When Money Crowds out Capital: Stagnation in a Liquidity Trap*

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Abstract

In this paper we analyze a monetary model with asset scarcity. A large and persistent deleveraging shock in the spirit of Eggertsson and Krugman (2012) leads to a persistent liquidity trap, with an increase in cash holdings and a long-term output decline. The long-term impact is a supply side effect independent of nominal rigidities, coming from the fact that saving is allocated to cash rather than physical capital. Quantitative easing is ineffective at the ZLB, but it can postpone the exit from a liquidity trap. An increase in debt may accelerate such an exit, but it may lower the capital stock because of higher interest rates. Monetary transfers are effective in stimulating employment in the short-run, but not in affecting the long term output level.

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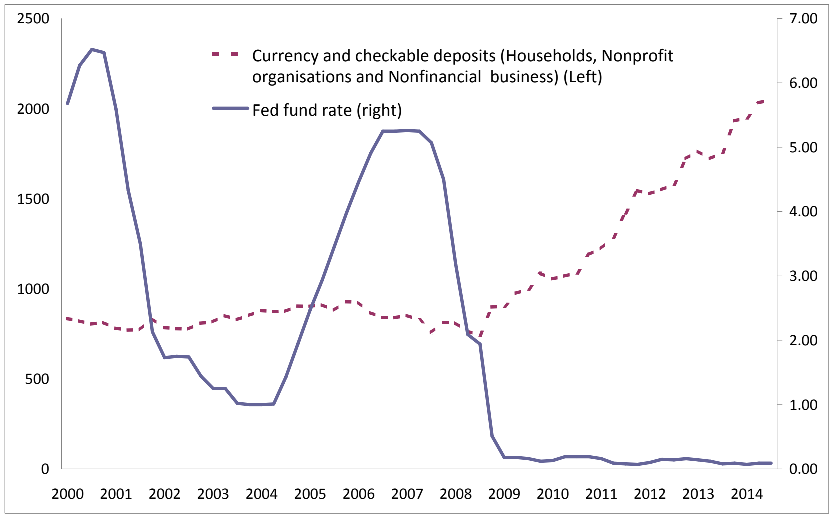


Figure 1: Fed funds rate and cash holdings in the US. Source: Federal Reserve Board of Saint-Louis and Flow of Funds.

1 Introduction

Since 2008, many advanced economies have hit the zero lower bound (ZLB). In addition to low growth and high unemployment, a striking feature of recent years in these economies has been the disappointing level of investment. The slow recovery in capital stock accumulation has led to downward revisions of estimates of potential output in coming years. As time went by, observers have begun to speculate whether these episodes might prove long-lasting. In a famous speech, Larry Summers suggested that the world economy might be suffering of "secular stagnation".[1] At the same time, there has been a dramatic increase in cash holding by private agents since 2009. Figure 1 shows this increase for the U.S. economy and illustrates how it coincides with the ZLB. Moreover, the increase in cash also coincides with a slowdown in capital accumulation. Figure 2 shows that capital growth sharply declines in 2009, when the ZLB is hit and cash starts increasing, while labour starts to fall earlier, in 2007-2008.

Can increased money holdings crowd out physical investment and contribute to lower growth? This is the question we investigate in this paper. We argue that in a liquidity trap

[1] See Teulings and Baldwin (2015) for an interesting collection of essays on secular stagnation.

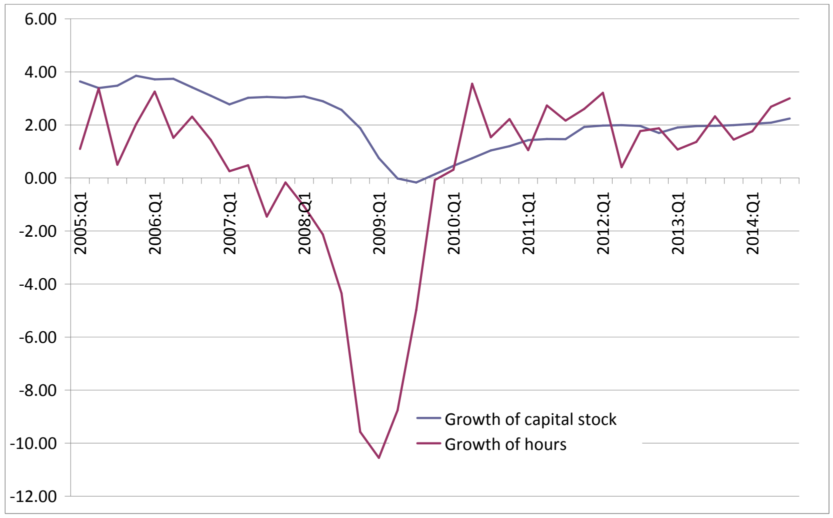


Figure 2: Capital and labor in the US. Source: Fernald (2014).

investment can be negatively related to money holdings. A liquidity trap arises when the supply of aggregate saving exceeds the demand for investment and the interest rate cannot adjust downward because of the ZLB. In that case there is an excess saving that is channeled through cash instead of investment. This decreases the capital stock and has a long-lasting impact on output. While this mechanism appears intuitive, it cannot be found in the existing literature. It implies that the liquidity trap has supply-side effects that contribute to a slower recovery. This has also interesting policy implications.

We consider a model where a liquidity trap can persist due a scarcity in asset supply, making excess saving persistent. This contrasts with the recent literature that has modeled long-lasting liquidity traps by assuming persistent nominal rigidities, implying long-run demand effects (see Schmitt-Grohe and Uribe, 2013, Eggertsson and Mehrotra, 2014, Begnigno and Fornaro, 2015). Also in contrast to most of the recent literature, we explicitly introduce money in the model. We then analyze the long-term implications of this model, assuming price flexibility.

In this framework, we consider a persistent deleveraging shock, modeled as in Eggertsson and Krugman (2012) by a tightening of borrowing constraints.[2] This shock decreases the supply

[2]With nominal rigidities, the literature has already shown that such a deleveraging shocks can lead to low

of saving instruments and triggers a decrease in the interest rate as an equilibrium adjustment. At the ZLB, outside money becomes a valuable saving instrument, which can accommodate the need for assets and substitutes to private bonds in the portfolio of savers. This translates into lower physical investment and a decrease in potential output in the new steady state, even in the absence of nominal rigidities. Our analysis shows that with prices flexible in the long run, there is a long-run supply-side effect in addition to the short-run demand effect. Interestingly, this negative long-run effect does not arise with some other types of shocks, such as an increase in the discount rate or a decrease in the growth rate.

More precisely, we build a model of heterogeneous agents with financial frictions. We introduce money in a model with credit-constrained investors and workers in the spirit of Woodford (1990) and Bacchetta and Benhima (2015). Investors find investment opportunities every other period, so that they alternate between investing phases and saving phases. In their investing phase, they use their past saving and borrow to invest, but this borrowing is limited by credit constraints. Agents can save either in real bonds or in money. But as long as the nominal interest rate is positive, money is dominated as an asset and is held only for transaction purposes. At the ZLB, bonds and money become substitutes and money can be held for saving purposes as well. As in Eggertsson and Krugman (2012), a deleveraging shock generates a decrease in the natural interest rate that makes the nominal interest rate hit the ZLB. However, in our model, the fall in interest rate lasts as long as the deleveraging shock. If this shock is permanent, then the ZLB persists in the steady state. Indeed, there is a lack of financial assets in the economy that prevents investors from moving away from their credit constraints through saving.

We show that the consequences of a deleveraging shock are very different outside the ZLB and at the ZLB. Outside the ZLB, a deleveraging shock has no effect on long-run capital accumulation and output (in our benchmark specification) as the interest rate can adjust. However, large deleveraging shocks that bring the economy to the ZLB have a negative effect on long-run capital accumulation and output. Unlike the previous literature, the ZLB generates a long-term decline of output even in the absence of nominal rigidities. This is because of a liquidity trap effect. At the ZLB, in the presence of a constant long-run growth rate of money

levels of output and employment in the short run, due to lower demand. Eggertson and Woodford (2003), Werning (2012) or Benigno et al. (2014) show this in New-Keynesian models.

(and hence constant long-run inflation), the real interest rate becomes inelastic. At the same time, money becomes a potential saving vehicle. As a result, while in the absence of ZLB the real interest rate would adjust to support investment, leaving the equilibrium long-run capital stock unchanged, in the ZLB savings are channelled to money, which crowds out investment.

We explore the effects of fiscal and monetary policy in our context. Because we introduce money explicitly, our framework also allows us to study the effects of quantitative easing and of government bonds. An obvious implication of our framework is that an increase in government debt may help exiting the ZLB by increasing the natural interest rate.[3] But higher interest rates may lower the capital stock outside of the ZLB. In contrast, quantitative easing in the form of large open-market operations may lengthen the ZLB period, while being ineffective in the liquidity trap. Credit easing would be a more effective policy. When we introduce short-term price stickiness in the model, we find that monetary transfers are effective in increasing employment in the short-run. On the other hand, monetary policy is not effective in affecting the long term output level.

The rest of the paper is organized as follows. Section 2 presents the basic model with infinitely-lived entrepreneurs and workers. Section 3 describes the steady state with flexible prices and the long-run effect of deleveraging shocks. Section 4 examines policy options. Section 5 studies several extensions of the benchmark model: nominal rigidities, endogenous money, idiosyncratic uncertainty, and preference and growth shocks. Section 6 concludes.

2 A Model with Scarce Assets and Money

We consider a monetary model where the supply of bonds may matter, due to the absence of Ricardian agents. Prices are flexible as we focus on the long run. In normal times, bonds dominate money and the real interest rate adjusts to balance the supply and demand for bonds. In a liquidity trap, however, bonds and money become perfect substitutes. The supply and demand of assets are then balanced by an adjustment in real money holdings (coming from either prices or money supply). These two adjustment mechanisms, through interest rates

[3] Such an effect can actually be found in a broad class of models where Ricardian equivalence does not hold. It is also currently discussed in policy circles, e.g., Kocherlakota, *Public Debt and the Long-Run Neutral Real Interest Rate,* President's Speech, Federal Reserve Bank of Minneapolis, August 19, 2015.

or money holdings, have different implications for investment and output, and therefore for policy. We show that in a liquidity trap real money holdings tend to increase, which may have a negative impact on capital and output in the long-run. This is in particular the case for a deleveraging shock, that we analyze in Section 3. In this section, we describe the model and the equilibrium.

2.1 The Setup

We model a monetary economy with heterogeneous investors, workers, and firms. There are three types of assets: bonds, money, and capital. We start by assuming that bonds are real bonds, that is, promises to pay one unit of final good in the next period. Denote by r_{t+1} their gross real rate of return expressed in units of final goods: a bond issued in period t is traded against $1/r_{t+1}$ units of final goods. The gross nominal return expressed in units of currency is $i_{t+1} = r_{t+1} E_t P_{t+1}/P_t$, where P_t is the price of the final good in units of currency in period t and E_t denotes the expectation as of time t. Money bears no interest rate; that is, it pays a gross nominal return equal to 1. While bond holdings can be both positive or negative, money holdings are non-negative. In addition, money provides transaction services and relaxes a cash-in-advance constraint faced by workers.

In normal times, when the gross nominal return i is strictly larger than 1, money is strictly dominated by bonds as a saving instrument. Then, only workers hold money, for transaction purposes. However, when $i = 1$, a situation that will obtain in a liquidity trap, money becomes as good a saving instrument as bonds and investors start holding money as well.

Investors Following Woodford (1990), investors find investment opportunities every other period, so that they alternate between a saving period and an investment period. This simple alternating approach is a convenient limit case allowing to capture idiosyncratic shocks in a very tractable way. Section 5 examines the more general case with idiosyncratic uncertainty and shows that the analysis is similar. Consequently, at each point in time there are two groups of investors, assumed of equal size one, investing and saving every other period. We call investors in their saving phase S-investors, or simply savers, and denote them by S, while investors in their investment phase are called I-investors and are denoted by I. We assume logarithmic utility in order to get closed form solutions. An individual investor i maximizes

$$U_t^i = E_t \sum_{s=0}^{\infty} \beta^s \log(c_{t+s}^i)$$

where c_t^i refers to her consumption in period t.

In period t, I-investors start with wealth $a_t + \frac{M_t^S}{P_t}$ where a_t and M_t^S are respectively real bond holdings and nominal money holdings inherited from their preceding saving phase. They get an investment opportunity, which corresponds to a match with a firm. I-investors consume c_t^I, issue b_{t+1} bonds, and invest k_{t+1} in the firm. We abstract from money demand M_{t+1}^I, as it is always zero in equilibrium in the benchmark model. Their budget constraint is

$$a_t + \frac{M_t^S}{P_t} = c_t^I - \frac{b_{t+1}}{r_{t+1}} + k_{t+1}. \tag{1}$$

In period t, S-investors start with equity k_t and outstanding debt b_t inherited from their preceding investment phase. They receive a dividend $\rho_t k_t$. Then, they consume c_t^S, buy a_{t+1} real bonds and save M_{t+1}^S in money. Their budget constraint is

$$\rho_t k_t - b_t = c_t^S + \frac{a_{t+1}}{r_{t+1}} + \frac{M_{t+1}^S}{P_t}. \tag{2}$$

In general, the return on capital is larger than r_t. Thus, I-investors are leveraged when they receive an investment opportunity. But they face a borrowing constraint as they can only pledge a fraction ϕ_t of dividends so that

$$b_{t+1} \leq \phi_t \rho_{t+1} k_{t+1}. \tag{3}$$

Firms There is a unit measure of one-period-lived firms, who are each matched with an I-investor. Firms use their investor's funds to buy capital k_t and produce output y_t with capital and labor through a Cobb-Douglas production function so that $y_t = F(k_t, h_t) = k_t^{\alpha} h_t^{1-\alpha} + (1-\delta)k_t$. Labor h_t is paid at the real wage w_t and all profits are distributed to I-investors, i.e., $\Pi_t = y_t - w_t h_t$. As the labor market is competitive, these profits are linear in k and can be rewritten as $\Pi_t = \rho_t k_t$, where ρ is the equilibrium return per unit of capital.[4] For expositional clarity, we assume full depreciation. The case of partial depreciation $\delta < 1$ is deferred to Appendix C.1. In equilibrium, profits are then simply $\rho_t k_t = \alpha y_t$.

Workers There is a mass one of workers who maximize

$$U_t^w = E_t \sum_{s=0}^{\infty} \beta^s u(c_{t+s}^w)$$

where c_t^w refers to workers' consumption. They have a fixed unitary labor supply, so that $h_t = 1$ in equilibrium. Their budget constraint is:

$$c_t^w + \frac{M_{t+1}^w}{P_t} + l_t^w = w_t + \frac{T_t^w}{P_t} + \frac{M_t^w}{P_t} + \frac{l_{t+1}^w}{r_{t+1}}, \tag{4}$$

where l^w is the amount of real bonds issued, M^w money holdings, and T^w a monetary transfer from the government.

Workers are subject to a cash-in-advance (CIA) constraint: they cannot consume more than their real money holdings. Assuming the bond market opens before the market for goods, these holdings are the sum of money carried over from the previous period, monetary transfers from the Government, and money borrowed on the bond market (net of debt repayment):

$$c_t^w \leq \frac{M_t^w + T_t^w}{P_t} + \frac{l_{t+1}^w}{r_{t+1}} - l_t^w. \tag{5}$$

Workers also face a borrowing constraint

$$l_{t+1}^w \leq \bar{l}_t^w y_{t+1}. \tag{6}$$

We assume that the borrowing limit is proportional to output. We allow for the case $\bar{l}^w < 0$, which represents forced savings by workers.

When $\beta r < 1$, which we will assume throughout the analysis, workers would prefer to dissave and always hold the minimum amount of money, so that the CIA is always binding. Together

[4] ρ is given by $\rho = F(1, 1/k(w)) + 1 - \delta - w/k(w)$ where $k(w)$ is the equilibrium capital-labor ratio defined by $w = F_h(k(w), 1)$.

with their budget constraint (4), this implies that their money holdings are simply equal to the wage bill: $M^w_{t+1}/P_t = w_t$. Since the wage bill is equal to $(1-\alpha)y_t$ in equilibrium, money demand by workers is given by:

$$M^w_t = (1-\alpha)P_t y_t. \tag{7}$$

Money supply and government policy Denote by M_t the money supply at the beginning of period t. In period t, the government can finance transfers to agents by creating additional money $M_{t+1} - M_t$ and by issuing real bonds l^g_{t+1}. For simplicity, we assume that the government only makes transfers to workers. The budget constraint of the government is:

$$\frac{M_{t+1}}{P_t} + \frac{l^g_{t+1}}{r_{t+1}} = \frac{M_t}{P_t} + \frac{T^w}{P_t} + l^g_t. \tag{8}$$

Several fiscal and monetary policies can be considered. As a benchmark case, we assume that the fiscal authority provides a real supply of bonds that is proportional to output $l^g_{t+1} = \bar{l}^g y_{t+1}$ and that the monetary authority control the growth of money

$$M_{t+1}/M_t = \theta_{t+1}. \tag{9}$$

Money growth is constant in the long run and equal to θ. Transfers to households then adjust to satisfy the budget constraint (8). The assumption of constant growth of money supply enables us to pin down steady-state inflation easily, as it will be equal to θ. We will thus be able to study the long-run effects of monetary policy.

We make the following parametric assumption:

Assumption 1 *$\theta > \beta$.*

Assumption 1 implies that the economy can only hit the zero lower bound in the steady state when $\beta r < 1$, that is with binding borrowing constraints. Indeed, in the steady state, the nominal gross interest rate is $i = r\theta$. With assumption 1, $i = 1$ implies $r = \theta^{-1} < \beta^{-1}$. This assumption is naturally satisfied as long as $\theta \geq 1$, that is with a non-negative steady-state inflation.

Market clearing for bonds and money The market for bonds clears so that

$$b_{t+1} + l^w_{t+1} + l^g_{t+1} = a_{t+1}. \tag{10}$$

Similarly, equilibrium on the money market is given by:

$$M^S_{t+1} + M^w_{t+1} = M_{t+1}. \tag{11}$$

Sequences of leverage The sequences of leverage $\{\phi_t, \bar{l}^w_t, \bar{l}^g_t\}$ are exogenous and deterministic. As a consequence, investors have perfect foresight, which will enable us to derive closed-form solutions. In Section 5, we introduce idiosyncratic uncertainty and show that our results survive.

2.2 Equilibrium

Asset scarcity and binding borrowing constraints We focus on equilibria where strong borrowing constraints prevent borrowers from supplying the saving instruments needed by savers. In such an "asset-scarce" economy, we will have $\beta r < 1$ in the long run, so the borrowing constraints are binding for workers and I-investors at the vicinity of the steady state, which we assume throughout.

A binding borrowing constraint for workers sets their supply of assets to $l^w_{t+1} = \bar{l}_t^w y_{t+1}$. We define the supply of bonds to S-investors by the rest of the economy, which includes workers and the government, by

$$l_{t+1} = l^w_{t+1} + l^g_{t+1} = \bar{l}_t y_{t+1} \tag{12}$$

where $\bar{l} = \bar{l}^g + \bar{l}^w$. In equilibrium, l_{t+1} is also the net position of investors. We will consider two cases: first, the case where investors are in effective autarky, that is $\bar{l} = 0$, which happens if workers have a no-liability constraint ($\bar{l}^w = 0$) and there are no government bonds ($\bar{l}^g = 0$); second, the case where investors are net borrowers vis-à-vis the rest of the economy ($\bar{l} < 0$), as this case is more realistic.

The zero lower bound and money demand The portfolio choice of S-investors can be summarized by the following complementary slackness condition:

$$M_{t+1}^S \left(r_{t+1} - \frac{P_t}{P_{t+1}} \right) = 0. \tag{13}$$

As long as $i > 1$, money has a strictly lower expected return than bonds and investors hold the minimum amount of money, which is zero. Then, we have $M^S = 0$. We refer to periods where $i > 1$ and investors hold no money as "cashless" periods.

When $i = 1$, that is $r_{t+1} = P_t/P_{t+1}$, bonds and money become perfect substitutes for savers, and they start holding money for saving purposes, so $M^S \geq 0$. We refer to periods where $i = 1$ and S-investors hold money as "liquidity trap" periods.

Euler equation of savers S-investors are typically unconstrained, so their Euler equation (with log-utility) is satisfied: $1/c_t^S = \beta r_{t+1}/c_{t+1}^I$. With log-utility, consumption is a fraction $1-\beta$ of wealth. Then, $c_{t+1}^I = (1-\beta)(a_{t+1} + M_{t+1}^S/P_{t+1})$ and $c_t^S = (1-\beta)(\rho_t k_t - b_t) = (1-\beta)(\alpha y_t - b_t)$. Assuming binding borrowing constraints (3) and (6), and using the market clearing condition for bonds (10), the Euler equation of S-investors can be rewritten

$$\beta\alpha(1 - \phi_{t-1})y_t = \frac{1}{r_{t+1}} \left[(\phi_t \alpha + \bar{l}_t) y_{t+1} + \frac{M_{t+1}^S}{P_{t+1}} \right]. \tag{14}$$

This Euler equation can also be interpreted as an equilibrium condition for saving instruments. The left-hand side (LHS) is the demand for saving instruments by S-investors. The first term on the right-hand side (RHS) is the supply of saving instruments by leveraged I-investors, driven by ϕ, and by workers and the government, driven by $\bar{l}$. Finally, the last term on the RHS corresponds to money used by S-investors as a saving instrument.

Aggregate budget constraint With log-utility, S-investors save a fraction β of their wealth. The budget constraints of I-investors and S-investors (1) and (2) become respectively $\beta(a_t + M_t^S/P_t) = k_{t+1} - b_{t+1}/r_{t+1}$ and $\beta(\alpha y_t - b_t) = M_{t+1}^S/P_t + a_{t+1}/r_{t+1}$. Aggregating these two constraints and using the bond market clearing condition (10), we find

$$k_{t+1} + \frac{M_{t+1}^S}{P_t} + \bar{l}_t \frac{y_{t+1}}{r_{t+1}} = \beta \left[(\alpha + \bar{l}_{t-1}) y_t + \frac{M_t^S}{P_t} \right] \tag{15}$$

This equation represents the aggregate resource constraint of of S- and I-investors. It depends on their net position vis-à-vis the rest of the economy $l = \bar{l}y$, as $a - b = l$. It is independent of ϕ, because the net position of investors as a whole ultimately depends on the net supply of bonds by the rest of the economy l.

Money market Substituting (7) into (11), we get

$$M_{t+1} = (1-\alpha) P_t y_t + M_{t+1}^S. \tag{16}$$

Money supply has to be equal to the demand for money for transaction purposes plus the demand for money for saving purposes.

Equilibrium The Euler equation (14), the aggregate resource constraint (15), and the money market equilibrium (16) describe a constrained equilibrium. The equilibrium is described formally in the following definition:

Definition 1 (Constrained equilibrium) *Consider an exogenous sequence of leverage* $\{\phi_t, \bar{l}_t^w\}_{t\geq 0}$, *a policy* $\{\theta_{t+1}, T_t^w, \bar{l}_{t+1}^g\}_{t\geq 0}$ *satisfying* (8), *and initial assets* $\{k_0, M_0\}$. *The associated constrained equilibrium is an allocation* $\{y_t, k_{t+1}, M_{t+1}, M_{t+1}^w, M_{t+1}^S\}_{t\geq 0}$ *and a price vector* $\{i_{t+1}, r_{t+1}, w_t, P_t\}_{t\geq 0}$ *satisfying* $i_{t+1} = r_{t+1} P_{t+1}/P_t$, $\bar{l}_{t+1} = \bar{l}_{t+1}^w + \bar{l}_{t+1}^g$, (7), (9), (11), (13), (14), (15), (16).

In the next section, we will focus on steady state equilibria. It will be useful to distinguish between cashless and liquidity-trap steady states. The definition of these steady states is made formally in the following definition:

Definition 2 (Cashless and liquidity-trap steady states) *A constrained steady state is a constrained equilibrium where* $\{\phi, \bar{l}^w, \theta, \tau^w, \bar{l}^g, y, k, m, m^w, m^S, i, r, w\}$ *are constant, where* $\tau^w = T^w/P$, $m = M/P$, $m^w = M^w/P$ *and* $m^S = M^S/P$. *A cashless steady state is a constrained* *steady state satisfying $i > 1$ and $m^S = 0$. A liquidity-trap steady state is a constrained steady state satisfying $i = 1$ and $m^S > 0$.*

Money and capital accumulation Regarding the evolution of real variables, it is convenient to consider the special case of autarkic investors with $\bar{l}^g = \bar{l}^w = \bar{l} = 0$. Rearranging the aggregate budget constraint of investors (15), we get

$$k_{t+1} = \beta \alpha y_t - \frac{M_{t+1}^S - \beta M_t^S}{P_t} \tag{17}$$

In a liquidity trap, increasing holdings M_{t+1}^S has a negative impact on capital accumulation. This is a key feature of this model. While investors use all their resources to buy capital in the cashless case, holding money as a saving instrument diverts some of their resources from investment in the liquidity trap.[5] In the long-run however, the effect of M^S on capital will also depend on the return of money.

3 The Long-term Impact of Deleveraging

This section studies the long-term effects of deleveraging. In our setting, a deleveraging shock on investors can be modeled by a drop in ϕ. Likewise, a deleveraging shock on workers can be modeled by a drop in $\bar{l}$ (coming from a drop in $\bar{l}^w$). We consider permanent shocks, which allows us to analyze changes in steady states. As Equation (14) makes clear, an unexpected deleveraging shock on investors affects the equilibrium for savings instruments through two channels. First, it reduces the supply of bonds on the RHS. Second, in subsequent periods, as savers start the period with less debt, it increases their wealth and hence their demand for saving instruments on the LHS. Both channels lead to an excess net demand of saving instruments by investors.

The equilibrium response is very different depending on whether the economy is in a cashless equilibrium or in a liquidity trap. In cashless equilibria, adjustment comes from a lower equilibrium interest rate which helps restore a higher supply of bonds. In the liquidity trap, when the

[5] In the cashless case capital accumulation equation does not depend on the interest rate (a consequence of logarithmic utility).

interest rate cannot adjust any more, the higher net demand for saving instruments by investors takes the form of higher money holdings instead. As we will see, this diverts resources away from investment and leads to lower capital and output in the long-run. A deleveraging shock on workers (a drop in $\bar{l}$) has similar effects on the interest rate and money holdings but only through a reduction in the bond supply on the RHS. This difference matters, as will become clear later: in the liquidity trap, a deleveraging shock on workers has no effect on capital and output.

3.1 The Effect of Investors' Deleveraging

Here we study the effect of a deleveraging shock affecting investors, that is, a drop in ϕ. We consider first the simpler case with $\bar{l} = 0$, which implies that investors are in autarky: S-investors lend to I-investors. Afterwards, we examine the case where investors have a net debt vis-à -vis the rest of the economy, when $\bar{l} < 0$.

Autarkic investors When $\bar{l} = 0$, the cashless dynamics of capital accumulation, given by (15) with $M^S = 0$, are independent of the real interest rate r. In the steady state:

$$k = \beta\alpha y = \beta\alpha k^{\alpha}. \tag{18}$$

This equation defines the long-term capital stock. It is independent of r, because of log-utility: agents save a constant fraction of their income each period, as the income effect offsets the substitution effect.[6]

Notice also that the borrowing capacity of I-investors, ϕ, does not directly affect the long-run capital stock. This is because in the aggregate, what matters is neither I-investor borrowing b nor S-investor saving a (both of which depend on ϕ), but only investors' net demand for saving $a - b$. This is ultimately equal to the net supply of bonds by the rest of the economy $\bar{l}y$, which is zero. After a deleveraging shock on investors (a decrease in ϕ), the capital stock is therefore unchanged. But this requires a change in the interest rate as an equilibrating mechanism. Indeed, for a given interest rate, this shock generates a decrease in the bond

[6] In fact, the case of log-utility is a realistic one when it comes to modeling the saving behavior of agents, as a unitary elasticity of intertemporal substitution is well within the estimated ranges.

supply b by I-investors and an increase in the demand for bonds a. However, the supply of bonds by the rest of the economy $\bar{l}$ remains unchanged, so the adjustment takes place through a decrease in interest rate, which enables I-investors to borrow more. This is clear from the Euler equation (14), which defines r in the cashless steady state as

$$r = \frac{\phi}{\beta(1-\phi)}. \tag{19}$$

Notice that a decrease in r implies a proportional decrease in $i = r\theta$ in the absence of a change in the steady state inflation rate θ. Therefore, a low steady-state inflation makes the economy at the risk of hitting the ZLB following a strong contraction of credit. In fact, i hits the zero-lower bound if $\phi/[\beta(1-\phi)] \leq 1/\theta$, as θ is the steady-state inflation rate. Similarly, a high enough ϕ brings the equilibrium interest rate at $1/\beta$. Beyond this, the credit constraint is not biding anymore.

If i hits the ZLB, then the equilibrium becomes a liquidity trap. In a steady state, the Euler equation (14) then becomes:

$$m^S = \alpha \left[(1-\phi)\frac{\beta}{\theta} - \phi \right] y. \tag{20}$$

where $m_t^s = M_t^S/P_t$ denotes real money holdings by savers. Here, an increase in net saving demand triggered by a deleveraging shock can be accommodated by an increase in real money holdings m^S: In Equation (20), m^S/y is decreasing in ϕ. Indeed, at the ZLB, bonds and money have the same return and money becomes a saving instrument. While in the cashless steady state, savings were eventually channelled to investment through bonds, this is not the case in the liquidity trap, and investment can be crowded out by money. If money had a high enough return, then in the long run this would not affect investors' financing capacities. However, at the ZLB, savings have a very low return $P_t/P_{t+1} = 1/\theta$, which might take out resources from investment, as suggested by (17), which becomes in a steady state

$$k = \beta\alpha y - (\theta - \beta)m^S. \tag{21}$$

From Assumption 1, we have $\theta > \beta$ and holding money indeed entails a resource cost that decreases investment and the stock of capital.

How does the adjustment in investor real money holdings m^S take place? From (16) taken in the steady state, we have $m = M/P = (1-\alpha)y/\theta + m^S$. Since workers' money holdings always equal their wage bill, investors' higher demand for money has to increase total real money holdings. For a given path of money supply, given by (9), this implies a downward shift in the path of prices $\{P_t\}$. At the ZLB, a deleveraging shock is disinflationary, which endogenously increase real money holdings to accommodate the higher net demand for saving instruments by investors.

Using this analysis, we establish the following Proposition:

Proposition 1 (Steady state with autarkic investors) *Define $\phi_T = \beta/(\theta+\beta)$ and $\phi_{max} = 1/2$. If $0 < \phi < \phi_{max}$, then there exists a locally constrained steady state with $r < 1/\beta$.*

(i) *If, additionally, $\phi \geq \phi_T$, then the steady state is cashless.*

(ii) *If $\phi < \phi_T$, then the steady state is a liquidity trap.*

(iii) *In the cashless steady state, the real interest rate r and the nominal interest rate i are increasing in ϕ, $m^S = 0$ and k is invariant in ϕ.*

(iv) *In the liquidity-trap steady state, the real interest rate r is invariant in ϕ, m^S/y is decreasing in ϕ and k is increasing in ϕ.*

Proof. See Appendix A. ■

This Proposition establishes under which condition on ϕ the steady state is cashless or a liquidity trap. It is illustrated in Figure 3, showing the levels of k, r , and m^S as a function of ϕ. For intermediate values of ϕ (between ϕ_T and ϕ_{max}), the cashless real interest rate r is higher than $1/\theta$, and the steady state is cashless as the nominal interest rate i is above the ZLB, as is illustrated by equilibrium C. When ϕ falls below ϕ_T, the steady state becomes a liquidity trap with $r = 1/\theta$. It is characterized by positive real money holdings among investors, for saving purposes, as illustrated by point T.

As long as the economy is in the cashless steady state (when $\phi > \phi_T$), a permanent deleveraging shock on investors (a decrease in ϕ), has no effect on investment and capital, but it has a negative effect on the real interest rate r, as illustrated by Figure 3. Deleveraging shocks therefore do not affect investment in the long run.

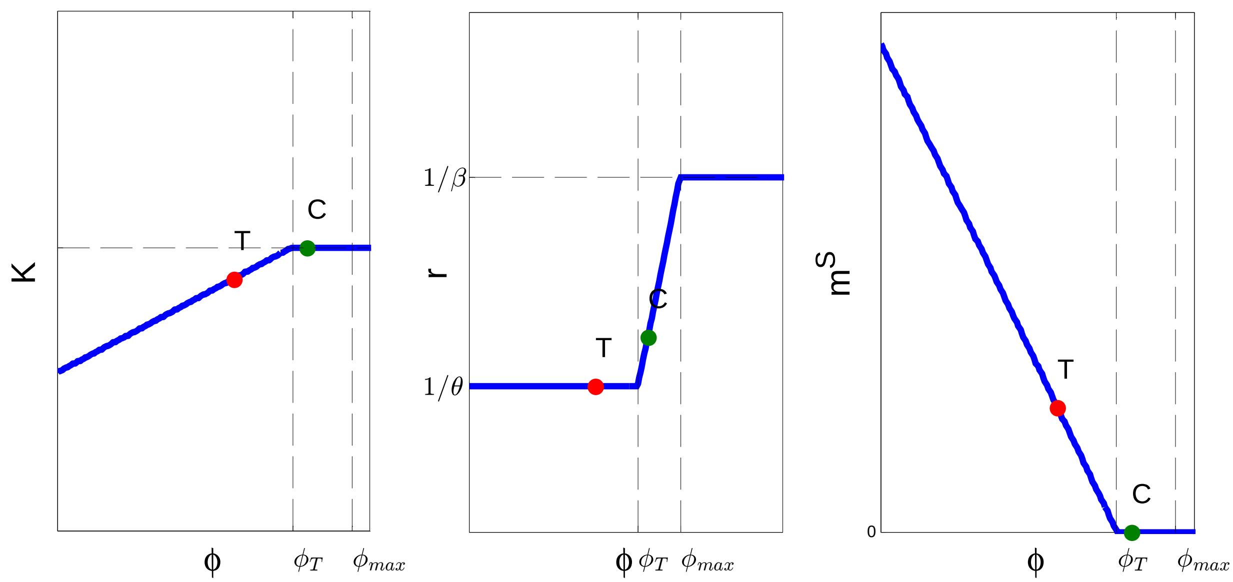


Figure 3: Steady states - Comparative statics w.r.t. ϕ, with $\bar{l} = 0$

Contrary to a deleveraging shock in cashless steady states, a deleveraging shock that is large enough to make the economy fall in a liquidity trap (by bringing ϕ below ϕ_T), has negative long-run effects on capital and output, as shown in Figure 3. At this point, the real interest rate r cannot adjust any more and adjustment comes from an increase in investors' real money holdings m^S instead. As described above, the resource cost of holding money has an adverse impact on investment and capital. The long-run investment then becomes sensitive to permanent deleveraging shocks. A permanent deleveraging shock is then consistent with secular stagnation, as in Eggertsson and Mehrota (2014). However, in Eggertsson and Mehrota (2014), this was driven by a fall in consumption demand, which can lead to long-run stagnation only in the presence of nominal rigidities. Here, the effects come from the supply side of the economy and hold in the absence of any nominal rigidity.

Investors are net debtors Whereas the case where investors are in autarky is a useful simplification, the case where investors are net debtors is more realistic ($\bar{l} < 0$). Indeed, using Flow-of-Fund data and the Survey of Consumer Finances, we establish that firms and households owning a business or participating to the stock market have a negative net position in interest-bearing assets in the US. Appendix B gives the details of our analysis. Note that in the presence of positive government debt ($\bar{l}^g > 0$), $\bar{l} < 0$ implies that workers have a positive net position ($\bar{l}^w < 0$). This is consistent with a high proportion of wealthy hand-to-mouth households, that is, households who own sizeable amounts of illiquid assets (like retirement accounts) but hold little liquid assets, as documented by Kaplan et al. (2014).

In that case, changes in the interest rate have an additional effect by redistributing resources between investors and workers. The steady-state cashless capital accumulation equation now becomes:

$$k = \beta\alpha y - \left(\frac{1}{r} - \beta\right)\bar{l}y. \tag{22}$$

The long-term capital stock depends on the interest rate. As the net wealth of investors is negative ($\bar{l} < 0$), a lower interest rate has a positive effect on capital accumulation, as it frees resources that would otherwise be used for interest payments. Besides, as shown by the cashless steady-state Euler equation:

$$r = \frac{\phi + \bar{l}/\alpha}{\beta(1-\phi)}, \tag{23}$$

the interest rate falls after a deleveraging shock in the cashless economy as before. Therefore, a deleveraging shock should *increase* the capital stock in the cashless economy.

In a liquidity trap however, a deleveraging shock still has a negative effect on capital. In that case, as money and bonds are perfect substitutes, capital accumulation is not affected by the net supply of bonds $\bar{l}$ per se, but by the total amount of financial savings $s = m^S + \bar{l}y$:

$$k = \beta\alpha y - (\theta - \beta)s. \tag{24}$$

where s is determined by the steady-state Euler equation taken in a liquidity trap, independently from the net supply of bonds $\bar{l}$:

$$s = \alpha\left[(1-\phi)\frac{\beta}{\theta} - \phi\right]y. \tag{25}$$

This equation is similar to (20), with total financial saving s replacing cash holdings m^S. After a deleveraging shock on investors, the interest rate remains fixed at $1/\theta$, whereas financial savings s increase. Since s has the same return as money in a liquidity trap, an increase in s takes resources away from investment as in the case of autarkic investors.

The main results are summarized in the following Proposition:

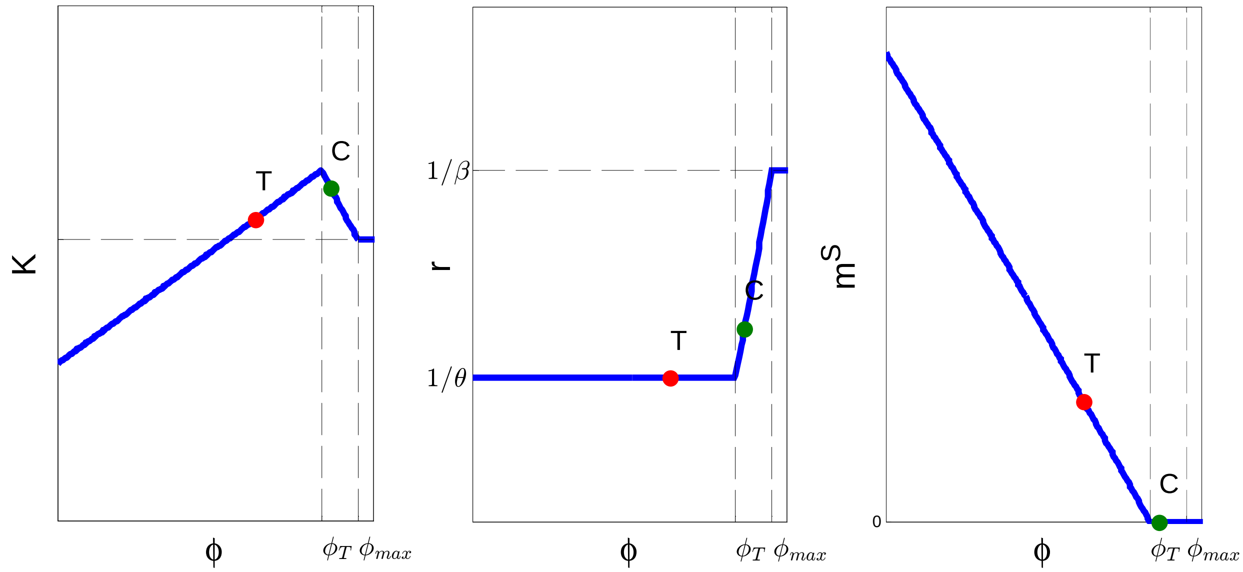


Figure 4: Steady states - Comparative statics w.r.t. ϕ, with $\bar{l} < 0$

Proposition 2 (Steady state when entrepreneurs are net debtors) *Define* $\phi_{min}(\bar{l}) = -\bar{l}/\alpha$, $\phi_{max}(\bar{l}) = (1 - \bar{l}/\alpha)/2$ *and* $\phi_T(\bar{l}) = (\beta - \theta\bar{l}/\alpha)/(\theta + \beta)$. *If* $\phi_{min} < \phi < \phi_{max}(\bar{l})$, *then there exists a locally constrained steady state with* $r < 1/\beta$.

(i) *If, additionally,* $\phi \geq \phi_T(\bar{l})$, *then the steady state is cashless.*

(ii) *If,* $\phi < \phi_T(\bar{l})$, *then the steady state is a liquidity trap.*

(iii) *In the cashless steady state, the real interest rate* r *and the nominal interest rate* i *are increasing in* ϕ, $m^S = 0$ *and if* $\bar{l} < 0$ ($\bar{l} > 0$), *then* k *is decreasing (increasing) in* ϕ.

(iv) *In the liquidity-trap steady state, the real interest rate* r *and the nominal interest rate* i *are invariant in* ϕ, m^S/y *is decreasing in* ϕ *and* k *is increasing in* ϕ.

Proof. See Appendix A. ■

Figure 4 represent the effect of ϕ on the steady state with a net supply of bonds from the rest of the economy ($\bar{l} < 0$). When ϕ is above ϕ_T, the steady state is cashless. When ϕ decreases while staying above ϕ_T, the supply of assets decreases, which has a negative effect on the equilibrium interest rate. Since investors are net debtors, this has a positive effect on the investors' income, which increases the long-run capital stock. When ϕ falls below ϕ_T, then the steady state state is a liquidity trap. As a result, the interest rate does not fall as a response to a deleveraging shock, thus not reestablishing the financing capacities of investors. Instead, investors start holding money, which has a negative effect on capital accumulation.[7]

3.2 Workers' Deleveraging

Notice that the interest rate in the cashless steady state is increasing in $\bar{l}$, as apparent from Equation (23). Indeed, a decrease in $\bar{l}$ corresponds to a deleveraging shock as well, except that deleveraging affects workers (or the government) and not investors. Workers' deleveraging (a fall in $\bar{l}$ through a fall in $\bar{l}^w$) can therefore also lead to the zero lower bound. This is illustrated in the right panel of Figure 5, where a decrease in $\bar{l}$ makes the economy switch from C, a cashless steady state, to T, a liquidity trap, through a fall in r.

The effect on r is similar to a deleveraging shock on investors, because a deleveraging shock on workers limits the economy's supply of assets. However, the effect on k is different. In particular, once the economy is in a liquidity trap, changes in $\bar{l}$ have no effect. Indeed, since the interest rate cannot adjust, the net demand for assets s is inelastic in a liquidity trap, so any decrease in the supply of assets to investors through $\bar{l}$ is matched by an increase through m^S. As before, higher real holdings of money obtain through a downward shift in the path of prices. The key difference between a deleveraging shock on workers and on investors is that the former affects the supply of assets to investors, while the latter affects their net *demand* for assets. Both are fully accommodated by an adjustment in real money holdings, but only a change in demand actually changes the asset holdings of investors, which is the source of disinvestment.

In the cashless steady state, as shown by Equations (22) and (23), the effect of $\bar{l}$ is more ambiguous, as a decrease in $\bar{l}$ both increases investors' net debt and decreases the interest rate. The first effect reduces the supplementary income investors get thanks to the low interest rate. The second effect increases that income. However, as established in the following Proposition, in the situation we consider, that is an economy that falls in a liquidity trap starting with net debt ($\bar{l} < 0$), k is decreasing in $\bar{l}$ in the neighborhood of the liquidity trap.

[7] Note that when investors are net creditors ($\bar{l} > 0$), the capital stock decreases in ϕ both in the cashless and liquidity trap steady state. However, this case is less realistic.

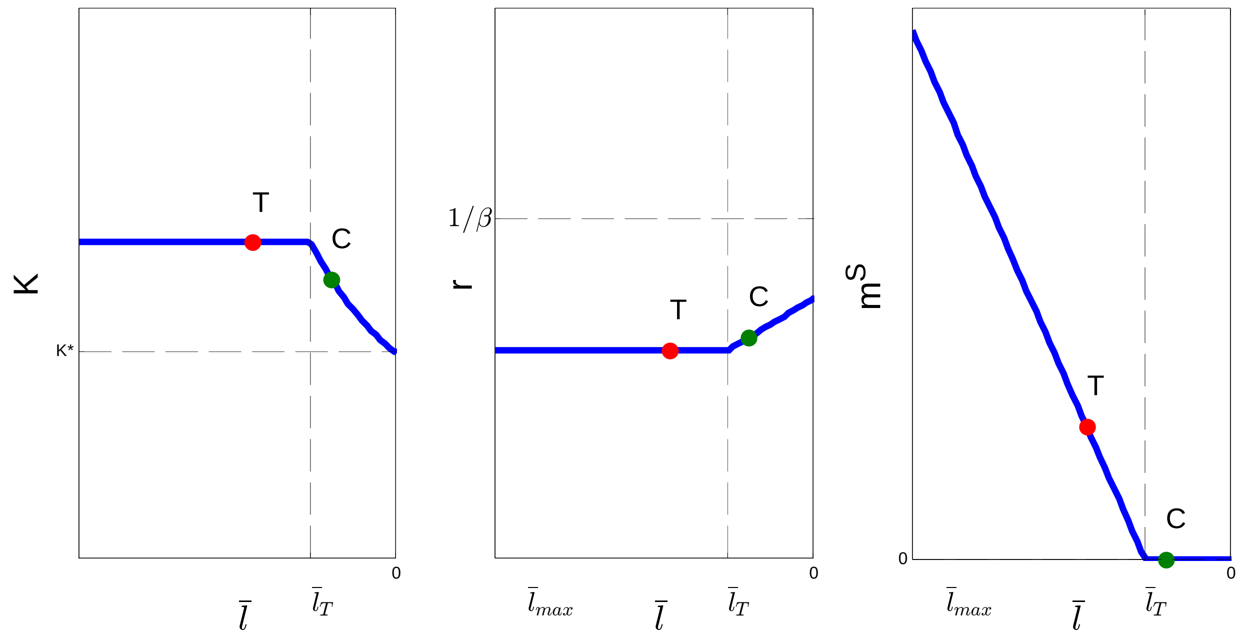


Figure 5: Steady states - Comparative statics w.r.t. $\bar{l}$, with $\bar{l} < 0$

Proposition 3 (Effect of workers' deleveraging with full depreciation) *Define* $\bar{l}_0(\phi) = \alpha\sqrt{\phi}(\sqrt{1-\phi} - \sqrt{\phi})$, $\bar{l}_{min}(\phi) = -\alpha\phi$, $\bar{l}_{max}(\phi) = \alpha(1-2\phi)$ *and* $\bar{l}_T(\phi) = \alpha\beta(1-\phi)/\theta - \alpha\phi$. *We have* $\bar{l}_{min} < \bar{l}_0 < \bar{l}_{max}$ *iif* $0 < \phi < 1/2$. *For* $\bar{l}_{min}(\phi) \leq \bar{l} \leq \bar{l}_{max}(\phi)$, *then there exists a locally constrained steady state with* $r < 1/\beta$.

(i) *If, additionally,* $\bar{l}_T(\phi) \leq \bar{l}$, *the steady state is cashless.*

(ii) *If* $\bar{l} < \bar{l}_T(\phi)$, *the steady state is a liquidity trap.*

(iii) *In the cashless steady state, the real interest rate* r *and the nominal interest rate* i *are increasing in* $\bar{l}$, $m^S = 0$ *and* k *is decreasing (increasing) in* $\bar{l}$ *for* $\bar{l} < \bar{l}_0$ ($\bar{l} > \bar{l}_0$).

(iv) *In the liquidity-trap state, the real interest rate* r *and the nominal interest rate* i *are invariant in* $\bar{l}$, m^S/y *is decreasing one for one in* $\bar{l}$ *and* k *is invariant in* $\bar{l}$.

(v) *if* $\phi > \beta/(\beta+\theta)$, *then* $\bar{l}_T < 0$, *so there exists cashless steady states with* $\bar{l} < 0$. *In that case,* $\bar{l}_0 > \bar{l}_T$ *so* k *is decreasing in* $\bar{l}$ *in the right neighborhood of* $\bar{l}_T$.

Proof. See Appendix A. ■

This is the situation illustrated in Figure 5. When switching from the cashless steady state C to the liquidity trap T, the economy experiences an increase in the capital stock.

4 Policy

We examine the policy implications of the model, first in exiting the liquidity trap and second in increasing output. The government can choose the growth rate of money θ, its debt/GDP ratio $\bar{l}^g$ or its primary deficit τ^w (equal to lump-sum transfers on workers). However, these three variables cannot be chosen independently as they are linked by the government budget constraint:

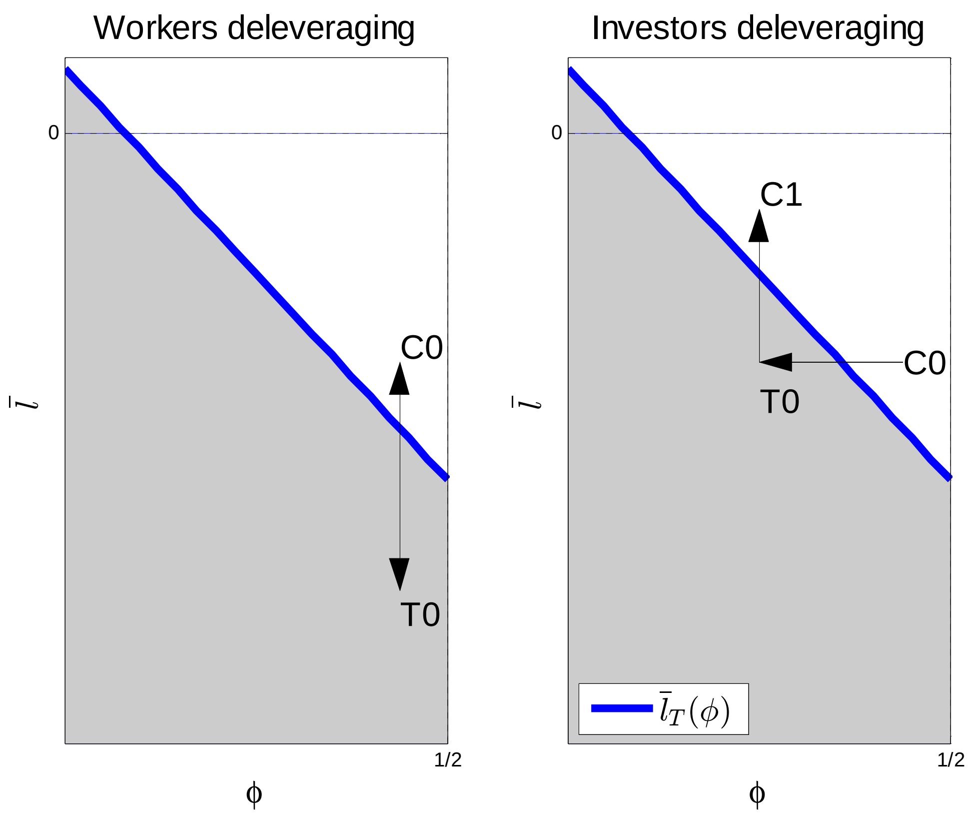
$$(\theta - 1)m + \bar{l}^g y \left(\frac{1}{r} - 1 \right) = \tau^w \tag{26}$$

The first term on the left-hand side is seignoriage and the second term is debt service. We characterize policy by $(\theta, \bar{l}^g)$ and let τ^w adjust to balance the budget constraint.

4.1 Getting Out of the Liquidity Trap

In line with the previous literature, a higher long-term inflation target helps avoiding or getting out of a liquidity trap (e.g., Krugman, 1998). Since the liquidity-trap steady state arises whenever the cashless steady-state interest rate is below $1/\theta$, it is sufficient to increase steady-state inflation θ. In constrast, the model has novel implications regarding the role of the public supply of bonds, which has implications for the impact of open market operations.

Public debt Proposition 3 has established that a decrease in $\bar{l}$ has a negative effect on the equilibrium real interest rate and hence can bring the economy to a liquidity trap. An increase in the supply of government bonds, by increasing $\bar{l}$, can have an opposite effect and bring the economy out of the liquidity trap. However, marginal changes in $\bar{l}$ have no effect as long as the economy remains in the liquidity trap, at least in the long run. Indeed, money and bonds are substitutes, so changes in the supply of bonds are matched by changes in the demand for real money balances. To accommodate for the change in the demand for real money, prices increase, unless the central bank intervenes to stabilize prices by increasing money supply.



(a) Workers' deleveraging

(b) Entrepreneurs' deleveraging

Figure 6: The effect of public debt

Only a massive increase in public debt, that compensates for the private deleveraging shock, can bring the economy out of a liquidity trap.

This is illustrated in Figure 6, which represents the $(\phi, \bar{l})$ space. The $\bar{l}_T(\phi)$ curve divides the graph between the cashless and the liquidity trap steady states. This curve is downward sloping: for a higher ϕ, the cashless steady state becomes compatible with a lower $\bar{l}$. In panels (a) and (b), C_0 represents the initial cashless steady state, where $\bar{l}$ is above $\bar{l}_T$. The economy then experiences a deleveraging shock that drives the economy to a long-term liquidity trap, T_0. In panel (a), the deleveraging shock affects workers (a drop in $\bar{l}$), which moves the steady state below the $\bar{l}_T(\phi)$ line, in the liquidity trap zone. In panel (b), the deleveraging shock affects investors (a drop in ϕ), which moves the steady state to the left of the $\bar{l}_T(\phi)$ line, in the liquidity trap zone. In both cases, a sufficient increase in public debt can bring the economy back to the cashless zone (back to C_0 in panel (a), to C_1 in panel (b)).

Fiscal policy In the baseline policy regime, the fiscal deficit τ^w adjusts to the policy mix $(\theta, \bar{l}^g)$. However, the fiscal deficit could also become the dominant policy parameter. In that case, either θ or $\bar{l}^g$ needs to adjust. In the cashless steady state, an increase in the fiscal deficit leads either to an increase in inflation (when $\bar{l}^g$ is exogenously set by the government) or a change in government debt (when θ is exogenously set). Government debt increases or decreases depending on whether r is larger or less than 1. However, in the liquidity trap, $\bar{l}^g$ becomes irrelevant. In that case, we can aggregate the demand for bonds and money to get:[8]

$$\theta(m + \bar{l}y) = \theta s + (1 - \alpha)y \tag{27}$$

We can then rewrite the government budget constraint (26) in the liquidity trap as

$$(\theta - 1)[(1 - \alpha)/\theta + s/y - \bar{l}^w] = \frac{\tau^w}{y}, \tag{28}$$

where s/y is given by (25). In the liquidity trap, the level of government debt $\bar{l}^g$ no longer appears in the government budget constraint. In fact $m + \bar{l}^g$ adjusts through m whenever $\bar{l}^g$ changes. Therefore, the composition of government liabilities changes without affecting the

[8]Equation (27) follows from (16) taken in a liquidity trap steady state, together with the definition of s.

government budget constraint. This is a by-product of the irrelevance result: in a liquidity trap, the composition of government liabilities does not matter.

This implies that with an increase in the fiscal deficit τ^w, only θ can adjust to maintain solvency by creating seigniorage. In that context, an increase in the fiscal deficit is necessarily inflationary. The fiscal deficit may therefore be effective in helping the economy getting out the liquidity trap, but only because it requires higher seignoriage which increases long-term inflation.[9]

Note that government spending, by increasing the deficit, would have similar effects. The results however, hinge partially on how the deficit is financed. We assume here that it is financed through taxes on workers. If, alternatively, it was partially financed through taxes on investors, then it would have negative effects on their investment capacities and hence on capital and output. This implication differs starkly from the literature on fiscal policy on government spending, which finds large fiscal multipliers. However, here we are focusing on long term effects, at a horizon where the effects of nominal rigidities unwind. Our results therefore do not exclude strong effects in the short run.

Quantitative Easing The above analysis implies that Quantitative easing (QE) has no effect per se in the liquidity trap steady state. QE consists in creating money through open market operations, i.e. increasing M by decreasing Pl^g. Since money and government bonds are perfect substitutes, this has no effect in our setting.[10] However, QE entails a decrease in the available amount of government bonds l^g, which worsens the excess demand for saving that maintains the economy at the ZLB. QE can therefore delay the exit from a liquidity trap.

This is represented in Figure 7. The economy experiences a deleveraging shock on ϕ that moves the steady state from C_0 to T_0, in the liquidity trap area. QE, by decreasing $\bar{l}$, then moves the steady state from T_0 to T_1. The economy therefore converges to T_1. If the level

[9] Note that this does not preclude changing the level of government debt through open-market operations, as $\bar{l}^g$ remains a free parameter. However, $\bar{l}^g$ per se will never be inflationary in a liquidity trap, because of the irrelevance result.

[10] Note that we abstract from some potential channels of QE. In particular, the perfect substitutability of money and bonds means that there is no broad portfolio balance channel that could lower term or risk premia. Similarly, there is no signalling effect on future rates. However, in our model, the liquidity trap is a steady state so there is no role for that channel. See Borio and Disyatat (2010) for a detailed description of the channels of QE.

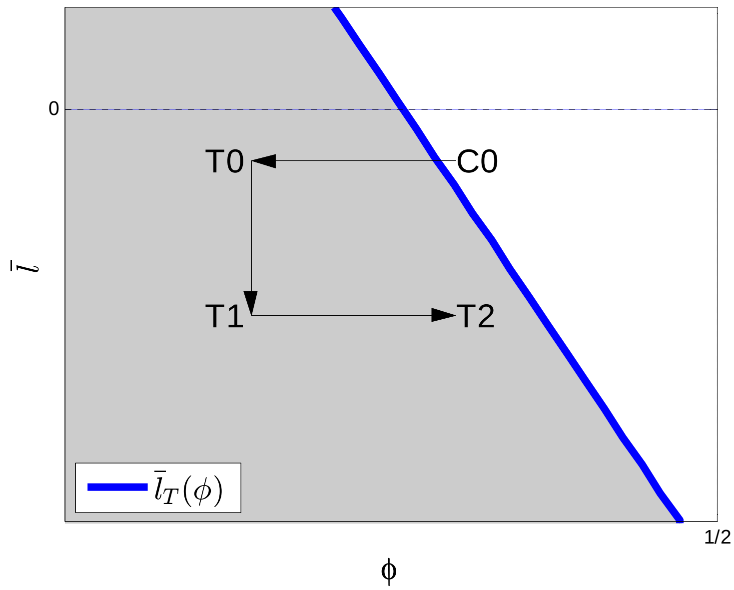


Figure 7: Open-market operations

of ϕ eventually comes back to its initial value, then the economy still converges to a liquidity trap in the long run, that is to T_2. Absent QE, the economy would have converged back to the cashless steady state C_0. It is thus important to time the exit from QE appropriately so that the economy does not linger in a liquidity trap.

Credit easing Our model does not account for the fact that QE sometimes goes hand-in-hand with credit easing aimed at improving credit conditions for the private sector, which can alleviate the effect of deleveraging. Credit easing would consist in the government issuing new debt Δl^g to lend an amount $\Delta\phi\alpha y$ to I-investors above the limit of their borrowing constraint, effectively relaxing this constraint. As $\Delta l^g = \Delta\phi\alpha y$, the government net debt does not change and stays equal to l^g. Credit easing can therefore be effective in getting out of the liquidity trap, as it helps re-leveraging investors after a deleveraging shock.

4.2 Stimulating Output

While inflation and public debt can help the economy get out of the liquidity trap, their effect on output is ambiguous.

Inflation Long-term inflation θ has a positive effect on the cashless nominal interest rate, which helps get the economy out of the liquidity trap. However, small increases in θ leaving the economy in the liquidity trap, have an ambiguous effect on capital, as suggested by the following Proposition:

Proposition 4 (Effect of steady-state inflation) *Consider $\phi_{min}(\bar{l})$ and $\phi_T(\bar{l})$ as defined in Proposition 2. If $\phi_{min}(\bar{l}) < \phi \leq \phi_T(\bar{l})$ and $\bar{l} \leq 0$, then the steady state is a liquidity trap and has the following properties:*

(i) *the real interest rate r is decreasing in θ;*

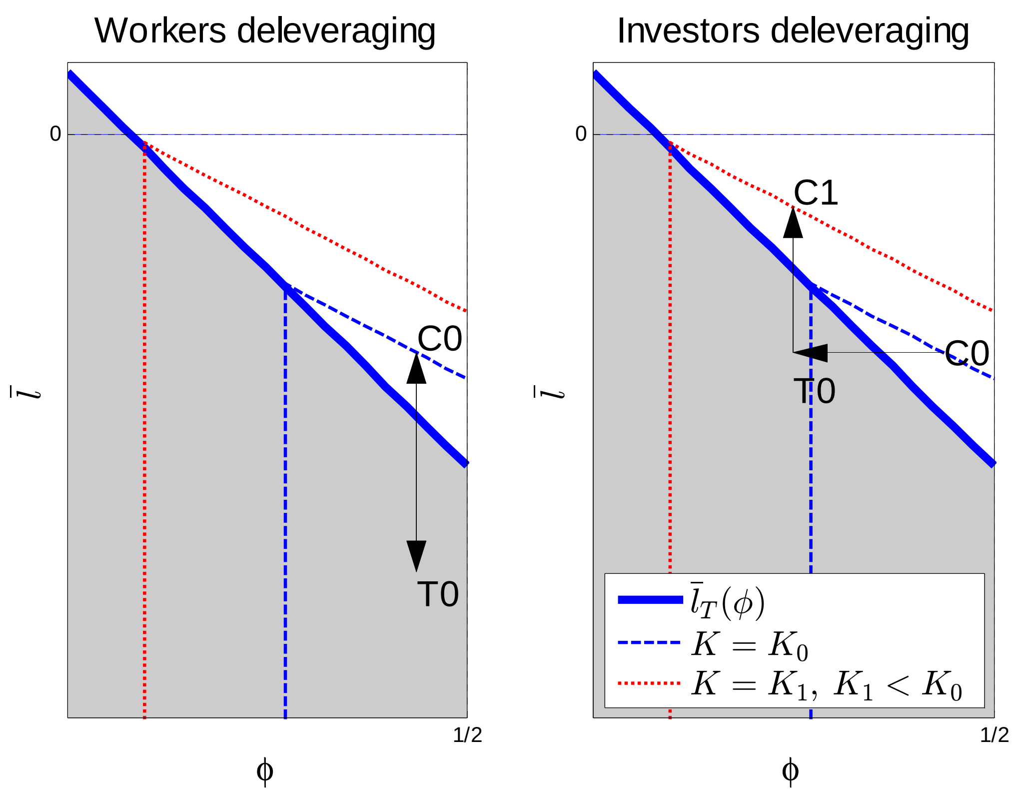
(ii) *the capital stock is decreasing in θ for $\phi_{min} < \phi < \frac{\beta^2}{\beta^2+\theta^2}$ and increasing in θ for $\frac{\beta^2}{\beta^2+\theta^2} < \phi < \phi_T(\bar{l})$;*

Proof. See Appendix A. ■

If investors are highly leveraged, the effect of inflation is positive because it deflates investors' debt as it decreases the real interest rate, and it improve their financing capacities. Otherwise, inflation affects negatively these financing capacities by deflating their savings.

Public debt Following a deleveraging shock that brings the economy to a liquidity trap, is an increase in government spending large enough to exit the trap then a good instrument to stimulate output? Not necessarily. In the case of a deleveraging shock on workers, an offsetting increase in government bonds can restore the previous steady state, as $\bar{l}^w$ and $\bar{l}^g$ have identical effects. This is illustrated in panel (a) of Figure 8, as public debt can restore the initial steady state C_0.

Things are different if the deleveraging shock affects investors. Suppose that investors are net debtors in the initial cashless steady state and that a deleveraging shock hits investors, bringing the economy in a liquidity trap. This is illustrated in panel (b) of Figure 8 by the switch from C_0 to T_0. The capital stock falls, as stated in Proposition 2. An increase in public debt can then take the economy out of the liquidity trap, but the stock of capital is lower than in the initial steady state. This is illustrated in the figure by the switch from T_0 to C_1. C_1 is below $\bar{l}_0(\phi)$, the threshold defined by Proposition 3, so it is in the area where k is decreasing in



(a) Workers' deleveraging

(b) Entrepreneurs' deleveraging

Figure 8: The effect of public debt

$\bar{l}$. In this area, an increase in $\bar{l}$ has a negative effect on the long-run capital stock. In that case, restoring the financing capacities of firms through credit easing would thus be a more effective way to stimulate output.

5 Extensions

5.1 Sticky Wages, Employment, and Output

Since the preceding analysis focused on the long term and studied steady states, the benchmark assumed flexible prices. But in order to discuss transitional dynamics in a meaningful way, we have to introduce nominal frictions. With nominal frictions, an increase in money demand by investors leads to a recession in the short run until the price level adjusts, as in the monetarist tradition. The intuition is best described by Equation (16), the market-clearing condition for money: $M_{t+1} = (1-\alpha)P_t y_t + M^S_{t+1}$. When the economy hits the ZLB, money demand by investors M^S increases. If the monetary authority does not react, adjustment has to come from a lower nominal output $P_t y_t$. As we have seen, in the long run, part of the adjustment comes from a downward shift in the price level, which accomodates the desired increase in *real* money holdings. If prices cannot adjust quickly, adjustment in the short run requires a drop in output. However, a sufficiently large monetary expansion directly accomodates investors' money demand, which stabilizes output in the short run and the price level in the longer run.

To examine this transitional dynamics more precisely, Appendix C.2 extends the model by introducing Calvo wage-setting and variable labor supply. The resulting model is similar to a standard New Keynesian framework, with one important difference: our model features an Euler equation that nests the traditional New Keynesian IS curve in the cashless economy, but has an additional monetary term in a liquidity trap.

The augmented IS curve We derive our augmented Phillips curve by substituting the money market equilibrium (16) into the Euler equation of savers (14):

$$\underbrace{\beta\alpha(1-\phi_{t-1})P_tY_t}_{\substack{\text{nominal demand}\\ \text{for assets by savers}}} + \underbrace{(1-\alpha)P_tY_t}_{\substack{\text{money demand}\\ \text{by workers}}} = \underbrace{(\phi_t\alpha+\bar{l}_t)\frac{P_{t+1}Y_{t+1}}{i_{t+1}}}_{\text{nominal supply of bonds}} + \underbrace{M_{t+1}}_{\substack{\text{money}\\ \text{supply}}}. \tag{29}$$

This relationship equates the total demand for assets to the total supply for assets in the economy. When $i > 0$, money and bond markets operate independently. Indeed, (16) becomes a quantity equation $M_{t+1} = (1-\alpha)P_t y_t$. The terms related to money demand and supply then drop from Equation (29), which simply becomes an equality between the demand for bonds by savers and the supply of bonds, i.e. an IS curve. Using the notation $\tilde{x}_t$ for log-deviations, it can be rewritten as the familiar IS curve of the New-Keynesian model, with the deleveraging shocks showing up as a natural rate shock:

$$\tilde{y}_t = \tilde{y}_{t+1} - \Big[\tilde{i}_{t+1} - \tilde{\pi}_{t+1} - \underbrace{\Big(\frac{\phi_0}{1-\phi_0}\tilde{\phi}_{t-1} + \frac{\alpha\phi_0}{\alpha\phi_0 + \bar{l}_0}\tilde{\phi}_t + \frac{\bar{l}_0}{\alpha\phi_0 + \bar{l}_0}\tilde{l}_t\Big)}_{\text{natural rate}}\Big],$$

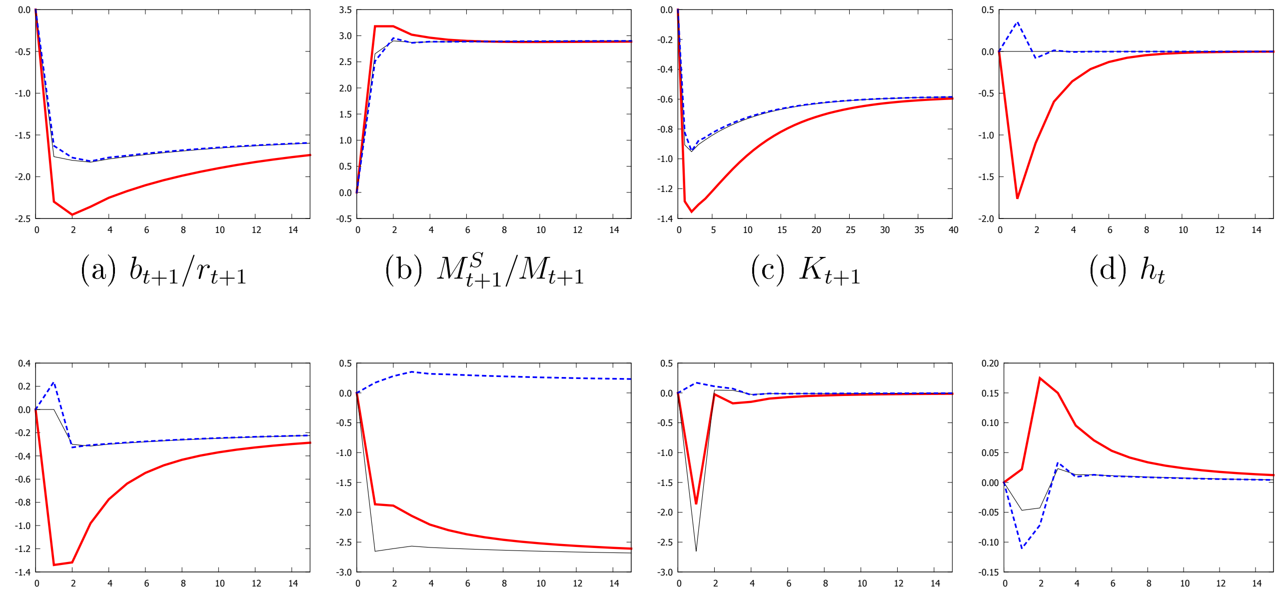
where $\tilde{\pi}$ is log-linearized inflation and the superscript 0 in $\bar{l}_0, \phi_0$ refers to values in the initial steady state. As in the New Keynesian model, monetary policy is able to fully offset deleveraging shocks as long as the economy does not hit the ZLB.

When $i_{t+1} = 1$, the simple quantity equation of the cashless dynamics ceases to hold, bonds and money are perfect substitutes and their corresponding markets merge. As in the standard New Keynesian model at the ZLB, a deleveraging shock (a negative shock to the natural rate) has to be accommodated by a drop in nominal output, which decreases the demand for assets. However, there is a noticeable difference: here, money supply M_{t+1} appears explicitly as a component of the asset supply. An increase in money supply M_{t+1} can therefore make the adjustment much easier.

To see this, iterate Equation (29) forward:

$$[(\beta\alpha(1-\phi_{t-1}) + 1 - \alpha)]\, P_t Y_t = \sum_{s=0}^{\infty}\left[\prod_{j=0}^{s-1}\frac{\theta(\alpha\phi_{t+j} + \bar{l}_{t+j})}{\beta\alpha(1-\phi_{t+j}) + 1 - \alpha}\right] M_{t+1}. \tag{30}$$

With constant values of ϕ and $\bar{l}$, the ratio of money supply to nominal output quickly converges (after one period) to $M_{t+1}/(P_t Y_t) = [\beta\alpha(1-\phi) - \theta(\alpha\phi + \bar{l}) + 1 - \alpha]$. The apparent velocity of money decreases after a deleveraging shock on ϕ or $\bar{l}$. If prices are very sticky and the stock of money does not change, then a deleveraging shock is fully absorbed by a drop in real output, until prices have adjusted enough to provide the desired real money holdings. However, an appropriate increase in money supply can potentially offset this transitory effect of the shock.



(a) b_{t+1}/r_{t+1} (b) M^S_{t+1}/M_{t+1} (c) K_{t+1} (d) h_t

(e) y_t (f) P_t (g) P_{t+1}/P_t (h) r_{t+1}

Figure 9: Transitory dynamics after a permanent tightening of the borrowing constraint at the ZLB. Thick red line: sticky wages. Thin black line: flexible wages. Dashed blue line: sticky wages with a permanent monetary expansion when the shock hits. All variables are relative deviation from initial steady state, in percent, except rates of return, inflation and M^s/M which are in absolute deviation from initial steady state, in percent.

Adjustment at the ZLB Figure 9 represents the response of the sticky wage model (with partial depreciation of capital $\delta < 1$) to a deleveraging shock on investors in the liquidity trap, in the case of autarkic investors. Here we assume a constant money supply, with $\theta = 1$. In period 0, the economy is in an initial steady state with $\phi = \phi_T$, that is, at the limit of the ZLB. In period 1, ϕ decreases permanently and unexpectedly by 1 percent. Outside of the ZLB, real variables would not react at all to such a shock, which would be entirely accommodated by a drop in the nominal interest rate i. The adjustment process is completely different in the liquidity trap.

Consider first the case of flexible wages, represented by the thin black line. When the shock hits, S-investors start demanding money (panel b) and the equilibrium adjustment comes from a drop in the price level (panel f). This works through a Pigou-Patinkin effect: the lower price level increases workers' real money holdings and makes them consume more, exchanging some of their money against goods to S-investors who want to hoard cash. This flexible price dynamics is different from models with a representative agent such as Krugman (1998) or models of moneyless economies such as Eggertsson and Krugman (2012): in these works, there is no Pigou effect and the price level has to decrease enough to generate inflation expectations that overcome the ZLB. By contrast, with heterogeneous agents and an explicitly modeled money demand, the price decrease has distributional effects which dampens the effect of the shock.

Consider now the case of sticky wages, represented by the thick red line in Figure 9. Because of staggered contracts, the nominal wage, and therefore the price level P_t, can only adjust gradually after the shock hits (panel f), triggering a long lasting deflationary process which raises the interest rate (paner g). Absent the drop in the price level, adjustment comes instead from lower employment and a lower output (panels d and e): total output falls because worker consumption cannot offset the fall in investment. With a lower production, capital accumulation drops sharply at impact (panel c). The demand for loans by I-investors is negatively affected by expected deflation, which raises the real interest rate, and by the lower expected return on capital (panel a). With a lower supply of saving instruments by investors, S-investors increase their demand for money even more than with flexible prices. As time goes by and prices gradually adjust, employment and output increase back to their flexible-price level and the economy converges to the liquidity trap steady state.

Therefore, with sticky wages, a deleveraging shock large enough to move the economy to the ZLB, creates a negative output gap in the short run, as in the existing New Keynesian literature. Contrary to that literature, the economy stays at the ZLB with a lower capital stock and lower level of output, even after wages have adjusted and the output gap has closed.

Alternative monetary policy Consider now a monetary expansion taking the form of transfers to workers. In the simulation represented by the dashed blue line, the government increases M once and for all when the shock hits. The increase is calibrated so that the nominal wage converges back to its initial value in the new steady state. As the figure shows, the resulting dynamics of real variables is very close to the dynamics with flexible wages. By increasing money supply, monetary policy substitutes to the fall in the price level that would obtain with flexible prices. Workers receiving monetary transfers feel richer exactly as they would with a lower price level. As a result they increase their consumption and sustain a higher level of output.

This result stands in sharp contrast to existing work, for instance Krugman (1998), where money creation taking the form of transfers has no effect at the ZLB with pre-set prices (see footnote 11 of this work). It comes from the non-ricardian structure of the model, which gives rise to the Pigou-Patinkin effect described above. However, if a policy of monetary transfers can be very effective in closing the output gap in the short-run of this model, it has no effect in the long run and therefore cannot prevent the long term output losses.

5.2 Nominal Government Bonds

We have assumed so far that government bonds were issued in real terms. In reality though, a large share of government bonds are nominal. In the model, assuming that bonds are nominal instead of real is innocuous, but the capacity of producing real saving instruments by issuing nominal bonds is somehow hampered in the liquidity trap as nominal bonds generate inflation.

In the cashless case, prices are determined by the stock of money through a classical quantity equation. Indeed, the money equilibrium (16) becomes $P = \theta M/(1-\alpha)y$ when $M^S = 0$. Therefore, for a given level of money M, the amount L^g of outstanding nominal bonds has no effect on the price level and directly determines the amount of real debt $l^g = L^g/P$. A 1% increase in nominal debt then translates into a 1% increase in real debt.

This is no longer true in the liquidity trap, as prices are now determined by the total stock of government nominal liabilities. From Equation (27), we now have $P = \theta(M+L^g)/[(1-\alpha)y + \theta(s-\bar{l}^w)]$. Because the market for money merges with the market for bonds, L^g is inflationary, just like money. However, the increase in prices following an increase in nominal debt, for M constant, is less than proportional, so the increase in nominal debt is not fully offset by the increase in prices. A 1% increase in nominal debt then does translate into an increase of real debt, though by less than 1%. The assumption of real bonds is therefore without loss of generality.

5.3 Financial Intermediation

In the benchmark model, money is modeled as *outside money* directly supplied by the Government. However, in practice, cash holdings usually take the form of deposits, which are a liability of banks, and could in principle be intermediated to capital investment. This extension shows that this is not the case. At the ZLB, banks are unable to channel deposits to credit constrained I-investors for the same reason that savers are unable to do it in the benchmark model. Instead, banks increase their excess reserves at the central bank.

Consider a simple model of *endogenous money*. The monetary authority now only controls base money M^0_{t+1}, which is assumed to be made entirely of banks' reserves. Total money M_{t+1} is made of deposits endogenously supplied by banks. In Equations (8) and (9) of the benchmark model, money supply M_{t+1} has then to be replaced by base money M^0_{t+1}.

There is a measure 1-continuum of banks owned by the representative worker. Banks receive a charter from the Government which allows them to issue deposits M_{t+1}, a zero nominal interest liability that can be used for transactions in the cash-in-advance constraint of workers. On their asset side, banks buy central bank reserves M^0_{t+1} and bonds for a nominal amount $M_{t+1} - M^0_{t+1}$. Banks maximize next-period profits, which they rebate (period by period) to households. In order to limit money creation, the bank charter subjects them to a reserve requirement: their buying of bonds cannot exceed a fraction μ of the net present value of deposits:[11]

$$M_{t+1} - M^0_{t+1} \leq \mu \frac{M_{t+1}}{i_{t+1}}.$$

The market clearing condition for bonds, given by Equation (10) in the benchmark model, has to be modified to account for bond demand by banks:

$$b_{t+1} + l^w_{t+1} + l^g_{t+1} = a_{t+1} + r_{t+1} \frac{M_{t+1} - M^0_{t+1}}{P_t}. \tag{31}$$

It is useful to define $\tilde{M}^0_{t+1}$, an indicator of *excess* reserves of banks, by:

$$\tilde{M}^0_{t+1} = M^0_{t+1} - \big(1 - \frac{\mu}{i_{t+1}}\big) P_t (1-\alpha) Y_t.$$

We obviously have $\tilde{M}^0_{t+1} = 0$ in the cashless equilibrium.[12] In the general case, the bond market

[11] While the precise form of the reserve requirement does not matter, this expression yields tractable results.

[12] When $i_{t+1} > 1$, banks issue as much money and buy as little reserves as they can and the reserve requirement is binding. Banks' reserves are then equal to $M^0_{t+1} = (1 - \mu/i_{t+1}) M_{t+1} = (1 - \mu/i_{t+1})(1-\alpha) P_t y_t$.

equilibrium can be rewritten

$$b_{t+1} + l^w_{t+1} + l^g_{t+1} = a_{t+1} + \mu \frac{P_t}{P_{t+1}}(1-\alpha)Y_t + \frac{M^S_{t+1} - \tilde{M}^0_{t+1}}{P_{t+1}}.$$

Note that a fraction μ of workers's money holdings for transaction purposes is channeled by banks to the bond market. At the zero-lower bound, banks are indifferent between buying bonds or reserves, the reserve requirement does not bind, and excess reserves $\tilde{M}^0 \geq 0$.

We can now rewrite the main equations of the benchmark model, the Euler equation (14) and the aggregate budget constraint (15) as

$$\beta\alpha(1-\phi_{t-1})y_t = \frac{1}{r_{t+1}}\left[(\phi_t\alpha + \bar{l}_t)y_{t+1} - \mu\frac{P_t}{P_{t+1}}y_t + \frac{\tilde{M}^0_{t+1}}{P_{t+1}}\right], \tag{32}$$

$$k_{t+1} + \frac{\tilde{M}^0_{t+1}}{P_t} + \bar{l}_t\frac{y_{t+1}}{r_{t+1}} - \mu\frac{P_t}{P_{t+1}}\frac{y_t}{r_{t+1}} = \beta\left[(\alpha + \bar{l}_{t-1})y_t - \mu\frac{P_{t-1}}{P_t}y_{t-1} + \frac{\tilde{M}^0_t}{P_t}\right]. \tag{33}$$

There are only two changes compared to the benchmark model. First, the net supply of bonds from the rest of the economy is decreased by the share μ of workers' deposits lent by banks to investors: $\bar{l}_t y_{t+1}$ has to be replaced by $\bar{l}_t y_{t+1} - \mu P_t/P_{t+1}(1-\alpha)y_t$. Second, money holdings by investors M^S is replaced by excess reserves $\tilde{M}^0$ at the Central Bank. In this extended model, the increase in cash holdings by investors at the zero lower bound shows up as an increase in excess reserves at the Central Bank. Results on the steady state of the benchmark model extend to the case of endogenous money with the simple change of parameter $\bar{l} \to \bar{l} - \mu(1-\alpha)/\theta$.

5.4 Idiosyncratic Uncertainty

The benchmark model with deterministic transitions between saving and investing phases can be easily extended to stochastic transitions. Appendix C.3 considers a 2-state Markov process where an investor with no investment opportunity at time $t-1$ receives an investment opportunity at time t with probability $\omega \in (0, 1]$; while an investor with an investment opportunity at time $t-1$ receives no investment opportunity at time t.

With this process, investment at the individual level is lumpy. The process converges to an invariant distribution with $1/(1+\omega)$ saving investors, and $\omega/(1+\omega)$ borrowing investors.

The benchmark model corresponds to the case $\omega = 1$ with an equal numbers of savers and borrowers. As ω decreases, the share of savers increases and the share of investors with an investment opportunity decreases.

Appendix C.3 shows that the Euler equation (14) for S-investors becomes::

$$\beta(1-\omega)\left[(\phi_{t-1}\alpha + \bar{l}_{t-1})y_t + \frac{M_t^S}{P_t}\right] + \beta\alpha(1-\phi_{t-1})y_t = \frac{1}{r_{t+1}}\left[(\phi_t\alpha + \bar{l}_t)y_{t+1} + \frac{M_{t+1}^S}{P_{t+1}}\right]. \quad (34)$$

This equation only differs from the benchmark model by the first term on the left hand side. This term represents the demand for saving instruments at time t from savers that were already savers at time $t-1$. The lower ω, the larger the share of savers, the higher this additional demand for saving instruments compared to the benchmark model. The term vanishes when $\omega = 1$. This is the only difference between the extended model and the benchmark. Indeed, we can aggregate the budget constraints of all investors, regardless of whether they save or borrow, to get the same aggregate budget constraint (15) as in the benchmark model.

Appendix C.3 then shows that results from the benchmark model extend to the case of idiosyncratic uncertainty.

5.5 Preference and Growth Shocks

In the existing literature, the shock that brings the economy to the ZLB is often assumed to be an increase in the factor of time preference. This shock, by increasing the agents' propensity to save, has a negative effect on the interest rate. A reduction in the average growth rate of productivity has also be put forward as a cause of the secular decrease in the interest rate and for hitting the ZLB. Here we examine whether these shocks are a reasonable source of the slowdown in investment observed in the liquidity trap. In fact, in an infinite-horizon model, the effect of a growth slowdown is isomorphic to an increase in the factor of time preference. We therefore restrict our analysis to the latter. We find that a permanent increase in β (alternatively, a permanent fall in steady-state growth), cannot generate a fall in investment when the economy falls into a liquidity trap.

To study the effect of β on output, we make the simplifying assumption of autarkic investors: $\bar{l} = 0$. This is without loss of generality as the investors' net debt matters only in the cashless economy. We derive the following Proposition:

Proposition 5 (Effect of β on the steady state with autarkic investors) *Define* $\beta_T = \theta\phi/(1-\phi)$ *and* $\phi_{max} = 1/2$.

If $0 < \phi < \phi_{max}$, *then there exists a locally constrained steady state with* $r < 1/\beta$.

(i) *If, additionally,* $\beta \leq \beta_T$, *then the steady state is cashless.*

(ii) *If* $\beta > \beta_T$, *then the steady state is a liquidity trap.*

(iii) *In the cashless steady state, the real interest rate* r *and the nominal interest rate* i *are decreasing in* β, $m^S = 0$ *and* k *is increasing in* β.

(iv) *In the liquidity-trap steady state, the real interest rate* r *is invariant in* β, m^S/y *is increasing in* β *and* k *is increasing in* β.

Proof. See Appendix A. ■

An increase in β makes the long-run interest rate fall, and eventually hit the zero-lower bound. In both the cashless and liquidity-trap steady states, an increase in β increases the investors' propensity to save, which increases the capital stock in the long run. As a result, whereas an increase in β can explain the emergence of a liquidity trap, it cannot explain the slowdown in investment. In the presence of trend growth, the same conclusions would hold in case of a growth slowdown. In particular, with lower trend growth, less investment is required to keep the capital stock on its trend. Therefore a given amount of saving leads to an upward shift of the capital intensity of production.

6 Conclusions

The liquidity trap that followed the Global Financial Crisis has been more persistent than was expected. In most countries, this has been accompanied by a slower-than-expected recovery and a surprising accumulation of money holdings. In this paper, we present a model consistent with these features. We model the liquidity trap by a scarcity of assets caused by firms' deleveraging. Since we focus on longer term issues, we mostly assume price flexibility. We find that a persistent deleveraging shock increases the demand for real money holdings, and that this leads to lower capital and output in the long term.

While most of our analysis is conducted in a stylized benchmark model, the main mechanism is robust to many extensions. The extensions considered in the paper include capital depreciation, idiosyncratic uncertainty, price rigidity, nominal bonds, or introducing financial intermediaries. For analytical convenience, we consider a permanent deleveraging shock for firms, but the results would be similar with a very persistent shock. However, long-term output declines in a liquidity trap only with firms' deleveraging. Other positive shocks to saving, like workers' deleveraging or an increase in the discount rate, may also lead to a liquity trap, but they do not depress output in the long run. Therefore it is crucial to determine the factors that have led to a liquidity trap. Interestingly, the evidence suggests that the balance sheets of both households and firms have played a role during the recession of 2007.[13]

Overall, our approach is complementary to keynesian analyses that stress the role of insufficient demand in a liquidity trap. While they describe a situation of negative output gap when the adjustment of prices is hampered by nominal rigidities, we show that low investment demand leads to lower potential output even after prices have fully adjusted.

Our framework also enables to examine policies that are complementary to more standard demand management. In this context, we find that quantitative easing is ineffective at the ZLB and can delay its exit. In contrast, an increase in public debt can shorten the liquidity trap period, but may crowd out investment when conditions normalize.

This analysis has been conducted in a closed economy. An obvious extension is to analyze the international spillovers of such policies.

A Proofs

We establish first the following Lemma:

Lemma 1 *The cashless and liquidity trap steady states are characterized as follows:*

[13]See Mian and Sufi (2010, 2012) for the evidence on households' deleveraging. See Chodorow-Reich (2014), Giroud and Mueller (2015), Bentolila et al. (2015) for the evidence on firms' deleveraging.

(i) *In a cashless steady state,*

$$r^* = \frac{\alpha\phi + \bar{l}}{\beta\alpha(1-\phi)}, \qquad k^* = \left[\beta\alpha - \bar{l}(1/r^* - \beta)\right]^{\frac{1}{1-\alpha}}, \qquad m^{S*} = 0.$$

(ii) *In a liquidity-trap steady state,*

$$\hat{r} = 1/\theta, \qquad \hat{k} = \left(\frac{\beta^2 + \phi(\theta^2 - \beta^2)}{\theta/\alpha}\right)^{\frac{1}{1-\alpha}}, \qquad \hat{m}^S = \alpha\left[(1-\phi)\frac{\beta}{\theta} - \phi - \bar{l}/\alpha\right]\hat{k}^\alpha.$$

Proof. In a steady state, the money market equilibrium implies that $P_{t+1}/P_t = \theta$. As a result, $i = r\theta$.

In a steady state with $i^* > 1$, (14) and (15) are satisfied with $M^S = 0$. Equation (14) taken at the steady state gives r^*. Besides, (15) in the steady state gives:

$$k^*/y^* = \beta\alpha - \bar{l}(1/r^* - \beta)$$

which yields our result for k^*. This proves result (i).

In a steady state with $i = 1$, (14) and (15) are satisfied with $r = \hat{r} = \theta^{-1}$, which yields $\hat{k}/\hat{Y} = \frac{\beta^2+\phi(\theta^2-\beta^2)}{\theta\alpha^{-1}}$, from which we derive $\hat{k}$, and $\hat{m}^S = \alpha\left[(1-\phi)\frac{\beta}{\theta} - \phi\right]\hat{k}^\alpha - \bar{l}\hat{k}^\alpha$. This proves result (ii). ■

A.1 Proof of Proposition 1

Consider a cashless steady state with $\bar{l} = 0$. According to Lemma 1, $r^* = \phi/[\beta(1-\phi)]$. We check that $0 < \beta r^* < 1$ as $\phi < \phi_{max}$ and that $i^* = \theta r^* \geq 1$ as $\phi \geq \phi_{max}$, which insures that the cashless steady state exists and is locally constrained. This proves result (i).

If $\phi < \phi_T$, then the steady state without money does not exist, as the implied nominal interest rate i^* would be below one. If there exists a steady state with $i = 1$, then it is a liquidity trap described by Lemma 1. According to Lemma 1, when $\bar{l} = 0$, $\hat{m}^S = \alpha\left[(1-\phi)\frac{\beta}{\theta} - \phi\right]\hat{k}^\alpha$, which is strictly positive when $\phi < \phi_T$. Besides, $\hat{r} = \theta$, which implies that $0 < \beta\hat{r} < 1$ under Assumption 1, and $\hat{r} > r^*$ for $\phi < \phi^T$. We also check that $\hat{k} = \left(\frac{\beta^2+\phi(\theta^2-\beta^2)}{\theta/\alpha}\right)^{\frac{1}{1-\alpha}} < k^* = (\beta\alpha)^{\frac{1}{1-\alpha}}$ for $\phi < \phi^T$. This proves result (ii).

Results (iii) and (iv) derive naturally from results Lemma 1.

A.2 Proof of Proposition 2

Consider a cashless steady state. Using Lemma 1, we check that $0 < \beta r^* < 1$ as $\phi_{min}(\bar{l}) < \phi < \phi_{max}(\bar{l})$ and that $i^* = \theta r^* > 1$ as $\phi > \phi_T(\bar{l})$, which insures that the cashless steady state exists and is locally constrained. This proves result (i).

If $\phi < \phi_T(\bar{l})$, then the steady state without money does not exist, as the implied nominal interest rate i^* would be below one. If there exists a steady state with $i = 1$, then it is a liquidity trap described by Lemma 1. According to Lemma 1, $\hat{m}^S = \alpha\left[(1-\phi)\frac{\beta}{\theta} - \phi - \bar{l}/\alpha\right]\hat{k}^\alpha$, which is strictly positive when $\phi < \phi_T(\bar{l})$. Besides, $\hat{r} = \theta$, which implies that $0 < \beta\hat{r} < 1$ under Assumption 1, and $\hat{r} > r^*$ for $\phi < \phi^T$. We also check that $\hat{k} = \left(\frac{\beta^2+\phi(\theta^2-\beta^2)}{\theta/\alpha}\right)^{\frac{1}{1-\alpha}} < k^* = k^* = \left[\beta\alpha - \bar{l}(1/r^* - \beta)\right]^{\frac{1}{1-\alpha}}$ for $\phi < \phi^T$. This proves result (ii).

Regarding the properties of r, i and m^S, results (iii) and (iv) derive directly from Lemma 1. To derive the properties of k, we replace r^* in k^* to obtain

$$k^* = \left(\alpha\beta - \bar{l}\left[\frac{\alpha\beta(1-\phi)}{\alpha\phi + \bar{l}} - \beta\right]\right)^{1/(1-\alpha)} \tag{35}$$

We can see that k^* is increasing in ϕ for $\bar{l} > 0$, decreasing for $\bar{l} < 0$.

A.3 Proof of Proposition 3

Results (i) and (ii) derive directly from Lemma 1.

Regarding the properties of r, i and m^S, results (iii) and (iv) derive directly from Lemma 1. To derive the properties of k, we use (35) and take the derivative of k with respect to $\bar{l}$. We find that k is decreasing in $\bar{l}$ whenever $P(\bar{l}) \geq 0$ with

$$P(\bar{l}) = \bar{l}^2 + 2\alpha\phi\bar{l} - \alpha^2\phi(1-2\phi)$$

This second-order polynomial admits two roots: $\bar{l}_{00} = -\alpha\phi - \alpha\sqrt{\phi}\sqrt{1-\phi}$ and $\bar{l}_0 = -\alpha\phi + \alpha\sqrt{\phi}\sqrt{1-\phi}$. As $\bar{l}_{00} < \bar{l}_{min}$, $\bar{l}_0$ is the only relevant solution. As a result, k is decreasing in $\bar{l}$ for $\bar{l}_{min} \leq \bar{l} \leq \bar{l}_0$ and increasing for $\bar{l}_0 \leq \bar{l} \leq \bar{l}_{max}$.

To show (iv), note that there exists cashless steady states with $\bar{l} < 0$ iif $\bar{l}_T(\phi) < 0$, which is the case when $\phi > \beta/(\beta+\theta)$. Besides, k is decreasing in $\bar{l}$ in the right neighborhood of $\bar{l}_T(\phi)$ iif $\bar{l}_0 > \bar{l}_T$, which is the case when $\phi > \beta^2/(\beta^2+\theta^2)$. Since $\theta/\beta > 1$ by assumption, we have $\phi > \beta/(\beta+\theta)$ implies $\phi > \beta^2/(\beta^2+\theta^2)$, hence the result.

A.4 Proof of Proposition 4

The proof derives from Lemma 1. To derive result (ii), we take the derivative of k with respect to θ and show that it is negative for $\phi_{min} < \phi < \frac{\beta^2}{\beta^2+\theta^2}$. $\bar{l} \leq 0$ is a sufficient condition for $\frac{\beta^2}{\beta^2+\theta^2} < \phi_T(\bar{l})$.

B Estimation of investors' net position

To calculate firms' net position in interest-bearing assets, we use the balance-sheet tables of the Nonfinancial Corporate Business (B.103) and of the Nonfinancial Nonorporate Business (B.104). We calculate the net positions as follows:

- Nonfinancial Corporate Business: Time and savings deposits (FL103030003) + Money market fund shares (FL103034003) + Security repurchase agreements (FL102051003) + Credit market instruments (FL104004005) + Trade receivables (FL103070005) - Credit market instruments (FL104104005) - Trade payables (FL103170005)- Taxes payable (FL103178000).

- Nonfinancial Corporate Business: Time and savings deposits (FL113030003) + Money market fund shares (FL113034003) + Credit market instruments (FL114004005) + Trade receivables (FL113070003) - Credit market instruments (FL114104005) - Trade payables (FL113170005) - Taxes payable (FL113178003).

We find large negative net positions of respectively -5000 Billion USD and -3500 Billion USD in 2013.

To calculate the net position of households owning a business or participating to the stock market, we use the 2013 Survey of Consumer Finances. We include only households who either own a business ($hbus = 1$) or own stocks ($hstocks = 1$). We compute their net position in interest-bearing assets as Certificates of deposit (*cds*) + Savings in bonds (*savbnd*) + Directly held bonds (*bond*) - Debt (*debt*). We exclude pensions and life insurance as these assets are usually not liquid and hence cannot be used for investment. The weighted average of these net positions is about -120 thousand USD.

C Extensions

C.1 Extension with Partial Capital Depreciation

We assume here that $\delta < 1$, so that capital depreciates only partially from period to period. For consistency, we focus on the case where investors are net debtors $\bar{l} \leq 0$. All our results generalize provided some mild condition on $\bar{l}$, as shown in the following Proposition:

Proposition 6 (Steady state when entrepreneurs are net debtors) *Define* $\phi_{max}(\bar{l}) = (1 - [1 - \beta(1-\delta)]\bar{l}/\alpha)/2$ *and* $\phi_T(\bar{l}) = (\beta - [\theta - \beta^2(1-\delta)]\bar{l}/\alpha)/[\theta + \beta - (\theta^2 - \beta^2)(1-\delta)\bar{l}/\alpha]$. *If* $\bar{l} \leq 0$ *and* $0 < \phi < \phi_{max}$*, then there exists a locally constrained steady state with* $0 < r < 1/\beta$.

(i) *If, additionally,* $\phi \geq \phi_T$*, then the steady state is cashless.*

(ii) *If,* $\phi < \phi_T$*, then the steady state is a liquidity trap.*

(iii) *In the cashless steady state, the real interest rate* r *and the nominal interest rate* i *are increasing in* ϕ *if* $\bar{l} > -1/\beta(1-\delta)$*, and increasing in* $\bar{l}$ *if* $\bar{l} > -1/[1+\beta(1-\delta)]$*,* $m^S = 0$*,* k *is decreasing in* ϕ *and decreasing in* $\bar{l}$ *in the neighborhood of* $\bar{l} = 0$*.*

(iv) *In the liquidity-trap steady state, the real interest rate* r *and the nominal interest rate* i *are invariant in* ϕ *and* $\bar{l}$*,* $m^S/\rho k$ *is decreasing in* ϕ *and* $\bar{l}$ *and* k *is increasing in* ϕ *and independent of* $\bar{l}$*.*

Proof. With partial depreciation, using $f(k) = [\rho(k) - (1-\delta)]k/\alpha$, we can show that the dynamic system at the cashless steady state satisfies

$$r\beta(1-\phi)\rho(k) = \phi\rho(k) + \frac{\bar{l}}{\alpha}[\rho(k) - (1-\delta)] \tag{36}$$

$$r = \beta r \left[\rho(k) + \frac{\bar{l}}{\alpha}\left(1 - \frac{1}{\beta r}\right)[\rho(k) - (1-\delta)]\right] \tag{37}$$

We derive r^* as follows. We use (37) to express ρ as a function of r and replace in (36). This gives $P(r) = 0$ where P is a second-order polynomial defined by

$$P(r) = \beta(1-\phi)\left[1+\beta(1-\delta)\frac{\bar{l}}{\alpha}\right]r^2 - \left(\phi + \frac{\bar{l}}{\alpha}\right)r + \phi(1-\delta)\frac{\bar{l}}{\alpha}$$

This polynomial admits two roots. We have $P(0) = \phi(1-\delta)\frac{\bar{l}}{\alpha} \leq 0$ as $\bar{l} \geq 0$ and $\phi > 0$, so it admits only one positive root, which we then take as our solution for r.

This solution is lower than $1/\beta$ as long as $P(1/\beta) > 0$. This is equivalent to $\phi < \phi_{max}$. Finally, $i = r\theta > 0$ is guaranteed by $P(1/\theta) < 0$, which implies $\phi > \phi_T$. In that case, the economy is cashless and follows 36 and 37. This proves result (i). Otherwise, the economy is in a liquidity trap and follows

$$\frac{\beta(1-\phi)}{\theta}\rho = \phi\rho + \frac{\bar{l}}{\alpha}[\rho - (1-\delta)] + \frac{m^S}{k} \tag{38}$$

$$1 = \beta\rho - (\theta - \beta)\left(\frac{\bar{l}}{\alpha}[\rho - (1-\delta)] + \frac{m^S}{k}\right) \tag{39}$$

This proves result(ii).

To establish result (iii), we totally differentiate P with respect to ϕ . Using the fact that r is the upper root of P so that $P'(r) > 0$, we can show that r is increasing in ϕ if and only if

$$\beta\left[1+\beta(1-\delta)\frac{\bar{l}}{\alpha}\right]r^2 + r - (1-\delta)\frac{\bar{l}}{\alpha} > 0$$

This is the case for $-1/\beta(1-\delta) \leq \bar{l} \leq 0$ as r is positive.

Similarly, we totally differentiate P with respect to $\bar{l}$. Using the fact that r is the upper root of P so that $P'(r) > 0$, we can show that r is increasing in $\bar{l}$ if and only if

$$r - (1-\phi)(1-\delta)\beta^2 r^2 - \phi(1-\delta) > 0$$

As $\beta^2 r^2 < 1$, a sufficient condition is $r > 1 - \delta$. This is the case for $\bar{l} > -1/[1+\beta(1-\delta)]$, as this guarantees $P(1-\delta) < 0$.

We then express r as a function of ρ using (36) and replace in (37). We find that $Q(\rho) = 0$ where Q is a second-order polynomial defined by

$$Q(\rho) = \beta\left[\left(\phi + \frac{\bar{l}}{\alpha}\right)\left(1 + \frac{\bar{l}}{\alpha}\right) - (1-\phi)\frac{\bar{l}}{\alpha}\right]\rho^2 - \left(\phi + \frac{\bar{l}}{\alpha}\right)\left[1 + 2\beta(1-\delta)\frac{\bar{l}}{\alpha}\right]\rho + (1-\delta)\frac{\bar{l}}{\alpha}\left[1 + \beta(1-\delta)\frac{\bar{l}}{\alpha}\right]$$

We select the upper root of this polynomial for similar reasons. We thus have

$$\rho = \frac{\left(\phi + \frac{\bar{l}}{\alpha}\right)\left[1 + 2\beta(1-\delta)\frac{\bar{l}}{\alpha}\right] + \sqrt{\left(\phi + \frac{\bar{l}}{\alpha}\right)^2 - \phi(1-\phi)\beta(1-\delta)\left[1 + \beta(1-\delta)\frac{\bar{l}}{\alpha}\right]}}{2\beta\left[\left(\phi + \frac{\bar{l}}{\alpha}\right)\left(1 + \frac{\bar{l}}{\alpha}\right) - (1-\phi)\frac{\bar{l}}{\alpha}\right]}$$

We compute $Q(1/\beta)$ and show

$$Q(1/\beta) = -[1 - \beta(1-\delta)]\frac{\bar{l}}{\alpha}\left[1 - 2\phi - [1 - \beta(1-\delta)]\frac{\bar{l}}{\alpha}\right]$$

This is positive if $\bar{l} < 0$, which implies that $\rho < 1/\beta$.

To study the effect of ϕ on k, we totally differentiate Q with respect to ϕ. Using the fact that ρ is the upper root of Q so that $Q'(\rho) > 0$, we can show that ρ is increasing in ϕ if and only if

$$(\beta\rho - 1) + 2\frac{\bar{l}}{\alpha}\beta[\rho - (1-\delta)] < 0$$

For $\bar{l} < 0$, $\beta\rho < 1$. Besides, as the non-negativity on k imposes $\rho \geq 1 - \delta$, then the second term is also negative in that case. As a result, ρ is increasing in ϕ, which implies that k is decreasing in ϕ.

Similarly, to study the effect of $\bar{l}$ on k, we totally differentiate Q with respect to $\bar{l}$. Using the fact that ρ is the upper root of Q so that $Q'(\rho) > 0$, we can show that ρ is increasing in $\bar{l}$ if and only if

$$\left[1 + 2\beta(1-\delta)\left(\phi + 2\frac{\bar{l}}{\alpha}\right)\right]\rho - 2\beta\left(\phi + \frac{\bar{l}}{\alpha}\right)\rho^2 - (1-\delta)\left[1 + 2\beta(1-\delta)\frac{\bar{l}}{\alpha}\right] > 0$$

This is the case both for $\bar{l} = 0$, for which $\rho = 1/\beta$. Therefore, ρ is increasing in $\bar{l}$ in the neighborhood of $\bar{l} = 0$. Since k is inversely related to ρ, k is decreasing in $\bar{l}$ in the neighborhood of $\bar{l} = 0$.

To derive result (iv), consider Equations (38) and (39), which describe the liquidity-trap steady state. They yield

$$\begin{aligned} \rho &= \tfrac{\theta}{\beta^2+(\theta^2-\beta^2)\phi} \\ \tfrac{m^S}{k} &= \left[\tfrac{\beta}{\theta}(1-\phi)-\phi-\tfrac{\bar{l}}{\alpha}\right]\rho+(1-\delta)\tfrac{\bar{l}}{\alpha} \end{aligned}$$

As $\theta > \beta$, ρ is decreasing in ϕ, which implies that k is increasing in ϕ. We can also see that ρ and hence k are independent of $\bar{l}$. Similarly, as $i = 1$ and $r = 1/\theta$ in a liquidity trap, i and r are independent of ϕ and $\bar{l}$. Regarding m^S/k, since ρ is decreasing in ϕ, then m^S/k is decreasing in ϕ. Finally, since ρ is independent of $\bar{l}$ and $\rho > 1-\delta$, then m^S/k is decreasing in $\bar{l}$. ■

C.2 Labor market with Calvo wage-setting

This section extends the model to a New Keynesian framework with sticky wages and endogenous labor supply. Workers supply labor h_t to a measure 1 continuum of employment agencies which produce differentiated labor and engage in monopolistic competition. Agency i transforms $h_{i,t}$ units of homogenous labor into $H_{i,t}$ units of variety i with nominal wage $W_{i,t}$. Employment agencies are owned by workers and transfer their profits to them period by period. A competitive sector then aggregates those differentiated varieties of labor into composite labor H_t with production function $H_t = \left[\int H_{i,t}^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}$. The corresponding aggregate nominal wage is $W_t = \left[\int W_{i,t}^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}$. Firms then hire this composite labor to produce $y_t = F(k_t, H_t)$.

Workers

The representative worker has a utility function $U_t^w = \mathrm{E}_t \sum_{s=0}^{\infty} \beta^s u(c_{t+s}^w, h_{t+s})$ and is subject to the budget constraint

$$c_t^w + \frac{M_{t+1}^w}{P_t} + l_t^w = w_t h_t + D_t + \frac{M_t^w}{P_t} + \frac{T_t^w}{P_t} + \frac{l_{t+1}^w}{r_{t+1}}, \tag{40}$$

where w_t is the real wage paid to workers by employment agencies. Compared to Equation (4), this budget constraint adds dividends D_t paid by employment agencies (which can be negative) as a source of income. Workers are also subject to the borrowing constraint (6) and the cash-in-advance constraint.

Denote respectively λ_t, μ_t, and γ_t the Lagrange multipliers on the budget constraint, the cash-in-advance constraint, and the borrowing constraint. Maximization implies the following first order conditions:

$$\begin{aligned} u'_c(c^w_t, h^w_t) &= \lambda_t + \mu_t, \\ -u'_h(c^w_t, h_t) &= w_t \lambda_t, \\ \lambda_t &= \beta \frac{P_t}{P_{t+1}} (\lambda_{t+1} + \mu_{t+1}), \\ \gamma_t &= \lambda_t + \mu_t - \beta r_{t+1} (\lambda_{t+1} + \mu_{t+1}). \end{aligned}$$

We can show that $\mu_t = \gamma_t + (i_{t+1} - 1)\lambda_t$. This implies that the cash-in-advance constraint is always binding, even at the zero lower bound, as long as the borrowing constraint is binding, which we assume throughout. The optimal decision by the worker is then given by:

$$c^w_t = \frac{M^w_t + T^w_t}{P_t} + l^w_{t+1} - r_t l^w_t \tag{41}$$

$$-u'_h(c^w_t, h_t) = w_t \lambda_t \tag{42}$$

$$l^w_{t+1} = r_{t+1} l^w_{t+1}, \tag{43}$$

with

$$\lambda_t = \beta \frac{P_t}{P_{t+1}} u'_c(c^w_{t+1}, h_{t+1}).$$

Employment agencies

Employment agencies are subject to a nominal friction, namely Calvo wage-setting. In each period, agencies can only reoptimize their wage $W_{i,t}$ with probability $1 - \eta$. With probability η, agencies simply adjust their nominal wage of the preceding period by the gross rate of steady state inflation θ: $W_{i,t} = \theta W_{i,t-1}$. We make the usual assumption that agencies receive a subsidy τ per unit of output, financed out of their profits by a lump sum tax. Later, we will set the subsidy to $\tau = (\epsilon - 1)^{-1}$ to offset the distortion from monopolistic competition in the steady state. This assumption, together with wage indexation, makes sure that the steady state of this model is identical to a flexible wage economy without monopolistic employment agencies.

Consider agency i. It faces a demand for its differentiated labor $H_{i,t} = \left(\frac{W_{i,t}}{W_t}\right)^{-\epsilon} H_t$. With probability η, it cannot reoptimize its wage $W_{i,t}$. Then, $W_{i,t} = \theta W_{i,t-1}$ and the associated value function is given by

$$V^S_{i,t}(W_{i,t}) = \lambda_t\Big((1+\tau)\frac{W_{i,t}}{P_t} - w_t\Big)H_{i,t} + \beta\lambda_{t+1}\Big(\eta V^S_{i,t}(\theta W_{i,t}) + (1-\eta)V^R_{i,t+1}\Big)$$

where V^R is the value of reoptimizing the nominal wage. It is given by:

$$V^R_{i,t} = \max_{W_i} \lambda_t\Big((1+\tau)\frac{W_i}{P_t} - w_t\Big)H_{i,t} + \beta\lambda_{t+1}\Big(\eta V^S_{i,t}(\theta W_i) + (1-\eta)V^R_{i,t+1}\Big),$$

where the maximization is subject to the demand for labor $H_{i,t}$. The optimal reset wage W^*_t is given by:

$$W^*_t = \frac{\Gamma^1_t}{\Gamma^0_t}P_t \tag{44}$$

where

$$\Gamma^0_t = \lambda_t H_t + \beta\eta\left(\frac{W_{t+1}}{\theta W_t}\right)^{\epsilon}\frac{\theta P_t}{P_{t+1}}\Gamma^0_{t+1},$$
$$\Gamma^1_t = \lambda_t H_t w_t + \beta\eta\left(\frac{W_{t+1}}{\theta W_t}\right)^{\epsilon}\Gamma^1_{t+1},$$

where we have set the subsidy to $\tau = (\epsilon - 1)^{-1}$.

Aggregating across agencies, we get the evolution of the nominal wage of composite labor W_t:

$$W_t = \left[\eta(\theta W_{t-1})^{1-\epsilon} + (1-\eta)(W^*_t)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}. \tag{45}$$

The aggregate demand for homogeneous labor by employment agencies is $\int H_{i,t}di = \Delta_t H_t$ with $\Delta_t = \int\left(\frac{W_{i,t}}{W_t}\right)^{-\epsilon} di$ capturing wage dispersion across agencies. Market clearing on the labor market yields:

$$\Delta_t H_t = h_t. \tag{46}$$

The evolution of price dispersion is given by

$$\Delta_t = \eta\Big(\frac{W_t}{\theta W_{t-1}}\Big)^{\epsilon}\Delta_{t-1} + (1-\eta)\Big(\frac{W^*_t}{W_t}\Big)^{-\epsilon}. \tag{47}$$

Finally, the aggregate dividends received by workers are given by $D_t = W_t H_t / P_t - w_t h_t$. Substituting this expression into (40), we get the budget constraint (4) of workers.

To close the model, the return paid by firms to investors is now $\rho_t = \alpha k_t^{\alpha-1} H_t^{1-\alpha} + (1-\delta)$ and the real wage paid by firms to employment agencies is given by

$$\frac{W_t}{P_t} = (1-\alpha)\left(\frac{k_t}{H_t}\right)^{\alpha}. \tag{48}$$

Cashless dynamics In cashless equilibria, the model can be log-linearize to get a standard New Keneysian framework. For simplicity, consider the case of autarkic investors. Denoting $\tilde{x}_t$ the log-deviation from an initial steady state associated with $\phi_t = \phi_0$, and choosing separable preferences for workers $u(c,h) = \log c - h^{1+\sigma^{-1}}/(1+\sigma^{-1})$, we get:

$$\tilde{y}_t = \tilde{y}_{t+1} - \Big[\tilde{i}_{t+1} + \tilde{\pi}_{t+1} - \underbrace{\Big(\tilde{\phi}_t + \frac{\phi_0}{1-\phi_0}\tilde{\phi}_{t+1}\Big)}_{\text{natural rate}}\Big], \tag{49a}$$

$$\tilde{\pi}^w_t = \beta\tilde{\pi}^w_{t+1} + \frac{(1-\eta)(1-\beta\eta)}{\eta}\Big[\tilde{\pi}_{t+1} + \tilde{y}_{t+1} + \frac{\sigma^{-1}}{1-\alpha}(\tilde{y}_t - \alpha\tilde{k}_t) - \frac{\alpha}{1-\alpha}(\tilde{k}_t - \tilde{y}_t)\Big], \tag{49b}$$

$$\tilde{\pi}^w_t = \tilde{\pi}_t + \frac{\alpha}{1-\alpha}[(\tilde{k}_t - \tilde{y}_t) - (\tilde{k}_{t-1} - \tilde{y}_{t-1})] \tag{49c}$$

$$\tilde{k}_{t+1} = \tilde{y}_t \tag{49d}$$

$$\tilde{\pi}_t = -(\tilde{y}_t - \tilde{y}_{t-1}), \tag{49e}$$

where $\tilde{\pi}$ is inflation, and $\tilde{\pi}^w$ nominal wage inflation. In the cashless equilibrium, the model behaves similarly to a conventional new-keynesian model: (**??**), the log-linearized version of the Euler equation, is the standard new-keynesian IS curve, where the deleveraging shock shows up as a natural rate shock; (**??**) is the new-keynesian Phillips curve for wages; (**??**) is the relation between wage and price inflation; (**??**) is the log-linear version of (17) with $M^S = 0$; and (49a) is implied by the constant money growth rate.

Replacing our assumption of a constant money growth rule by a more usual Taylor rule is straightforward. For example, it could be replaced by

$$\tilde{i}_{t+1} = \tilde{\phi}_t + \frac{\phi_0}{1-\phi_0}\tilde{\phi}_{t+1} + \psi\tilde{\pi}_t \tag{50}$$

where the first two terms represent the log-deviation of the natural rate from the initial steady state.

Table 1: Calibration

Parameter	Value
β	0.94
δ	0.10
α	0.33
η	0.75
ϵ	13
σ	1

Calibration The model is calibrated as described in Table 1. We define a period to be a year. All parameters are standard, except the discount rate β. Indeed, in the cashless autarkic steady state of this model, the inverse of the discount rate is equal to the rate of return on capital, not to the interest rate r. We set β to match a rate of return of 6 percent per year annually.

C.3 Idiosyncratic Uncertainty

In this Appendix we examine a stochastic transition between saving and investing phases. We assume the following 2-state Markov process for individual investors:

- an investor with no investment opportunity at time $t-1$ receives an investment opportunity at time t with probability $\omega \in (0, 1]$,
- an investor with an investment opportunity at time $t-1$ receives no investment opportunity at time t.

While investors face some risk at the individual level, they do not face risk at the aggregate level, as the fraction of investors with investment opportunity is always ω.

A modified aggregate Euler equation of savers Consider an investor i, and denote Ω_t^i her wealth at the beginning of period t. With log utility, her consumption c_t^i is a fraction $1-\beta$ of wealth Ω_t^i, and the Euler equation of an (unconstrained) saver is $1/c_t^i = \beta r_{t+1} \mathrm{E}_t[1/c_{t+1}^i]$, which implies $1/\Omega_t^i = \beta r_{t+1} \mathrm{E}_t[1/\Omega_{t+1}^i]$. For an investor in her saving phase in period t, wealth in period $t+1$ is given by $\Omega_{t+1}^i = a_{t+1}^i + M_{t+1}^i/P_{t+1}$. As there is no aggregate risk, P_{t+1} is known in t, so Ω_{t+1}^i is known in t and we have $\beta\Omega_t^i = \Omega_{t+1}^i/r_{t+1}$. Aggregating over saving investors, we get

$$\beta \int S_t(i)\Omega_t^i di = \frac{1}{r_{t+1}} \int S_t(i)[a_{t+1}^i + M_{t+1}^i/P_t]di = \frac{1}{r_{t+1}} \left(a_{t+1} + \frac{M_{t+1}^S}{P_{t+1}} \right) \tag{51}$$

where $S_t(i)$ is an indicator equal to 1 if investor i has no investment opportunity at time 1 and 0 if she has, and a and M^S denote aggregate bond and money holdings by savers, as in the benchmark model. To compute the left-hand side of (51), note that investors in their saving phase at time t are made of a fraction $1-\omega$ of investors in their saving phase at time $t-1$ and all investors in their investment phase at time $t-1$. The latter enter period t with wealth $\Omega_t^i = \rho_t k_t^i - b_t^i$. This implies:

$$\begin{aligned} \int S_t(i)\Omega_t^i di = (1-\omega) \int S_{t-1}(i)\Omega_t^i di + \int [1 - S_{t-1}(i)]\Omega_t^i di \\ = (1-\omega)\left(a_t + \frac{M_t^S}{P_t} \right) + \rho_t k_t - b_t, \end{aligned}$$

where k and b are aggregate capital and aggregate debt of borrowers. As long as $\rho_t > r_t$, which will be the case in equilibrium, investors with an investment opportunity will leverage up as much as possible until they hit their borrowing constraint. Thus, we have $b_t^i = \phi_{t-1}\rho_t k_t^i$, which aggregates to $b_t = \phi_{t-1}\rho_t k_t = \phi_{t-1}\alpha y_t$. Substituting these expressions back into Equation (51), and using the market-clearing condition (10), we find:

$$\beta(1-\omega)\left[(\phi_{t-1}\alpha + \bar{l}_{t-1})y_t + \frac{M_t^S}{P_t} \right] + \beta\alpha(1-\phi_{t-1})y_t = \frac{1}{r_{t+1}} \left[(\phi_t\alpha + \bar{l}_t)y_{t+1} + \frac{M_{t+1}^S}{P_{t+1}} \right]. \tag{52}$$

This equation extends Equation (14) from the benchmark model to the case of idiosyncratic uncertainty. It only differs by the first term on the left hand side. This term represents demand for saving instruments at time t from savers that were already savers at time $t-1$. The lower ω, the larger the share of savers, the higher this additional demand for saving instruments compared to the benchmark model. The term vanishes when $\omega = 1$.

This is the only difference between the extended model and the benchmark. Indeed, we can aggregate the budget constraints of all investors, regardless of whether they save or borrow, to get the same aggregate budget constraint (15) as in the benchmark model.

Steady state with autarkic equilibrium This extended model behaves quite similarly to the benchmark model. Consider for example the case of autarkic investors ($\bar{l} = 0$) treated in Proposition 1 for the benchmark model. In the extended model, the steady state is determined by:

$$\beta(1-\omega)(\phi\alpha y + m^S) + \beta\alpha(1-\phi)y = \frac{1}{r}(\phi\alpha y + m^S),$$
$$k + (\theta - \beta)m^S = \beta\alpha y.$$

When $\beta/(\theta + \omega\beta) \leq \phi < 1/(1+\omega)$, the steady state is cashless with $m^S = 0$, a constant capital stock $k = (\beta\alpha)^{1/(1-\alpha)}$ as in the benchmark model, and

$$r = \frac{\phi}{\beta(1-\omega\phi)}.$$

A lower ω is associated with a lower interest rate: because there are more savers, channeling saving to investment is more difficult and requires a lower interest rate compared to the benchmark model. The interest rate is still strictly increasing in ϕ, but $dr/d\phi$ is increasing in ω: with a larger share of savers (i.e. a lower ω), the interest rate is lower but less responsive to ϕ. Note also that the upper bound on ϕ in the cashless equilibrium is larger than $\phi_{\max} = 1/2$: it is easier to have binding borrowing constraints when there are more savers. Likewise, the lower bound is larger than ϕ_T: it is easier to be in the liquidity trap equilibrium when there are more savers.

When $0 < \phi < \beta/(\theta + \omega\beta)$, the steady state is a liquidity trap with $r = 1/\theta$, and

$$k^{1-\alpha} = \alpha\,\frac{\omega\beta^2 + \phi\big(\theta^2 - \beta[\omega\beta + (1-\omega)\theta]\big)}{\theta - (1-\omega)\beta},$$
$$m^S = \alpha\left[\frac{(1-\omega\phi)\beta - \phi\theta}{\theta - (1-\omega)\beta}\right]y.$$

A lower ω, that is, a higher share of savers, leads to a stronger demand for money m^S/y and a lower stock of capital k. In the liquidity trap, we get the unusual result that more saving actually leads to less investment. As in Proposition 1, k is strictly increasing in ϕ, and m^S/y is strictly decreasing in ϕ. In addition, $dk/d\phi$ is decreasing in ω and $d(m^S/y)/d\phi$ is increasing in ω. A larger share of savers (i.e. a lower ω) implies steeper slopes of k and m^S/y with respect to ϕ.

Overall, the results we have in the benchmark model become stronger when investment opportunity arrive randomly to saving investors instead of in deterministic way.

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