where Ω is a global bilateral input-output matrix with ij elements $\frac{p_i X_{ij}}{p_j Q_j}$ equal to the share of intermediate inputs from country i in gross output in country j. Equation (18) describes how demand for value added from each source country depends on prices and the level of demand for final goods in all countries.

1.3 The Value-Added Real Effective Exchange Rate

Two additional steps turn Equation (18) into a real effective exchange rate formula. First, following standard practice, we set changes in real final demand $\hat{F}$ to zero. This means that the real exchange rate measures the influence of price changes on demand, holding levels of final demand constant. Second, we adopt a country-specific normalization so that weights on price changes sum to one.[17] This normalization ensures that the real effective exchange rate depreciates by x% when all foreign prices increase by x% relative to the domestic price. We discuss intuition for this type of normalization in Section 2.2.1.

To perform the normalization, we split the weighting matrix attached to price changes into the product of a weight matrix, with weights that sum to one, and second matrix containing country-specific normalizations. To keep the notation simple, we define a shorthand notation for the weighting matrix:

$$T \equiv I - [diag(p_i Q_i)]^{-1}[I-\Omega]^{-1}[diag(p_i Q_i)] S_F M_2 W_F [I-\Omega']^{-1}[diag(s_i^v)].$$

[16]The assumption that the elasticity of substitution is the same in aggregation of final and intermediate inputs is common in the trade literature. The assumption that elasticity for aggregation of factors and the composite input – i.e., V_i and X_i – is the same as the elasticity among inputs themselves is less standard. We discuss relaxation of both assumptions in Section 5.

[17]This follows standard practice laid out in McGuirk (1987) and Bayoumi, Jayanthi, and Lee (2006), among others.

Then the change in demand for real value added induced by a change in prices is:

$$\hat{V}_i = -\eta \bar{T}_i \sum_j \frac{T_{ij}}{\bar{T}_i} \hat{p}^v_j, \tag{19}$$

where T_{ij} is the ij element and $\bar{T}_i \equiv \sum_j T_{ij}$ is the row sum of T. We then define the value-added real effective exchange rate as:

$$\Delta \log(VAREER_i) \equiv \sum_j \frac{T_{ij}}{\bar{T}_i} \hat{p}^v_j. \tag{20}$$

The parameters $\bar{T}_i$ and η translate changes in the VAREER into changes in demand for real value added.

2 Interpreting Demand for Value Added

The key to interpreting the VAREER formula is understanding how demand for value added depends on price changes, as encoded in Equation (18). We approach this from two complementary directions. First, we provide a general interpretation emphasizing that demand for value added takes the CES form under the assumptions above, as if preferences were defined directly over consumption of value added. Second, we discuss variations of Equation (18) in two special cases. We then close this section by describing how the value-added REER formulas we present differ from current practice.

2.1 CES Demand for Value Added

Equation (18) says that each country faces a single CES demand schedule for the value added it produces, as if each country sells value added to a single world market.[18] To see this, let us rewrite Equation (18) as:

$$\begin{aligned}
&\hat{V} = -\eta \left(\hat{p}^v - \hat{P}^w\right) + \hat{F}^w \\
&\text{with} \quad \hat{P}^w \equiv [diag(p_i Q_i)]^{-1}[I-\Omega]^{-1}[diag(p_i Q_i)] S_F M_2 W_F [I-\Omega']^{-1}[diag(s^v_i)] \\
&\text{and} \quad \hat{F}^w \equiv [diag(p_i Q_i)]^{-1}[I-\Omega]^{-1}[diag(p_i Q_i)] S_F M_2 \hat{F}.
\end{aligned} \tag{21}$$

The vectors $\hat{P}^w$ and $\hat{F}^w$ contain the aggregate price levels and final demand levels for each country in exporting to the hypothetical world market. We note here that the hypothetical

[18] Our interpretation here mimics logic in Anderson and Yotov (2010).

world market is different for each source country, and elaborate below.

Demand for value added from country i falls when the price of its own value added rises, all else constant, with an elasticity of η. How much demand falls given this price change depends on how each country's own price compares to the aggregate price level of value added on the hypothetical world market. The change in the aggregate price level as perceived by country i is the i^{th} element of $\hat{P}^w$, which is a source-specific weighted average of price changes for value added originating from all countries.

The weighting scheme for mapping from $\hat{p}^v$ to $\hat{P}^w$ has two components. The first component is: $W_F[I-\Omega']^{-1}[diag(s_i^v)]$. Using the price indices for gross output and the composite input, one can show that $\hat{p}=[I-\Omega']^{-1}[diag(s_i^v)]\hat{p}^v$.[19] So then the term $[I-\Omega']^{-1}[diag(s_i^v)]$ converts prices of value added into prices for gross output. Combining this with final expenditure share weights W_F yields the price level for final demand in each destination market as weighted average of prices of value added from all sources. The weighting scheme takes the form: $\hat{P}_j=\sum_k\left(\frac{p_k^v V_{kj}}{P_j F_j}\right)\hat{p}_k^v$, where V_{kj} is the amount of real value added from k embodied in final goods absorbed in j.[20]

The second component is: $[diag(p_iQ_i)]^{-1}[I-\Omega]^{-1}[diag(p_iQ_i)]S_FM_2$. Each ij element records the share of gross output from each source country i used directly or indirectly to produce final goods absorbed in destination j. These weights are equal to the share of value added from source i absorbed embodied in final goods in destination j: $\frac{p_i^v V_{ij}}{p_i^v V_i}$. That is, they are export shares measured in value added terms. These shares measure the importance of destination j in demand for value added from source i. The level of perceived demand ($\hat{F}^w$) is also computed using these value-added export shares.[21]

Combining these elements, we can re-write Equation (21) in summation notation as:

$$\begin{aligned}
&\hat{V}_i=-\eta\left(\hat{p}_i^v-\hat{P}_i^w\right)+\hat{F}_i^w\\
&\text{with}\quad \hat{P}_i^w=\sum_j\left(\frac{p_i^v V_{ij}}{p_i^v V_i}\right)\hat{P}_j\quad\text{where}\quad \hat{P}_j=\sum_k\frac{p_k^v V_{kj}}{P_j F_j}\hat{p}_k^v,\\
&\text{and}\quad \hat{F}_i^w=\sum_j\left(\frac{p_i^v V_{ij}}{p_i^v V_i}\right)\hat{F}_j.
\end{aligned}\tag{22}$$

Setting $\hat{F}_j=0$ for all j, this can be manipulated to write the definition of the VAREER in

[19]This uses Equations (13) and (17), along with the fact that $diag(s_i^x)W_X=\Omega'$.

[20]Note that $\sum_k\frac{p_k^v V_{kj}}{P_j F_j}=1$, since final goods are 100% value added attributable to some source country.

[21]This weighting scheme is identical to the final demand weights in Bems, Johnson, and Yi (2010). In that paper, we assumed that technology and preferences were both Leontief (i.e., $\eta=0$). Hence demand for value added depended on value-added exports weighted changes in final demand, but was independent of price changes in that paper. An alternative way to interpret Bems et al. is that we assumed that price changes were zero (i.e., $\hat{p}=0$). Equation (18) generalizes this result by dropping this restrictive assumption.

a form that mimics commonly used formulas:

$$\hat{V}_i = -\eta \bar{T}_i \underbrace{\left[\sum_{j \neq i} \left(\frac{1}{\bar{T}_i} \sum_k \left(\frac{p_i^v V_{ik}}{p_i^v V_i} \right) \left(\frac{p_k^v V_{jk}}{P_k F_k} \right) \right) \left(\hat{p}_i^v - \hat{p}_j^v \right) \right]}_{\Delta \log(VAREER_i)} \tag{23}$$

$$\text{with} \quad \bar{T}_i = 1 - \sum_k \left(\frac{p_i^v V_{ik}}{p_i^v V_i} \right) \left(\frac{p_k^v V_{ik}}{P_k F_k} \right),$$

where we have used the fact that $\sum_j \sum_k \left(\frac{p_i^v V_{ik}}{p_i^v V_i} \right) \left(\frac{p_k^v V_{jk}}{P_k F_k} \right) = 1$ to define $\bar{T}_i$.[22] Thus, the VAREER captures a normalized version of the relative price change $\hat{P}^w - \hat{p}^v$.

One final point is that this CES-demand interpretation suggests an alternative way to derive the main REER result. Specifically, the same formulas can be derived from preferences specified directly over value added coming from different countries, as in $F_i = \left(\sum_j (\omega_{ji}^v)^{1/\eta} V_{ji}^{(\eta-1)/\eta} \right)^{\eta/(\eta-1)}$, where ω_{ji}^v is now a value-added preference weight. These preferences generate a CES demand system that can be combined with market clearing conditions for value added to yield Equation (22). Thus, one can in the end re-interpret the VAREER formula as derived from an Armington demand system for value added.[23]

2.2 Interpretation in Two Special Cases

To aid in understanding the value-added REER formula, we discuss variations on Equations (18) and (22) in two special cases. The first case has no intermediate inputs in production, which facilitates interpretation of the weighting of price changes in REER formulas. The second case allows for input trade, but assumes imports are used to produce exports for only one bilateral pair. This case allows us to discuss how vertical specialization influences computation of REERs in a simple concrete case.

[22] We follow convention here and write the real exchange rate index so that increases mean appreciation and decreases mean depreciation.

[23] We have chosen to take the alternative approach of specifying the framework in gross terms for two reasons. First, in the abstract, it is difficult to motivate the assumption that consumers have preferences defined directly over value added from particular sources. One contribution of our derivation is then to show that in fact these direct preferences over value added emerge from a gross model under certain assumptions. Second, CES preferences over value added emerge from the framework only under the assumption that the elasticity of substitution is the same in production and preferences. The full gross framework allows us to explore the robustness of our value-added approach to relaxation of this assumption, as in Section 5.

2.2.1 Case I: no intermediate inputs

Suppose that we apply Equation (18) in a model with no intermediates, so that Ω is a matrix of zeros. In this example, value added is equal to gross output, so $Q_i = V_i$ and hence $p_i = p_i^v$. Further, exports consist entirely of final goods, which are themselves produced entirely out of domestic value added: $V_i = \sum_j F_{ij}$.

Demand for value added is given by: $\hat{V} = -\eta \left[I - S_F M_2 W_F\right] \hat{p}^v + S_F M_2 \hat{F}$. Setting $\hat{F}$ to zero and re-writing this in summation notation, we get:

$$\hat{V}_i = -\eta \hat{p}_i^v + \eta \sum_j \left(\frac{p_i^v F_{ij}}{p_i^v V_i}\right) \hat{P}_j \quad \text{with} \quad \hat{P}_j = \sum_k \left(\frac{p_k^v F_{kj}}{P_j F_j}\right) \hat{p}_k^v \tag{24}$$

And note that we could replace $p_i^v F_{ij}$ with gross exports EX_{ij} or $p_i^v V_i$ with $p_i Q_i$ in the formula because the model makes no distinction between final goods, gross output, or value added.[24] Essentially, $\frac{EX_{ij}}{p_i^v V_i}$ is the share of value-added exports in total value added in this case.

To see how the weighting system works, let us suppose that only the price of output in country i changes: $\hat{p}_i^v \neq 0$ and $\hat{p}_k^v = 0 \ \forall \ k \neq i$. In this event, the amount by which aggregate prices rise in each destination is $\hat{P}_j = \left(\frac{p_i^v F_{ij}}{P_j F_j}\right) \hat{p}_i^v$. Note that the increase in the destination market price is larger when i has a large market share in j. A large destination market share softens the extent to which country i loses market share in the destination, because it is essentially competing against itself. When competition is stiff and country i has only a small share of the destination, then any change in its price leads to a large decline in demand in the destination. To aggregate individual changes in competitiveness across markets, each market is weighted according to how much country i sells to the destination.

This basic weighting scheme underlies construction of REERs. However, REERs do not use these weights directly, but rather use a modified version of these weights that sum to one. To illustrate the purpose of this normalization, consider a different experiment. Suppose that all foreign prices double ($\hat{p}_k^v = 1 \ \forall \ k \neq i$), but price in country i is unchanged. Then $\hat{V}_i = \eta \sum_j \left(\frac{p_i^v F_{ij}}{p_i^v V_i}\right) \left[1 - \left(\frac{p_i^v F_{ij}}{P_j F_j}\right)\right]$, where we have used the fact that $\sum_k \frac{p_k^v F_{kj}}{P_j F_j} = 1$. Then the effective relative price change is less than one.[25] Thus, a doubling of foreign prices does not lead to a doubling of the effective relative price. The conventional normalization introduced in Section 1.3 eliminates this problem.

[24] As written, Equation (24) also holds in a case with domestic inputs, but no trade in inputs. In that alternative case, $p_i^v F_{ij} = EX_{ij}$ continues to hold, but $p_i^v V_i \neq p_i Q_i$.

[25] To be clear: $\sum_j \left(\frac{p_i^v F_{ij}}{p_i^v V_i}\right) \left[1 - \left(\frac{p_i^v F_{ij}}{P_j F_j}\right)\right] < 1$.

2.2.2 Case II: restricted input trade

We now turn to a case in which there are no domestic intermediates, but there is restricted trade in inputs. We assume that country 1 exports inputs to country 2, and no other country exports or imports inputs. Put differently, $\Omega_{12} > 0$ is the only non-zero element of Ω. In this event, Equation (22) is the correct formula for demand, so we focus on interpreting it in this special case.

Starting with destination price indexes ($\hat{P}_j$), computing $W_F[I - \Omega']^{-1}[diag(s_i^v)]$ yields the weights to attach to value added prices. These can be written in the form:

$$\hat{P}_j = \left(\frac{p_1 F_{1j} + p_2 F_{2j}\Omega_{12}}{P_j F_j}\right)\hat{p}_1^v + \left(\frac{p_2 F_{2j}(1-\Omega_{12})}{P_j F_j}\right)\hat{p}_2^v + \sum_{k\neq 1,2}\left(\frac{p_k F_{kj}}{P_j F_j}\right)\hat{p}_k^v. \tag{25}$$

Here the weight on $\hat{p}_1^v$ is adjusted upwards and the weight on $\hat{p}_2$ is adjusted downward relative to the share of final goods imported from each country by j. This reflects the fact that country 1 ships inputs to country 2 that are embodied in final goods shipments F_{2j}. Therefore, the fraction Ω_{12} of F_{2j} is value added originating in country 1.[26]

These price indexes get weighted by $[diag(p_i Q_i)]^{-1}[I - \Omega]^{-1}[diag(p_i Q_i)]S_F M_2$ in constructing the hypothetical world price index. For country 1, demand for real value added can be written as:

$$\hat{V}_1 = -\eta\hat{p}_1^v + \eta\sum_j\left(\frac{p_1 F_{1j} + \Omega_{12} p_2 F_{2j}}{p_1 Q_1}\right)\hat{P}_j, \tag{26}$$

where $\hat{P}_j$ is given by Equation (25).

How do we interpret the destination weights? Note that $p_1 Q_1 = p_1^v V_1$ and $p_1 F_{1j} + \Omega_{12} p_2 F_{2j} = p_1^v V_{1j}$ for country 1, so these destination weights are simply equal to the share of value added from country 1 consumed in country j (i.e., $\frac{p_1^v V_{1j}}{p_1^v V_1}$). Some of the value added from country 1 ($p_1^v V_{1j}$) is consumed directly in final goods shipped from country 1 ($p_1 F_1 j$), and some of it is consumed indirectly embodied in final goods shipped from country 2 ($\Omega_{12} p_2 F_{2j}$).

Turning to country 2, demand for real value added can be written as:

$$\begin{aligned}\hat{V}_2 &= -\eta\hat{p}_2^v + \eta\sum_j\left(\frac{p_2 F_{2j}}{p_2 Q_2}\right)\hat{P}_j,\\ &= -\eta\hat{p}_2^v + \eta\sum_j\left(\frac{(1-\Omega_{12})p_2 F_{2j}}{(1-\Omega_{12})p_2 Q_2}\right)\hat{P}_j.\end{aligned} \tag{27}$$

From the first to the second line, we simply multiply and divide the destination weight by

[26] Linking this back to notation in Equation (22), $p_1 V_{1j} = p_1 F_{1j} + p_2 F_{2j}\Omega_{12}$, $p_2 V_{2j} = p_2 F_{2j}(1-\Omega_{12})$, and $V_{kj} = p_k F_{kj}$ for $k \neq 1, 2$.

$1 - \Omega_{12}$ to convert the gross output share $\frac{p_2 F_{2j}}{p_2 Q_2}$ into a value added share $\frac{p_2^v V_{2j}}{p_2^v V_2}$. So these weights also equal the share of value added from country 2 consumed in j.

In both cases, destinations are weighted by value-added trade shares, which means that these shares tell us how important destination j is as a source of demand for country i. Further, the share of value added from i in final spending in j captures how important price changes in i are in determining the price level in j. The takeaway from this example is trade measured in value added terms captures how production linkages influence evaluations of competitiveness. When inputs are traded, neither final goods shipments nor gross exports suffice to evaluate competitiveness.

2.3 Conventional Real Effective Exchange Rates

Stepping back, we pause to discuss current approaches to construction of real exchange rate indexes.[27] Constructing the REER requires making choices about (a) how to measure relative price changes, and (b) what weights to attach to those price changes.

Starting with prices, there is wide agreement among data providers: the Federal Reserve, OECD, ECB, and IMF all construct their main index using aggregate consumer price indexes (CPIs).[28] This choice is typically motivated by pragmatism, not theory.[29] The motivation for constructing REERs – i.e., measuring competitiveness (with or without intermediates) – suggests that one would like to measure the price of output supplied by a country to world markets. Since the CPI includes the price both of what a country produces as well as what it consumes through imports, it seems ill-suited to this purpose.[30] In contrast, we use value-added prices, as proxied by GDP deflators, which are more consistent with the underlying theory.

There is much less agreement among data providers about what weights to use in construction of the REER index. Most use weights intended to capture the implications of price changes for competitiveness (i.e., demand) as we do. However, details regarding how weights are constructed and how/whether different sectors are included differ substantially.

[27] On methods used, see Loretan (2005) for the Federal Reserve, De Clercq, Fidora, Lauro, Pinheiro, and Schmitz (2012) for the ECB, and Durand, Simon, and Webb (1992) for the OECD. References for the IMF are included below.

[28] Some use unit labor costs and/or producer price indexes in supplemental indexes.

[29] For example, CPIs are available for a wide range of countries at high frequencies. Moreover, measuring consumer prices via retail surveys is conceptually more straightforward than measuring the price of real value added. Finally, GDP deflators are revised, whereas CPIs are typically unrevised. We recognize all these advantages of using the CPI in measuring relative prices. The counterargument to using them is that theoretical consistency is also an important objective. Further, in the empirical work below, we show that differences between GDP deflators and CPIs appear to contain real information.

[30] Even if all goods in the consumer price index were produced domestically (or tracked domestically produced goods prices one-for-one), CPI weights reflect consumption shares, not production shares.

One point of common ground is that all data providers use gross trade data to construct the weights.

The REER index that most closely matches the approach we take is provided by the IMF.[31] The IMF constructs weights using an analog to Equation (23) for manufactures and the bulk of services.[32] In the language of Section 2, the IMF constructs both destination weights and market shares using data on gross sales. Gross sales for country i to country j is measured using gross export data (i.e., $Sales_{ij} \equiv EX_{ij}$), and gross sales for country i to itself is constructed as gross output minus total gross exports ($Sales_{ii} = p_i Q_i - \sum_{j \neq i} EX_{ij}$). Using these gross sales and CPIs, we can define an IMF-style analog to Equation (23):

$$\Delta \log(REER_i) = \left[\sum_{j \neq i} \left(\frac{1}{\tilde{T}_i} \sum_k \left(\frac{Sales_{ij}}{p_i Q_i} \right) \left(\frac{Sales_{jk}}{\sum_l Sales_{lk}} \right) \right) \left(\hat{p}_i^{cpi} - \hat{E}_{i/j} - \hat{p}_j^{cpi} \right) \right]$$
$$\text{with} \quad \tilde{T}_i = 1 - \sum_k \left(\frac{Sales_{ii}}{p_i Q_i} \right) \left(\frac{Sales_{ik}}{\sum_l Sales_{lk}} \right), \tag{28}$$

where $E_{i/j}$ is the nominal exchange rate that converts CPIs into a common currency. Henceforth we refer to $REER_i$ as the 'conventional REER.'

Given discussion in previous sections, the natural question here is: what does this conventional REER measure? Unfortunately, we do not have a good answer to this question. Specifically, the framework above does not yield this REER formula to describe demand for real value added or gross output under any assumptions about input use. The discussion above demonstrates that it does not emerge under the assumption either that there are no inputs are used in production or that both domestic and foreign inputs are used in production. We show in Appendix B that it also does not emerge if only domestic inputs are used in production. Further, no assumptions suggest using consumer prices to proxy for either gross output or value added prices, as in Equation (28). As we discuss below, the use of consumer prices in place of gross output prices is not an innocuous substitution.

[31] The theory underlying the IMF index is described in McGuirk (1987), Desruelle and Zanello (1997), and Bayoumi, Jayanthi, and Lee Jayanthi (2005).

[32] The IMF treats commodities and tourism differently. The index assumes that commodities are homogeneous and adjusts weights accordingly, and it uses information on tourist arrivals to construct tourism trade shares. It then aggregates weights for manufactures/services, commodities, and tourism based on shares in total trade. Finally, it is worth noting that the IMF does not directly use data for the bulk of trade in services. Rather it calculates bilateral weights for manufactures, and applies these shares to both manufactures and services.

3 Data

To compute the conventional and value-added REERs, we need two main pieces of data. First, we need trade measured in both gross and value-added terms. For this, we rely on a new dataset tracking trade in value added over the 1970-2009 period developed in Johnson and Noguera (2012b). Second, we need data on price changes, which we take from standard sources. We describe both pieces of data in turn.

3.1 Trade and Input-Output Data

Johnson and Noguera (2012b) assembles an annual sequence of global input-output tables covering 42 countries and a composite rest-of-the-world region from 1970 to 2009.[33] We briefly summarize their data sources and general approach here, and refer the interested reader to that paper for details.

Raw data on production, trade, demand, and input-output linkages comes from several sources, including the OECD Input-Output Database, the UN National Statistics Database, the NBER-UN Trade Database, and the CEPII BACI Database. Johnson and Noguera harmonize these sources at the four-sector level to create a sequence of internally consistent annual national input-output tables and bilateral trade datasets.[34] In each year, they then link the national input-output tables together using the bilateral trade data to form a synthetic global input-output table.[35] This global table tracks shipments of both final and intermediate goods between countries.

The resulting framework contains all the non-price information needed to parameterize the model in Section 1.1 and calculate trade in value added at the four sector level. In the benchmark calculation below, we aggregate value added trade flows across these four sectors and compute real exchange rates for the aggregate economy directly. For aggregation to be innocuous, we must implicitly assume that relative price changes within countries are not important for aggregate real exchange rate dynamics. In an extension, we compute

[33] The 42 countries include the OECD plus major emerging markets (including Brazil, China, India, Mexico, and Russia). Remaining countries are aggregated into a composite rest-of-the-world. Because they do not have input-output data for these countries, they assume that all exports from the 42 countries to the rest-of-the-world are absorbed there. Further, data for the Czech Republic, Estonia, Russia, Slovakia, and Slovenia only becomes available from the early 1990's. These countries are included in the rest-of-the-world composite during the first two decades. Overall, the rest-of-the-world region accounts for about 10-15% of world trade and GDP in most years.

[34] The four composite sectors include agriculture and natural resources, non-manufacturing industrial production, manufacturing, and services.

[35] To do this, Johnson and Noguera make two proportionality assumptions within each sector. First, they split imports from each source country between final and intermediate use by applying the average split across all sources for that destination. Second, they split those imported intermediates across purchasing sectors by applying shares of total imported intermediate use in the destination.

real exchange rates using two sectors: goods (including agriculture and natural resources, non-manufacturing industrial production, and manufacturing) and services.

3.2 Price Data

We take price data from several sources. The two most important pieces of data are the value-added (GDP) deflators and consumer price indexes. We take consumer price data from the IMF's World Economic Outlook, and obtain value added deflators from the World Bank's World Development Indicators. These are available for each of the 42 countries in our sample. In our calculations, we set price changes in the rest-of-the-world to zero due to data availability and aggregation problems. Finally, in one figure below we use data on price indexes for gross output, which we obtain from the EU KLEMS database.

4 Computing the Value-Added REER

To recap, our VAREER differs from the conventional REER in two ways: the weights attached to bilateral relative price changes, and the measure of prices changes themselves. We open this section by comparing data on weights and prices directly. We then combine these data to compute both conventional and value-added REERs and decompose differences between them.

4.1 Differences in Weights

We compute weights attached to bilateral price changes as in Equation (23) for the VAREER and Equation (28) for the conventional REER. These weights are a normalized combination of destination weights and market share weights. Because we are interested in comparing the two approaches, we focus on differences in the weights in this section. In Table 1, we report differences between the VAREER weight and the conventional REER weight across alternative destinations for each source country in 2005.[36] Because weights for both the VAREER and REER are normalized to sum to one, the rows of the table sum to zero. Thus, this table captures reallocation in weights across bilateral partners.

Bilateral weights generally move in intuitive directions, falling among partners for which the ratio of value-added to gross bilateral trade is relatively low.[37] Consider a few examples.

[36] We report results for a few large countries separately and group remaining countries into composite regions.

[37] See Johnson and Noguera (2012a, 2012b, 2012c) for extensive analysis of the ratio of value-added to gross trade. The results we document below regarding how VAREER versus REER weights adjust within/outside regions, for pairs with RTAs, and with distance all echo those results.

For the United States, the weight attached to Canada and Mexico falls by 6 percentage points and rises correspondingly across other partners. For France, the weight on Eurozone partners falls by 7 percentage points, with the largest increase in weight on the United States. Similar patterns hold for other European countries. Asian countries (e.g., Japan and Korea) see declines in weights attached China. In all these cases, gross trade flows and conventional REER weights are inflated relative to value added weights due to production sharing.

Flipping perspective in the table, destinations systematically differ in whether they receive larger or smaller weights. Weights attached to the United States systematically rise, consistent with the idea that the U.S. is downstream in the production chain. Weights attached to the Eurozone systematically fall for both Eurozone and Other EU source countries, reflecting the tight integration of the European production structure and hence inflation in gross cross-border flows. Weights attached to China sometimes rise and sometimes fall, with declines concentrated among Asian source countries.

One message that emerges from this discussion is that adjustments appear to be larger within regions than across them. We document this result explicitly in Table 2. For each country, we compute weights attached to broad regions (Asia, EU, NAFTA, and Other) by summing across partners within those regions, and then we compute the mean weight across source countries within each region. Results indicate that the typical Asian country sees a decline in weights attached to other Asian countries of 5.5 percentage points. The declines for EU and NAFTA countries with intra-regional trade partners is similar, at 5-6 percentage points.

There are several reasons why geography matters for adjustment of weights. First, regional trade agreements (RTAs) are associated with increased production sharing, lower value-added to export ratios, and hence declines in weights. In our data, the typical country has a VAREER weight between 4-5 percentage points lower for countries with which it has an RTA relative to countries with whom it has no RTA.[38] Second, distance tends to be an impediment to development of cross-border supply chains. Distance to trade partners is positively correlated with the difference between VAREER and conventional REER weights. That is, partners that are nearby tend to have the largest declines in VAREER relative to REER weights, mostly driven by large negative adjustments among relatively close partners with population-weighted distances of less than 5000km.

Looking through time, the reassignment of weights is more important now than in the past. To document this, we compute the 'city-block distance' between trade weights mea-

[38] We use data on trade agreements assembled by Scott Baier and Jeffrey Bergstrand: `http://www.nd.edu/~jbergstr/`. We define an RTA to be a free trade agreement or stronger.

sured in gross and value added terms for each year and each country as:

$$d_{it} = \sum_j \left| w_{ij}^v - w_{ij}^g \right|,$$

where $w_{ij}^v = \frac{1}{T_i} \sum_k \left(\frac{p_i^v V_{ik}}{p_i^v V_i} \right) \left(\frac{p_k^v V_{jk}}{P_k F_k} \right)$ and $w_{ij}^g = \frac{1}{\tilde{T}_i} \sum_k \left(\frac{Sales_{ij}}{p_i Q_i} \right) \left(\frac{Sales_{jk}}{\sum_l Sales_{lk}} \right)$.

Figure 1 plots this measure over time for the Germany, Japan, and the United States along with the cross-country median in each year. The extent of reassignment increased slowly during the 1970-1990 period and then rose rapidly over the last two decades, coincident with sharp changes in the value added of trade during this later period [Johnson and Noguera (2012b)].[39] This increase implies that to the extent that changing weights matters, we would expect this to be more important in recent data.

In constructing the VAREER, we have to make a choice about whether to use fixed or variable weights. For the remainder of the calculations in this section, we opt for fixed weights computed in the year 2000. The reason for this is that it closely mimics the fixed weight formulas used by the IMF. For reasons that will be clear below, we do not believe that time-varying weights will have a large influence on the VAREER. [Aside: we plan to report the index with time-varying weights in future drafts.]

4.2 Differences in Prices

Turning from weights to prices, we compare value-added deflators (used in the VA REER index) and consumer price indexes (used in the conventional REER index). In Figure 2, we plot the proportional difference between the aggregate GDP deflator and CPI for several representative countries from 1990-2009. For each country, we normalize the relative price of value added to consumer prices to be one in 2000, so the axis should be read as the cumulative percentage change in value added relative to consumer prices from 2000 levels.

As is evident, there are large and persistent differences in the alternative price measures. In Japan and the United States, the price of value added falls relative to consumer prices over the period, though relative prices level off for the United States after 2000. Spain and the United Kingdom see rising prices of value added relative to consumer prices. Finally, Korea sees value added prices first rise then fall relative to consumer prices. Suffice it to say that these differences imply that switching to prices of value added is likely to generate differences between the VAREER and conventional REER.

To interpret differences between CPI and GDP deflator, it is instructive to decompose the difference into (a) differences between value added versus gross output prices ($\hat{p}^v - \hat{p}$),

[39] The increase in reassignment is robust to using alternative distance metrics.

and (b) differences between gross output and consumer prices ($\hat{p} - \widehat{p^{cpi}}$):

$$\widehat{p^{v}} - \widehat{p^{cpi}} = \underbrace{\hat{p}^{v} - \hat{p}}_{\text{VA terms of trade}} + \underbrace{\hat{p} - \widehat{p^{cpi}}}_{\text{approximation}} . \tag{29}$$

The first component ($\hat{p}^v - \hat{p}$) captures differences between gross output and value added prices. These differences grow out of the gross versus value added distinction in our framework. Gross output prices are a weighted average of the prices of value added in all countries, with weights that correspond to cost shares for inputs used directly or indirectly to produce gross output from different source countries.[40] For example, if the value added price (e.g., wage) in country i increases, the price of gross output in i will increase less than proportionally since that country uses imported inputs in production. This logic implies that $\hat{p}^v - \hat{p}$ captures changes in the value added terms of trade.

The second component $\hat{p} - \widehat{p^{cpi}}$ captures differences between each country's gross output price and its consumer prices. As mentioned above, conventional REER calculations use consumer prices rather than gross output prices for pragmatic reasons. Therefore, we think this price gap as simply reflecting an approximation error that arises when the consumer price changes are a bad proxy for gross output price changes. There are several reasons why we might expect this approximation to be imperfect. First, the terms of trade factor in here as well. Consumer prices are weighted averages of gross output prices from all countries (i.e., $\hat{P} = W_F\hat{p}$), so changes in the gross output terms of trade drive a wedge consumer prices and a country's own gross output price.[41] Second, the CPI measures consumer prices rather than supply-side prices. So, further deviations can be attributed to differences in weights that the CPI assigns to components of total demand. For example, CPI assigns zero weight to expenditures on nonresidential investment.

To illustrate how the gap between value added and consumer prices breaks down in practice, we plot the components of Equation (29) in Figure 3, focusing on the same six countries depicted in Figure 2. Both components are important in explaining differences between value added and consumer prices, though the relative importance of each component differs across figures. For example, gross output and value added prices track each other closely in Germany, but growth in consumer prices persistently outstrips growth in either prices for value added or output over this period. The opposite is true in the United Kingdom. Other countries like Spain see the exact opposite pattern, where gross output and CPI prices track each other, and the gap between value added and gross output prices is large.

[40] To be precise, we noted above that $\hat{p} = [I - \Omega']^{-1}[diag(s_i^v)]\hat{p}^v$. See also the appendix.

[41] Note that the first component was affected by terms of trade in value added prices, while here the relevant variable is terms of trade in gross prices.

Overall, this evidence points to both the distinction between gross output and value added, as well as the approximation of output prices with consumer prices, as important in understanding gaps between value added and consumer prices. Explaining differences in price measures in detail for individual countries lies outside the scope of this paper. However, one thing we can do in a straightforward way is compare gross output prices in data versus our model.

In our framework, gross output prices are a weighted average of prices of value added, with weights given by the cross-country structure of input-output linkages. We compute these model-based prices by feeding observed GDP deflators for all countries into the framework, and plot the resulting gross output prices along with data on gross output prices in Figure 4.[42] Despite the crudeness of this exercise, model-based prices replicate both short- and medium-term patterns in the data reasonably well. This suggests to us that deviations between value added and gross output prices identified above reflect variation in the value-added terms of trade.

4.3 Differences in REER Indexes

Combining data on weights and prices, we compute the VAREER and conventional REER indexes. To focus our discussion, we plot results for some important and much-discussed exchange rates over the 1995-2009 period: the United States and China in Figure 5 and selected Eurozone countries in Figures 6 and 7. We plot REER series for all countries in the web appendix.

For each country, we create two figures to emphasize different aspects of the data. One figure plots the level of the log real exchange rate index over time, with the level of each series normalized to zero in 1995. This figure is useful for gauging the cumulative effect of deviations between REER and VAREER growth over time. The second figure plots the actual annual growth rates in the indexes (i.e., log changes in indexes), along with the difference in growth rates. In reading these figures, it is helpful to recall that an increase in the index represents an appreciation of the real exchange rate.

Starting with Figure 5, we compare real exchange rates for China and the United States. For China, there are sizable fluctuations in the conventional REER, but no obvious trend.

[42] We perform this exercise using a two-sector version of the framework, splitting goods and services. We then feed in separate value added deflators for goods and services. We also performed this calculation using aggregate GDP deflators only (results not reported here). We found that disaggregating to two sectors substantially improved the fit of the model-based prices with actual data. We therefore suspect further disaggregation would allow the model to fit the data even better. Also, it is useful to note that we are using data on gross output and value added deflators from separate sources, so internally consistent data from a single source would also likely improve the fit.

Since the late 1990s or early 2000s, the REER depreciated then appreciated, but only back to its initial 2000 level. In contrast, the VAREER registers a significant appreciation of the Chinese real exchange rate during the 2000s (on the order of 15-20 percentage points) because it depreciates less sharply during the early 2000s and appreciates more sharply thereafter. Put differently, there is a persistent gap between changes in the VAREER and REER for China, as depicted in the right panel. The United States is a mirror reflection of these relative price movements. Like China, the US VAREER and REER diverge after 2000, but the United States sees a larger depreciation in the VAREER than is picked up by the conventional REER (on the order of 5-10 percentage points since 2000).

Turning to Figures 6 and 7 for Europe, we also see significant divergences between VAREERs and REERs. Starting with Germany, the VAREER depreciates more strongly over the post-1995 period than does the conventional REER. At the same time, the VAREER tends to appreciate more strongly than the VAREER in the Portugal, Italy, Ireland, Greece, and Spain (i.e., the PIIGS) over the post-1995 period. The details of these adjustments naturally vary across countries. For example, the divergence in VAREER vs. REER levels for Ireland can be traced to the late 1990's, a period in which the REER appreciates while the VAREER actually depreciates in our data. In contrast, for Spain, the divergence materializes during the early 2000s, during which the pace of appreciation of the VAREER outstrips the appreciation as measured by the REER.

Suffice it to say, the VAREER appears to register stronger appreciations/depreciations in directions consistent with the general narrative of global imbalances during this period. For the US and China, the VAREER moves more strongly in directions that are consistent with rebalancing. For Europe, in contrast, the VAREER moves more strongly in directions that are consistent with the build-up of imbalances prior to the onset of the debt crisis. At face value, it thus appears that the VAREER contains useful information on price developments above and beyond that contained in the conventional REER.

To summarize the remainder of the data succinctly, we plot changes in VAREERs versus REERs in Figure 8 over two time horizons (1970-2009 and 1990-2009). Consistent with the previous figures, VAREER changes are correlated with changes in the REER, but there are substantial deviations from the 45-degree line. Over the long time horizon (1970-2009), changes in the VAREER tend to be slightly smaller than changes in the REER, with points clustered above the 45-degree line in the lower left quadrant and above in the upper right quadrant. This pattern is weaker in the post-1990 data, however.

4.4 Decomposing Differences in REER Indexes

Digging below the surface of these results, we turn to an examination of the comparative roles that changing weights versus changing prices play in explaining deviations in the VAREER from the conventional REER. Using $\Delta \log(VAREER_i) = \sum_{j \neq i} w_{ij}^v \left(\hat{p}_i^v - \hat{E}_{i/j} - \hat{p}_j^v\right)$ and $\log(REER_i) = \sum_{j \neq i} w_{ij}^g \left(\hat{p}_i^{cpi} - \hat{E}_{i/j} - \hat{p}_j^{cpi}\right)$, we can write the difference as:

$$\begin{aligned}\log(VAREER_i) - \log(REER_i) = & \underbrace{\sum_{j \neq i} \left(w_{ij}^v - w_{ij}^g\right)\left(\hat{p}_i^v - \hat{E}_{i/j} - \hat{p}_j^v\right)}_{\text{weight effect}} \\ & + \underbrace{\sum_{j \neq i} w_{ij}^g \left[\left(\hat{p}_i^v - \hat{E}_{i/j} - \hat{p}_j^v\right) - \left(\hat{p}_i^{cpi} - \hat{E}_{i/j} - \hat{p}_j^{cpi}\right)\right]}_{\text{price effect}}.\end{aligned} \tag{30}$$

We plot the levels of these two terms along with the overall difference between the VAREER and REER in Figure 9 for the eight countries in previous figures. In all cases, the price effect clearly is the dominant force driving differences between the VAREER and the REER. Thus, the shift from using consumer prices to value added prices, not the shift from gross to value-added trade weights, explains the bulk of the differences.

To understand why shifting weights does not have a large effect on the real exchange rate, we highlight two features of the calculation. First, note that the reassignment of weights is a zero sum exercise, i.e., $\sum_{j \neq i} \left(w_{ij}^v - w_{ij}^g\right) = 0$. Therefore, price changes that are uniform across partners do not have any differential effect on gross versus value-added REERs. For example, if the nominal exchange rate depreciates against all partners by the same amount, this has the same effect on gross and value-added REERs. Second, even if price changes are not identical across destinations, there must be systematic variation between weight reassignments and changes in relative prices for the reassignment to matter.[43] In the data, this correlation is small for nearly all countries, roughly in range $(-0.2, .1)$ over the 1980-2009 period. Much larger correlations would be needed to generate significant effects from shifting weights alone.

This line of reasoning implies that even considerably larger cross-country reassignments of trade weights, as in continued expansion of global supply chains, may not have big additional effects. For example, if future weight changes are proportional to historical changes, then this simply re-scales the weight effect, as in multiplying $\left(w_{ij}^v - w_{ij}^g\right)$ by a constant. If changes

[43] That is, $\sum_{j \neq i} \left(w_{ij}^v - w_{ij}^g\right)\left(\hat{p}_i^v - \hat{E}_{i/j} - \hat{p}_j^v\right) \neq 0$ requires $\left(w_{ij}^v - w_{ij}^g\right)$ to be positively/negatively correlated with $\left(\hat{p}_i^v - \hat{E}_{i/j} - \hat{p}_j^v\right)$.

are not proportional to historical changes, then they must be systematically biased in a way that strengthens the correlation between weight reassignment and bilateral relative price changes. In other words, our finding that reassignment of trade weights generates small changes in REERs is likely to continue to hold even as vertical specialization deepens.

4.5 Mapping Price Changes to Demand for Value Added

In both the conventional and value-added REER formulas, we have normalized the weights to sum to one. Measuring changes in demand for value added then requires us to multiply the value-added REER by the normalization term and an elasticity: $\hat{V}_i = -\eta \bar{T}_i \Delta \log(VAREER_i)$. To enable this computation, we present the normalization terms for the VAREER and REER in Figure 10. One point to note is that the normalization term for the VAREER tends to be larger than the normalization for the conventional REER. This will tend to amplify the competitiveness implications of changes in relative prices. Another point is that these normalization terms vary widely across markets, with more open economies having generally larger $\bar{T}_i$'s. Not surprisingly, this means that a given relative price change has a bigger effect on demand for these countries.

5 Alternate REERs with Relaxed Assumptions

To derive the main real exchange rate formula, we assumed that the elasticity of substitution is the same in final demand and production. Further, in computing the exchange rate, we implicitly assumed that we can ignore relative price dynamics within countries and compute real effective exchanges using aggregate data alone. In this section we examine the consequences of relaxing these assumptions. Relaxing these assumptions yields modified formulas, but the resulting real effective exchange rate series are quantitatively similar to the VAREER.

5.1 Heterogeneous Elasticities

[To Be Completed.]

5.2 Two Sectors: Splitting Goods and Services

[To Be Completed.]

6 Conclusion

With the rise of global production chains, academics and policymakers have become aware that there are important gaps between cross-border shipments of goods and the implicit value added content of those shipments. This paper draws out the implications of shifting from a 'gross view' to a 'value-added view' of the production process for measuring international relative prices. Starting from a model of gross production and trade flows with cross-border input linkages, we derived a real exchange rate formula that uses value-added data alone. Comparing this value-added REER to the conventional REER, we found important differences. We hope that this initial work stimulates further efforts to integrate this approach into official statistics.

While our paper has focused on developing the methodology for computing value-added REERs, work comparing how the VAREER versus REER perform in applications is an important next step. For example, we expect the VAREER to be helpful in thinking about rebalancing, since the aggregate trade balance is essentially a net (i.e., value-added, not gross) concept.[44] We also believe the VAREER, and our value-added approach more generally, will be useful in calibrating macroeconomic models. For example, international business cycle models are typically written down as if they are models of value added, specifying production functions for value added and hence implicitly defining preferences over value added.[45] Yet, they are usually calibrated using a mixture of gross and value added data. Differences between value-added and conventional REERs indicate that it is important to treat value-added and gross flows in a consistent manner in data and models. We are therefore working on a project that revisits the calibration of macro-models using value added data.

[44] As discussed in Johnson and Noguera (2012a), the aggregate gross trade balance for a country is equal to that country's aggregate value-added trade balance. At the bilateral level, gross and value-added trade balances do diverge, however.

[45] See Bems (2012) and Johnson (2012) for further discussion.

References

Anderson, James E., and Yoto V. Yotov. 2010. "The Changing Incidence of Gravity." *American Economic Review*, 100(5): 2157–2186.

Armington, Paul S. 1969. "A Theory of Demand for Products Distinguished by Place of Production." *IMF Staff Papers*, 16(1): 159–178.

Bayoumi, Tamim, Sarma Jayanthi, and Jaewoo Lee. 2006. "New Rates from New Weights." *IMF Staff Papers*, 53(2): 272–305.

Bems, Rudolfs. 2012. "Intermediate Inputs, External Rebalancing, and Relative Price Adjustments." Unpublished Manuscript, International Monetary Fund.

Bems, Rudolfs, Robert C. Johnson, and Kei-Mu Yi. 2010. "Demand Spillovers and the Collapse of Trade in the Global Recession." *IMF Economic Review*, 58(2): 295–326.

Bems, Rudolfs, Robert C. Johnson, and Kei-Mu Yi. 2011. "Vertical Linkages and the Collapse of Global Trade." *American Economic Review: Papers and Proceedings*, 101(3): 308–312.

Bennett, Herman Z., and Ziga Zarnic. 2009. "International Competitiveness of the Mediterranean Quartet: A Heterogeneous-Product Approach." *IMF Staff Papers*, 56(4): 919–957.

Chinn, Menzie. 2006. "A Primer on Real Effective Exchange Rates: Determinants, Overvaluation, Trade Flows and Competitive Devaluation." *Open Economies Review*, 17: 115–143.

De Clercq, Maarten, Michael Fidora, Bernadette Lauro, Christina Pinheiro, and Martin Schmitz. 2012. "Revisting the Effective Exchange Rates of the Euro." ECB Occasional Paper Series No. 134.

Desruelle, Dominique, and Alessandro Zanello. 1997. "A Primer on the IMF's Information Notice System." IMF Working Paper 97/71.

Durand, Martine, Jacques Simon, and Colin Webb. 1992. "OECD's Indicators of International Trade and Competitiveness." OECD Economics Department Working Papers No. 120.

Erumban, Abdul Azeez, Bart Los, Robert Stehrer, Marcel Timmer, and Gaaitzen de Vries. 2011. "Slicing Up Global Value Chains: The Role of China." Unpublished Manuscript, University of Groningen.

Hummels, David, Jun Ishii, and Kei-Mu Yi. 2001. "The Nature and Growth of Vertical Specialization in World Trade." *Journal of International Economics*, 54: 75–96.

Johnson, Robert C. 2012. "Trade in Intermediate Inputs and Business Cycle Comovement." NBER Working Paper No. 18240.

Johnson, Robert C., and Guillermo Noguera. 2012*a*. "Accounting for Intermediates: Production Sharing and Trade in Value Added." *Journal of International Economics*, 82(2): 224–236.

Johnson, Robert C., and Guillermo Noguera. 2012*b*. "Fragmentation and Trade in Value Added over Four Decades." NBER Working Paper No. 18186.

Johnson, Robert C., and Guillermo Noguera. 2012*c*. "Proximity and Production Fragmentation." *American Economic Review: Papers and Proceedings*, 102(3): 407–411.

Lorentan, Mico. 2005. "Indexes of the Foreign Exchange Value of the Dollar." *Federal Reserve Bulletin*, 91: 1–8.

McGuirk, Anne. 1987. "Measuring Price Competitiveness for Industrial Country Trade in Manufactures." IMF Working Paper No. 87/34.

Spilimbergo, Antonio, and Athanasios Vamvakidis. 2003. "Real Effective Exchange Rate and the Constant Elasticity of Substitution Assumption." *Journal of International Economics*, 60(2): 337–354.

Yi, Kei-Mu. 2003. "Can Vertical Specialization Explain the Growth of World Trade?" *Journal of Political Economy*, 111: 52–102.

Yi, Kei-Mu. 2010. "Can Multi-Stage Production Explain the Home Bias in Trade?" *American Economic Review*, 100(1): 364–393.

Table 1: Difference in Trade Weights in 2005

	Partner Country or Region								
Source Country	United States	Canada Mexico	Eurozone	Other EU	China	India	Japan	Other Asia	Other
Argentina	2.4	0.3	0.4	1.1	-0.6	0.1	1.4	-0.2	0.8
Australia	2.9	0.5	1.4	0.5	-2.0	-0.2	0.1	-3.3	-0.1
Austria	2.8	0.4	-6.6	-0.3	0.7	0.2	1.0	0.3	1.1
Belgium	3.0	0.5	-8.0	0.8	0.6	0.4	0.9	0.3	1.2
Brazil	1.7	0.0	-0.8	1.1	-0.7	0.0	0.9	-0.5	0.5
Canada	-4.6	0.3	1.7	0.4	0.1	0.1	1.0	0.2	0.6
Switzerland	2.7	0.4	-7.4	1.5	0.7	0.1	0.9	0.0	0.8
Chile	2.6	-0.4	-0.2	1.1	-1.8	0.2	0.8	-1.1	1.3
China	2.5	0.1	0.0	0.7		0.0	-0.4	-3.5	0.5
Czech Republic	3.0	0.5	-7.6	0.0	0.5	0.2	0.9	0.3	1.9
Germany	3.0	0.5	-4.9	-1.3	0.2	0.3	1.1	0.2	0.7
Denmark	3.1	0.4	-2.9	-3.6	0.4	0.2	1.0	0.2	0.8
Spain	2.5	0.1	-6.4	0.5	0.5	0.2	0.9	0.2	1.2
Estonia	2.5	0.4	-2.8	-2.4	0.4	0.2	0.8	0.3	0.2
Finland	2.8	0.4	-1.2	-2.7	0.2	0.2	1.0	0.1	-1.0
France	2.6	0.5	-7.0	0.6	0.4	0.2	1.0	0.2	1.2
United Kingdom	2.9	0.2	-5.6	-0.3	0.4	0.1	1.0	0.1	0.8
Greece	2.0	0.4	-5.1	0.2	0.6	0.1	1.0	-0.2	0.6
Hungary	2.7	0.4	-6.7	-0.4	0.5	0.2	0.8	0.3	1.7
Indonesia	3.3	0.6	1.2	1.0	-1.5	-0.1	-0.6	-4.7	0.6
India	1.8	0.4	0.1	0.1	-1.7		1.2	-1.4	-0.7
Ireland	1.4	0.6	-1.6	-3.2	0.4	0.2	0.8	0.2	0.8
Israel	0.0	0.6	-1.7	0.3	0.3	-0.1	1.0	-0.3	-0.4
Italy	2.6	0.4	-5.6	0.2	0.4	0.2	1.0	0.1	0.5
Japan	2.9	0.4	1.5	0.9	-3.4	0.2		-3.6	0.6
Korea	2.7	0.4	1.1	0.9	-5.0	0.1	0.1	-1.0	0.5
Mexico	-5.1	0.3	1.3	0.9	0.3	0.2	1.0	0.1	1.0
Netherlands	2.8	0.5	-5.3	0.1	0.2	0.3	1.0	0.1	0.2
Norway	4.3	-0.4	-1.9	-5.9	0.5	0.3	0.9	0.2	1.7
New Zealand	1.9	0.2	0.5	0.3	-0.6	0.2	0.5	-1.1	-2.1
Poland	2.9	0.4	-4.7	-1.5	0.5	0.2	1.0	0.3	0.5
Portugal	2.1	0.3	-7.8	0.9	0.9	0.2	0.9	0.4	1.7
Romania	2.4	0.4	-4.2	-1.0	0.5	0.1	0.9	0.3	0.2
Russia	3.4	0.6	-4.1	-0.6	-0.7	0.2	1.0	-0.4	0.4
Slovakia	2.8	0.5	-0.7	-6.4	0.8	0.2	1.0	0.2	1.2
Slovenia	2.7	0.4	-7.7	-0.1	0.9	0.2	1.0	0.5	1.7
Sweden	2.8	0.4	-3.0	-2.3	0.5	0.2	1.0	0.2	-0.1
Thailand	2.4	0.4	1.0	0.7	-2.1	0.1	-0.9	-1.9	0.0
Turkey	2.2	0.4	-3.4	-0.6	0.3	0.1	1.0	-0.2	-0.2
United States		-5.8	1.5	1.3	-0.1	0.2	1.6	-0.4	1.3
Vietnam	2.2	0.5	0.7	0.6	-1.7	0.2	0.5	-3.6	0.2
South Africa	2.3	0.5	-1.4	-0.8	-0.7	-0.3	0.5	-0.6	0.2

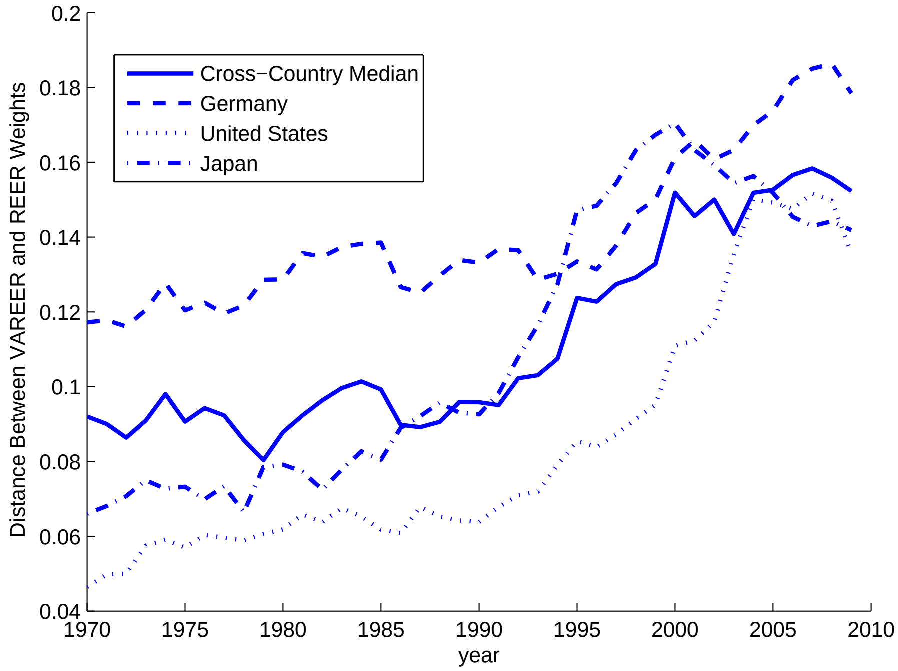
Note: Each entry records the difference in VAREER Weights minus REER Weights for source country to alternative destinations. Changes in weights are expressed in percentage points. Columns do not sum to zero for some countries due to rounding.

Table 2: Difference in Trade Weights in 2005, by Region

	Partner Region			
Source Region	Asia	EU	NAFTA	Other
Asia	-5.52	1.73	3.08	0.71
EU	1.67	-6.08	3.04	1.38
NAFTA	1.22	2.36	-4.95	1.37
Other	-0.48	-1.70	2.61	-0.43

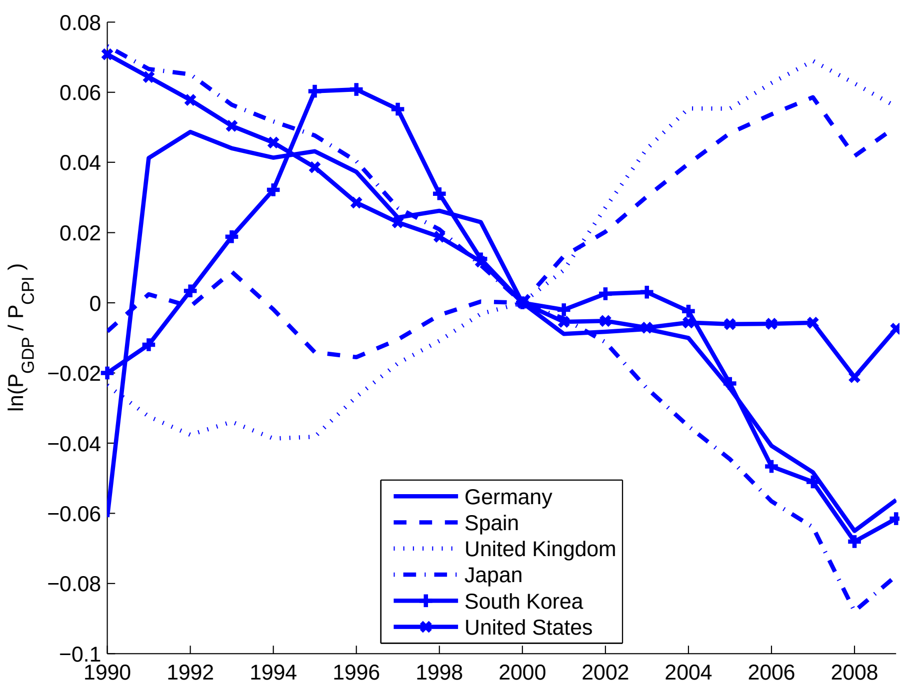
Note: Each entry records the total change in trade weights (VAREER weights minus REER weights) for partners in each destination region, averaged across source countries within each region. Changes in weights are expressed in percentage points. Columns do not sum to zero due to rounding.

Figure 1: Reassignment of Trade Weights over Time



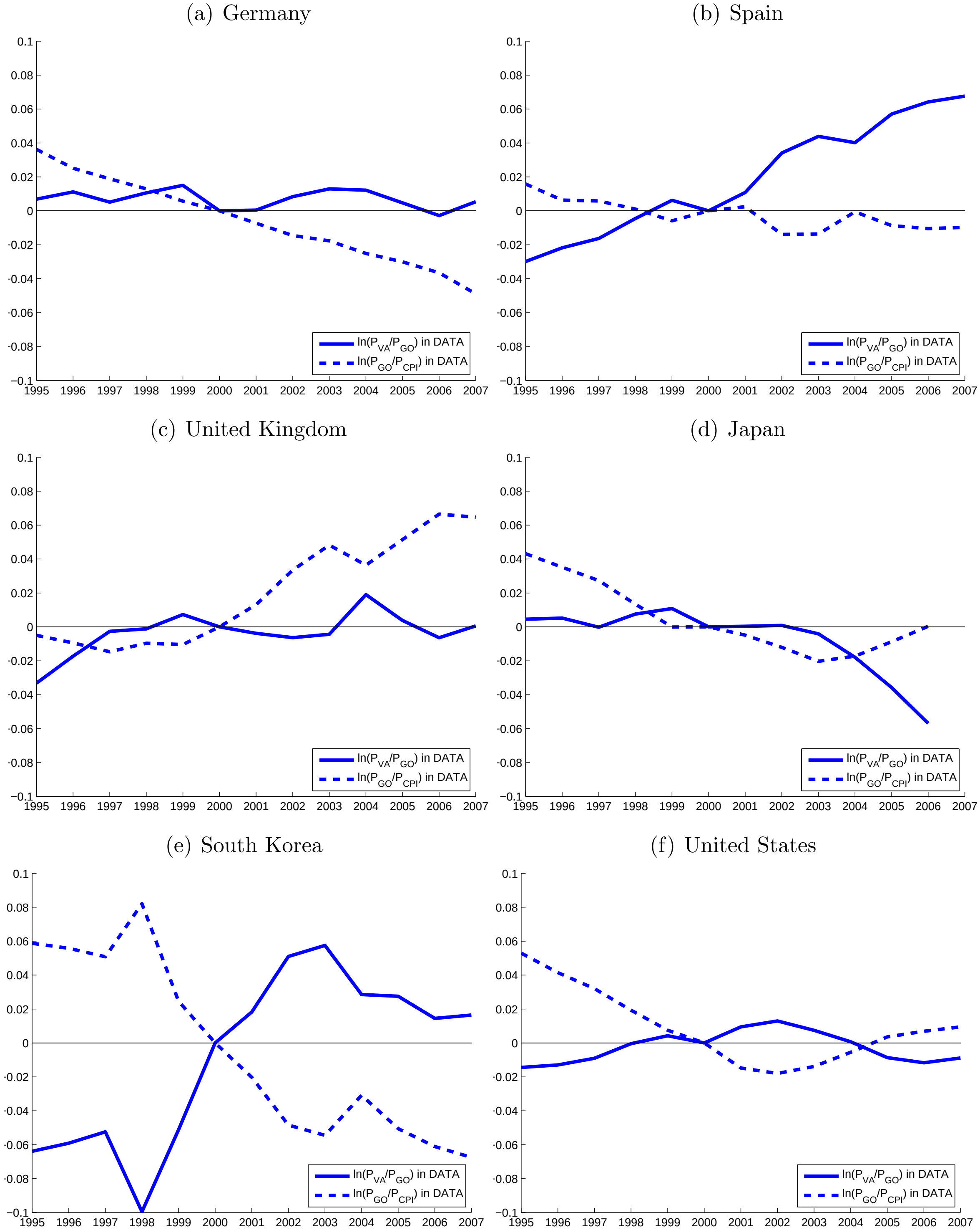
Note: Distances are measured using the city-block metric: $d_{it} = \sum_j \left| w^v_{ijt} - w^g_{ijt} \right|$.

Figure 2: Difference Between GDP and CPI Price Deflators (1990-2009)



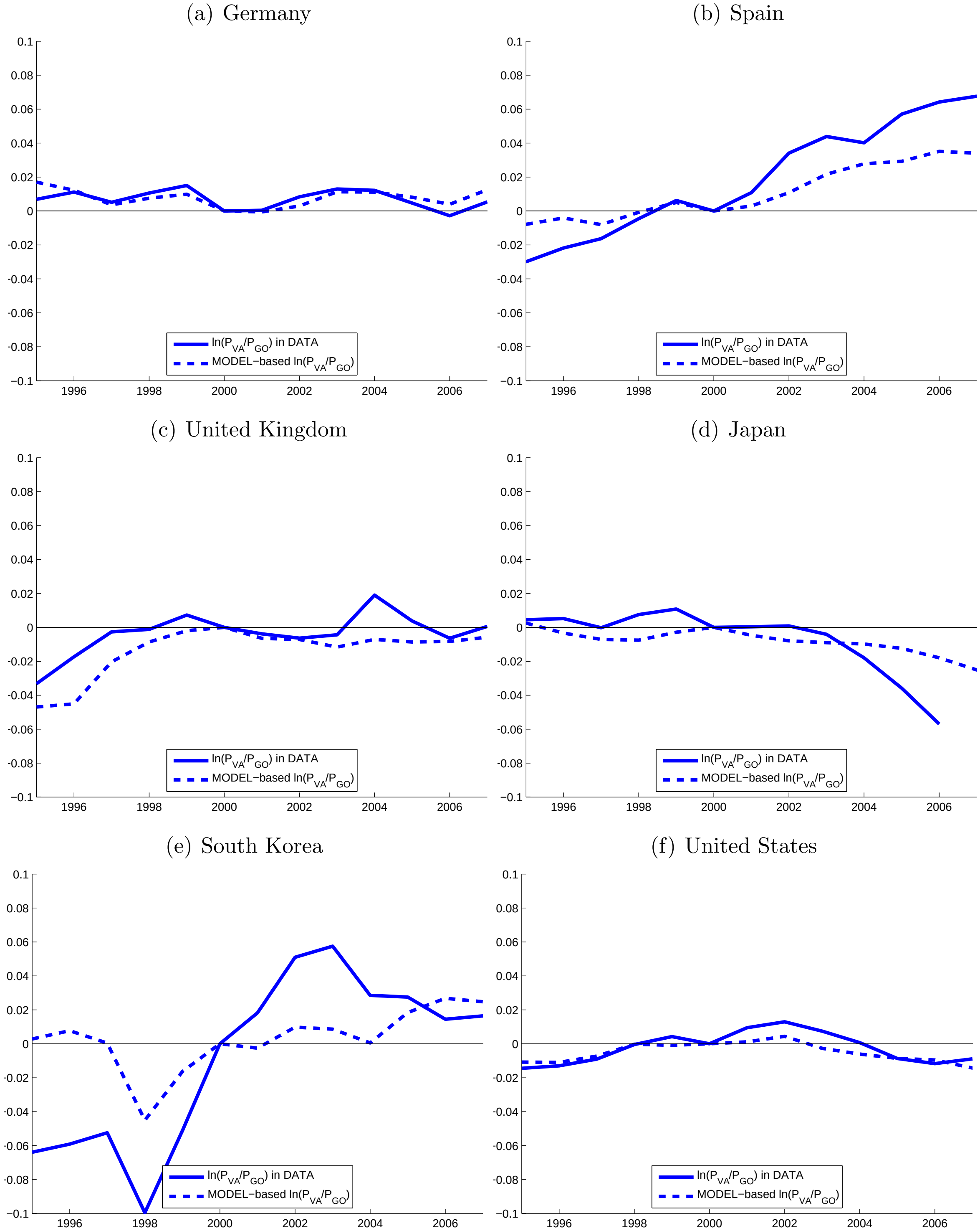
Note: Log relative price of GDP to CPI is normalized to zero in 2000.

Figure 3: Decomposition of Differences Between GDP and CPI Price Deflators



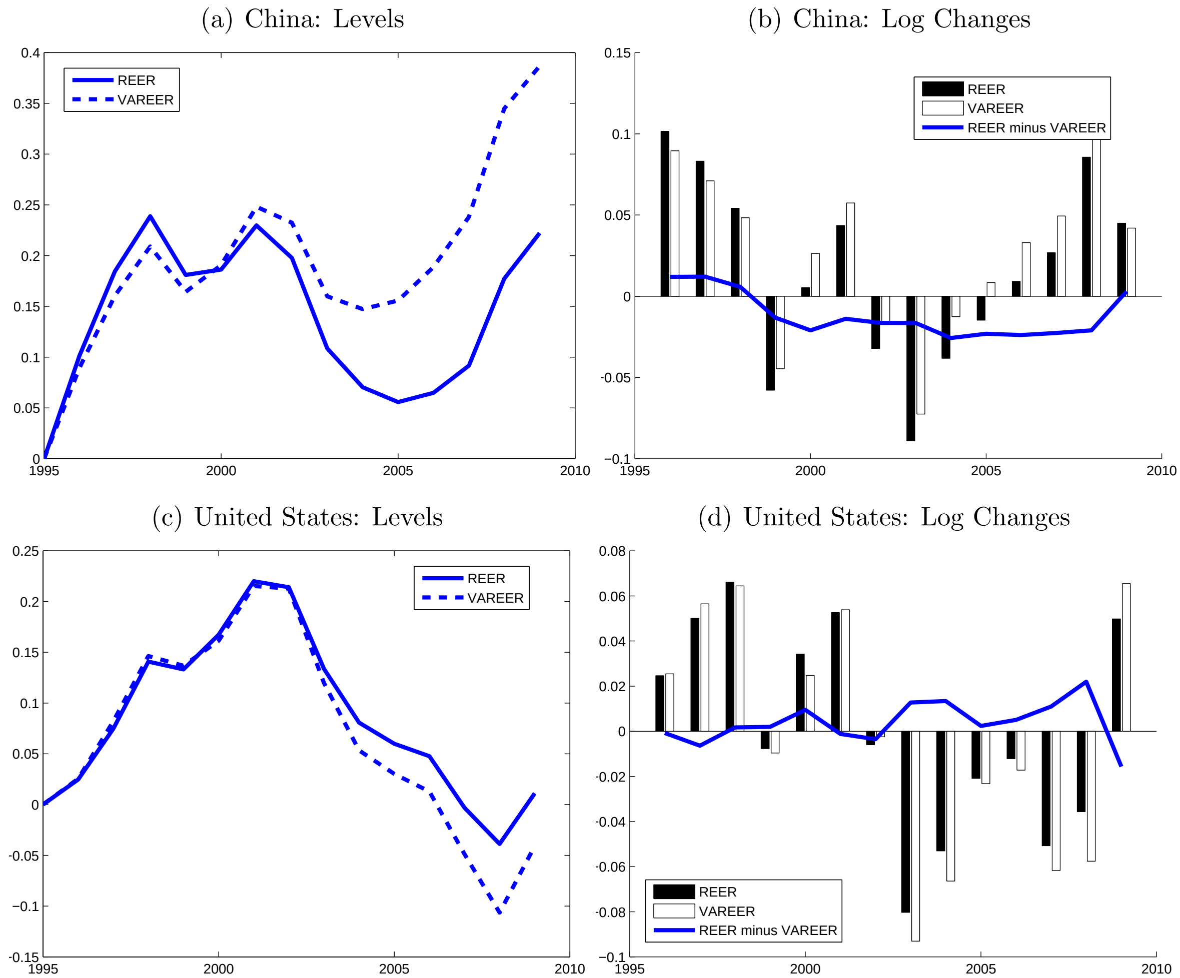
Note: Log relative price of GDP to CPI is normalized to zero in 2000.

Figure 4: Relative Price of Value Added to Gross Output in Data vs. Model



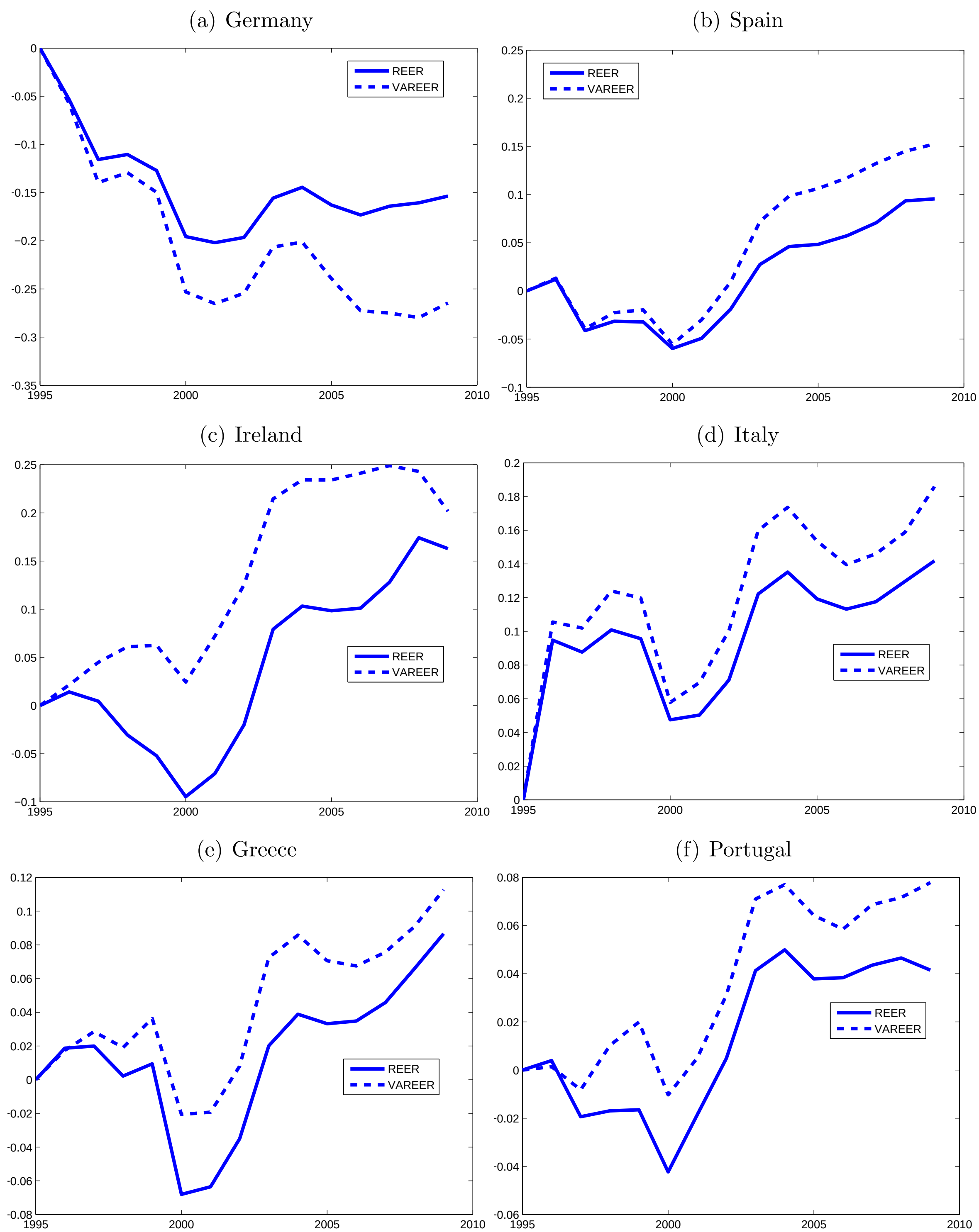
Note: Log relative price of GDP to CPI is normalized to zero in 2000.

Figure 5: Real Effective Exchange Rates for United States and China



Note: The level of the log(VAREER) and log(REER) are normalized to zero in 1995.

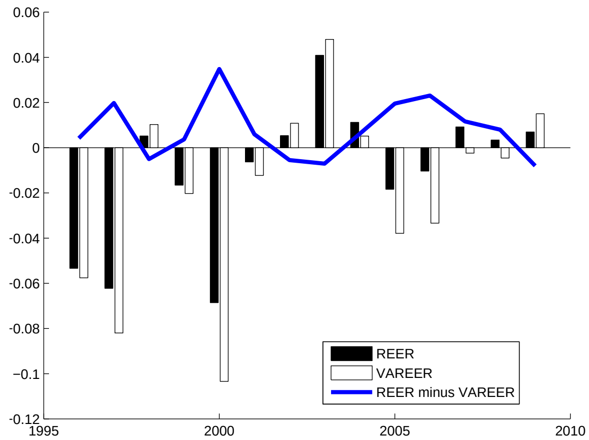
Figure 6: Real Effective Exchange Rates for Selected European Countries, in Levels



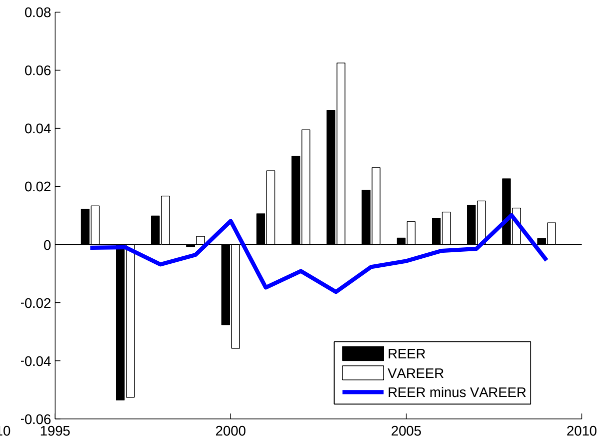
Note: The level of the log(VAREER) and log(REER) are normalized to zero in 1995.

Figure 7: Real Effective Exchange Rates for Selected European Countries, Log Changes

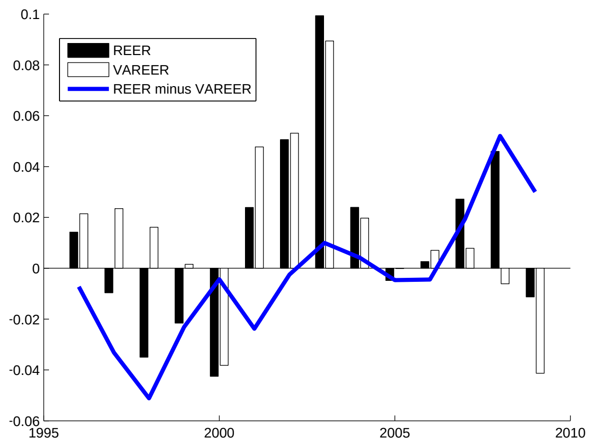
(a) Germany



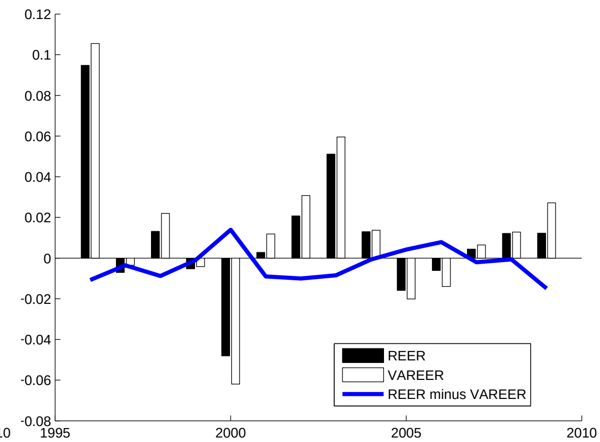
(b) Spain



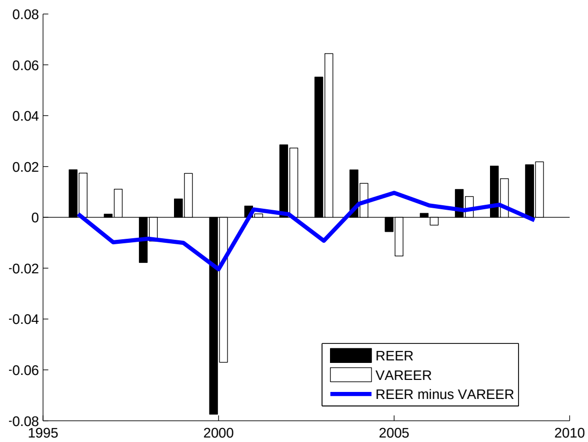
(c) Ireland



(d) Italy



(e) Greece



(f) Portugal

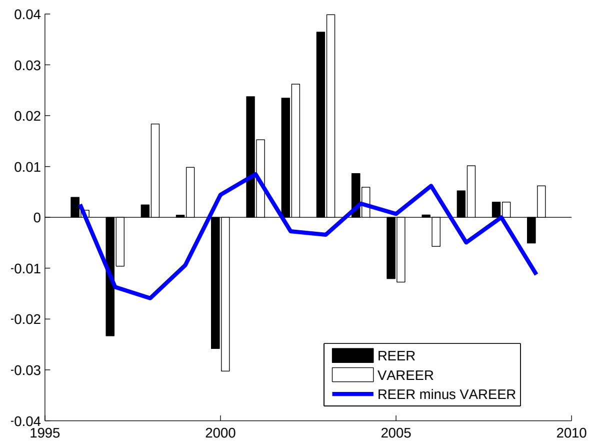
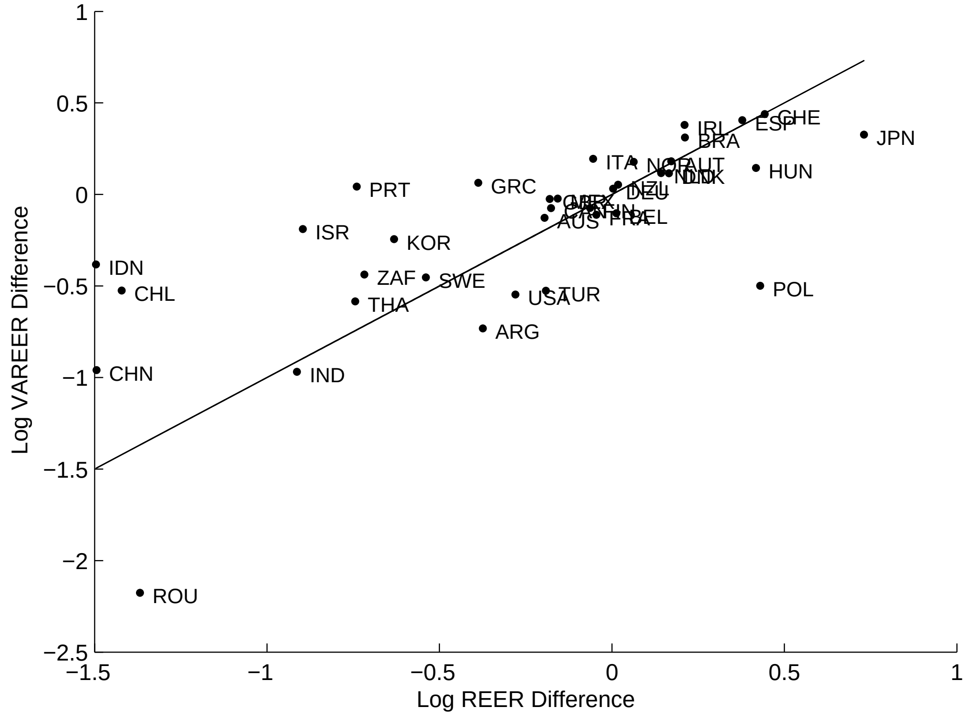
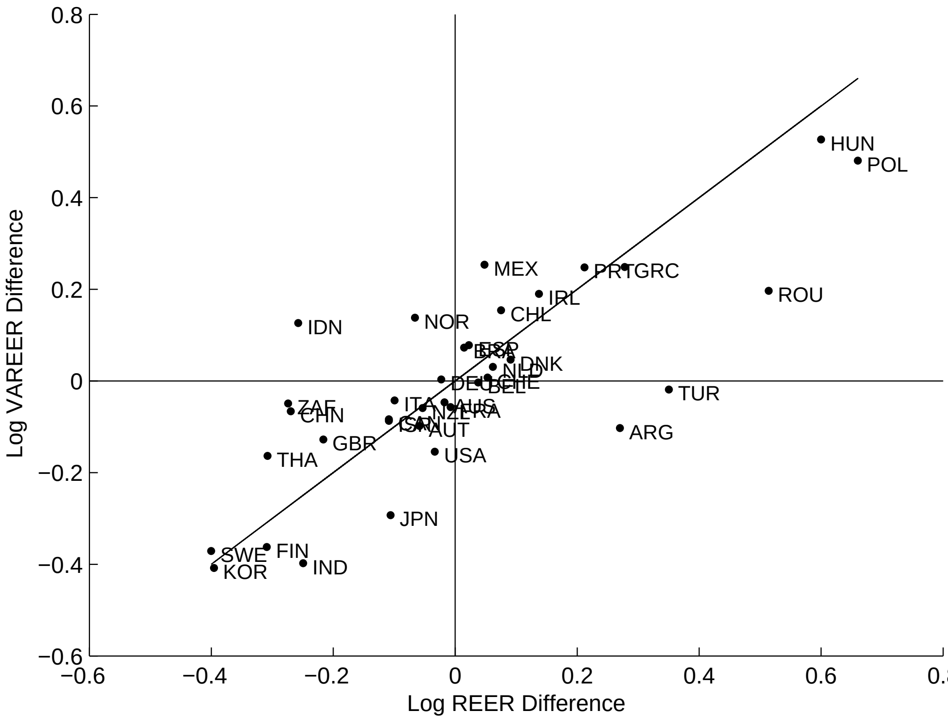


Figure 8: Changes in Real Effective Exchange Rates

(a) Log Changes from 1970-2009



(b) Log Changes from 1990-2009



Note: The solid line in both figures is the 45-degree line.

Figure 9: Decomposing Differences Between Value-Added and Conventional REERs

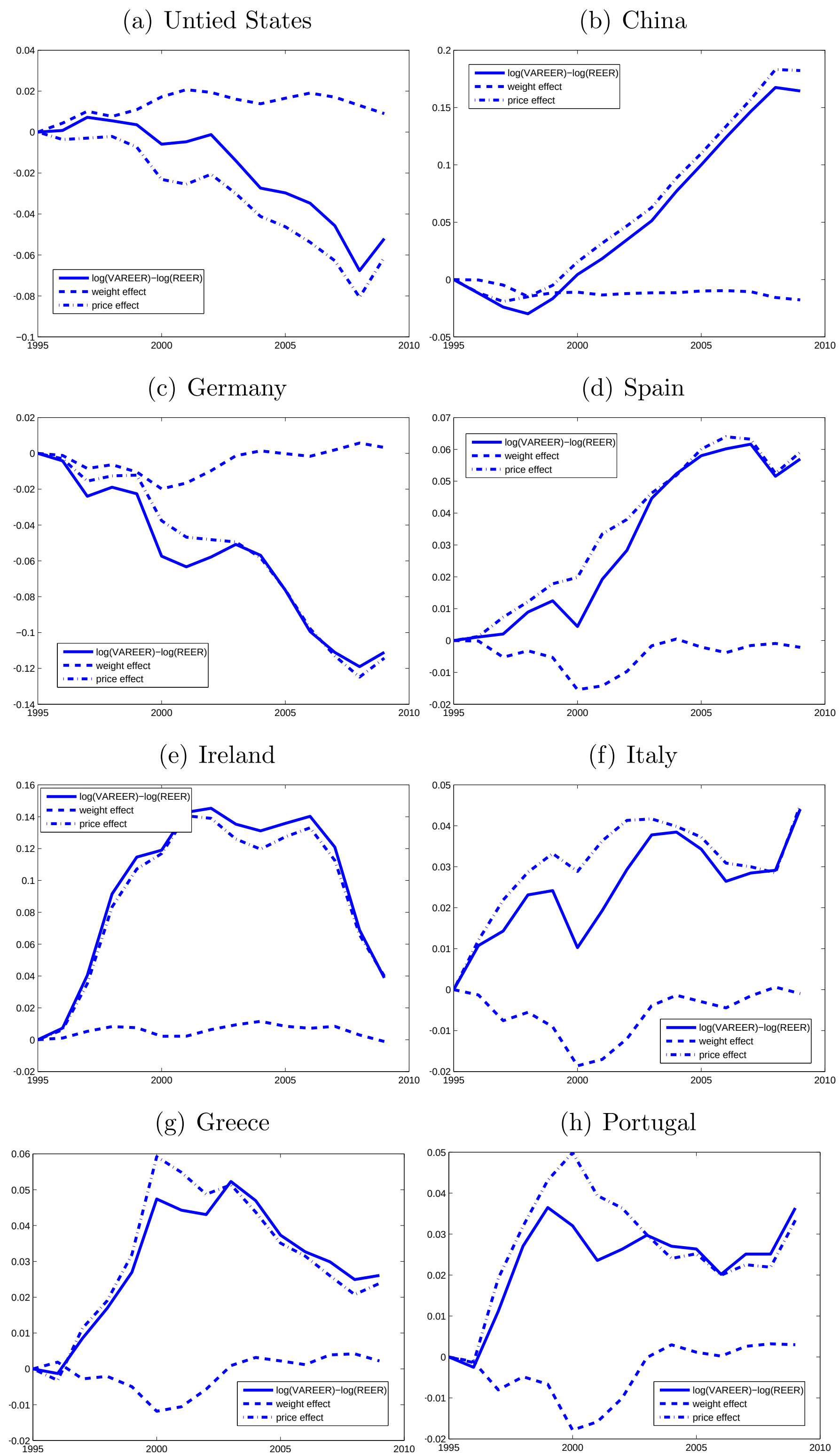
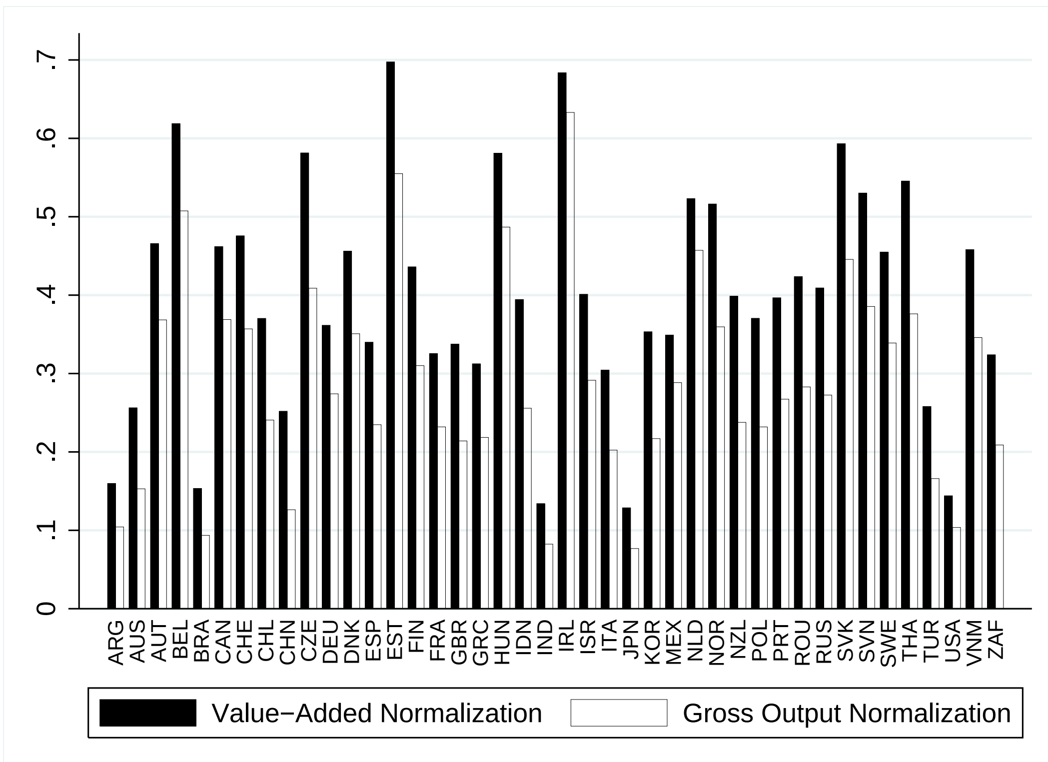


Figure 10: Normalization Terms for Real Effective Exchange Rates in 2000



A VAREER Derivation Appendix

In this appendix, we discuss the two main steps in moving from Equations (9)-(17) to Equation (18). As noted in the text, there are two steps. First, we compute demand for gross output. Second, we convert demand for gross output into demand for value added, and replace prices of gross output with value-added prices.

A.1 Demand for Gross Output

To compute the total change in demand for gross output, we combine the market clearing conditions in Equation (14) with the final and intermediate inputs first order conditions in Equations (9)-(12). This yields:

$$\begin{aligned}\hat{Q} &= S_F[-\eta M_1 \hat{p} + \eta M_2 W_F \hat{p} + M_2 \hat{F}] \\ &\quad + S_X\left[-\eta(M_1 - M_2)\hat{p} + M_1 \hat{Q}\right],\end{aligned} \tag{31}$$

An important feature of these market clearing conditions is that the change in gross output $\hat{Q}$ shows up on both sides of the equation. This is obviously because intermediate input demand depends on gross output itself, a standard feature of an input-output accounting framework. We want to purge the demand for gross output of this circularity, so we isolate gross output on one side of the equation:

$$[I - S_X M_2]\hat{Q} = \eta S_F M_2 W_F \hat{p} - \eta S_F M_1 \hat{p} - \eta S_X (M_1 - M_2)\hat{p} + S_F M_2 \hat{F} \tag{32}$$

Then note that: $[I - S_X M_2] = [diag(p_i Q_i)]^{-1}[I - \Omega][diag(p_i Q_i)]$, where Ω is the global bilateral input-output matrix (defined in the main text). Using this observation, and collecting terms on the right hand side yields:

$$\begin{aligned}\hat{Q} &= \eta[diag(p_i Q_i)]^{-1}[I - \Omega]^{-1}[diag(p_i Q_i)]S_F M_2 W_F \hat{p} \\ &\quad - \eta[diag(p_i Q_i)]^{-1}[I - \Omega]^{-1}[diag(p_i Q_i)][S_F M_1 + S_X(M_1 - M_2)]\hat{p} \\ &\quad + [diag(p_i Q_i)]^{-1}[I - \Omega]^{-1}[diag(p_i Q_i)]S_F M_2 \hat{F}.\end{aligned} \tag{33}$$

This is the analog to a standard demand CES equation for gross output in our context. Here demand for total gross output depends on prices for gross output $\hat{p}$, mediated by the elasticity of substitution and trade shares, as well as changes in final demand in all countries, recorded in $\hat{F}$. We are not primarily interested in demand for gross output, however. We want to understand how demand for real value added produced by each country depends on prices and demand. With traded intermediate inputs, demand for real value added behaves differently than demand for gross output. We thus turn to explaining this difference.

A.2 From Gross Output to Real Value Added

To convert demand for gross output into demand for real value added, we use the production side of the framework. We need to examine the production function because agents can substitute between using real value added and inputs to produce of gross output in response to relative prices changes. Moreover, we need to link prices for gross output to prices for value added.

Starting with the production function, note that we can substitute out for the composite input in Equation (15)to get: $\hat{Q} = [diag(s_i^v)]\widehat{V} + W_{QX}W_X\mathbb{X}$. Then we substitute out for $\mathbb{X}$ using Equation (16) to get:

$$[I - W_{QX}W_XM_2]\hat{Q} = [diag(s_i^v)]\widehat{V} - \eta W_{QX}W_X(M_1 - M_2)\hat{p}. \tag{34}$$

We then note that $[I - W_{QX}W_XM_2]$ reduces to a very simple matrix with elements $1 - \sum_j w_{ji}^x = s_i^v$ along the diagonal. Then, we can re-write this expression as:

$$\hat{Q} = \widehat{V} - \eta[diag(s_i^v)]^{-1}W_{QX}W_X(M_1 - M_2)\hat{p}. \tag{35}$$

As for prices, we note that $\hat{p} = diag(s_i^v)\hat{p}_i^v + diag(s_i^x)W_X\hat{p}$, and $diag(s_i^x)W_X = \Omega'$. Therefore, gross output prices can be written as the following weighted average of prices of value added:

$$\hat{p} = [I - \Omega']^{-1}[diag(s_i^v)]\hat{p}^v. \tag{36}$$

We then substitute Equations (35) and (36) into Equation (33) to write demand for value added as a function of prices of value added. The resulting expression initially looks daunting, but it simplifies to the more manageable expression:

$$\begin{aligned}\hat{V} = -\eta &\left[I - [diag(p_iQ_i)]^{-1}[I-\Omega]^{-1}[diag(p_iQ_i)]S_FM_2W_F[I-\Omega']^{-1}[diag(s_i^v)]\right]\hat{p}^v \\ &+ [diag(p_iQ_i)]^{-1}[I-\Omega]^{-1}[diag(p_iQ_i)]S_FM_2\hat{F}.\end{aligned} \tag{37}$$

This is Equation (18) in the main text.

B Special Case: domestic inputs only

In this appendix, we examine the framework in Section 1.1 under the assumption that domestic inputs are used in production, but there is no input trade. In this case, Ω is a diagonal matrix with elements ω_{ii} equal to the share of domestic intermediates in gross output in each country.

With this assumption, it is straightforward to show that $\hat{p}_i = \hat{p}^v$. Since all intermediates are drawn from domestic gross output, the gross output price simply tracks the value added price one-for-one. Holding final demand constant, the formula for demand for value added yields:

$$\begin{aligned}\hat{V}_i &= -\eta\hat{p}^v + \eta\sum_j\left(\frac{(1-\omega_{ii})^{-1}p_iF_{ij}}{p_iQ_i}\right)\hat{P}_j \\ \text{with} \quad \hat{P}_j &= \sum_k\left(\frac{p_kF_{kj}}{P_jF_j}\right)\hat{p}_k^v.\end{aligned} \tag{38}$$

The basic interpretation here is similar to the case above without intermediates, with minor modifications. The weights attached to foreign price levels equal the share of value added exports to the destination in total value added. These weights look superficially different because gross output is not equal to value added here.

In this example, gross output equals final goods plus domestic intermediates, which implies that $\sum_j \frac{p_i F_{ij}}{p_i Q_i} < 1$. The $(1-\omega_{ii})^{-1}$ adjustment in the formula above takes final goods and converts them into the amount of gross output needed to produce those final goods. Then the weights on individual destination markets records the amount of gross output needed to produce final goods shipped to a given destination $(1-\omega_{ii})^{-1} p_i F_{ij}$ as a share of gross output $p_i Q_i$. Noting that the ratio of value added to gross output is $1-\omega_{ii}$, then these can be interpreted simply as the ratio of value added exports – equal in this case to gross shipments of final goods $p_i F_{ij}$ to total value added $(1-\omega_{ii}) p_i Q_i$. For destinations $i \neq j$, then $p_i F_{ij} = Sales_{ij}$. However, the weight on the domestic market is not equal to $Sales_{ii}$, but rather simply final goods shipments $p_i F_{ij}$.

It can be further shown that demand for gross output is proportional to demand for real value added in this case: $\hat{Q}_i = \hat{V}_i$. One way to see this is to work through Equation (33) when Ω is diagonal. A more straightforward way to see this is to look directly at the production side of the economy. Starting with the first order conditions in Equation (4), note that the demand for domestic inputs (hence all inputs) is proportional to gross output under the maintained assumption that $\gamma = \rho$, so $\hat{X}_i = \hat{X}_{ii} = \hat{Q}_i$. Then substituting into the production function in Equation (15) yields $\hat{Q}_i = \hat{V}_i$.