DSGE-CH: A dynamic stochastic general equilibrium model for Switzerland

Nicolas A. Cuche-Curti, Harris Dellas and Jean-Marc Natal

No. 5 2009

SCHWEIZERISCHE NATIONALBANK
BANQUE NATIONALE SUISSE
BANCA NAZIONALE SVIZZERA
BANCA NAZIUNALA SVIZRA
SWISS NATIONAL BANK

Swiss National Bank Economic Studies

Copies of Swiss National Bank Economic Studies may be obtained from:

Swiss National Bank, Library, Fraumünsterstrasse 8, P.O. Box, CH-8022 Zurich
Fax: +41 44 631 81 14
E-mail: library@snb.ch

This publication is also available on the SNB website (www.snb.ch).

ISSN 1661-142X (printed version)
ISSN 1661-1438 (online version)

Swiss National Bank Economic Studies

No. 5 2009

DSGE-CH: A dynamic stochastic general equilibrium model for Switzerland*

Nicolas A. Cuche-Curti[a], Harris Dellas[b] and Jean-Marc Natal[c]

* We thank Alex Aschwanden, Fabrice Collard, Alexander Dentler, Michel Peytrignet, Enzo Rossi, Marcel Savioz, Markus von Allmen and two anonymous referees for their valuable comments, suggestions and support. The views expressed in this paper do not necessarily reflect those of the Swiss National Bank.

a Swiss National Bank and University of St. Gallen. Swiss National Bank, P.O. Box, CH-8022 Zurich, nicolas.cuche-curti@snb.ch

b University of Bern and CEPR. University of Bern, P.O. Box, CH-3001 Bern, harris.dellas@vwi.unibe.ch

c Swiss National Bank, P.O. Box, CH-8022 Zurich, jean-marc.natal@snb.ch

Contents

Abstract

This paper presents a DSGE (dynamic stochastic general equilibrium) model of the Swiss economy used since 2007 in the monetary policy decision process at the Swiss National Bank. In addition to forecasting the likely course of main macro variables under various scenarios for the Swiss economy, the model DSGE-CH serves as a laboratory for studying business cycles and examining the effects of actual and hypothetical monetary policies. The microfounded model DSGE-CH represents Switzerland as a small open economy with optimizing economic agents facing several real and nominal rigidities and exogenous foreign and domestic shocks. The comparison of the model's implications with the real world indicates that DSGE-CH performs well along standard dimensions. It captures the overall stochastic structure of the Swiss economy as represented by the moments of its key macroeconomic variables; furthermore, it has appropriate dynamic properties, as judged by its impulse response functions. Finally, it quite accurately replicates the historical path of major Swiss variables.

JEL classification: E27, E52, E58
Keywords: DSGE, forecasting, small open economy, Switzerland

Zusammenfassung

Dieses Dokument stellt ein DSGE-Modell (dynamisches, stochastisches, allgemeines Gleichgewicht) der Schweizer Volkswirtschaft vor, das seit 2007 im geldpolitischen Entscheidungsprozess der Schweizerischen Nationalbank eingesetzt wird. Zur Prognose der unter verschiedenen Szenarien zu erwartenden Entwicklung wichtiger makroökonomischer Variablen der Schweizer Volkswirtschaft wird das Modell DSGE-CH herangezogen und dient darüber hinaus der Erforschung von Konjunkturzyklen sowie der Untersuchung der Auswirkungen tatsächlicher und hypothetischer Geldpolitiken. Das mikrofundierte Modell DSGE-CH bildet die Schweiz als kleine, offene Volkswirtschaft mit optimierenden Wirtschaftssubjekten ab, die verschiedenen realen und nominalen Rigiditäten sowie exogenen Schocks aus dem In- und Ausland ausgesetzt sind. Ein Vergleich der Modellimplikationen mit der Realität zeigt, dass DSGE-CH an üblichen Kriterien gemessen gut funktioniert. Es bildet die gesamte stochastische Struktur der Schweizer Volkswirtschaft gut ab, dargestellt durch die Momente ihrer wichtigsten makroökonomischen Variablen, und zeigt, nach seinen Impuls-Antwort-Funktionen zu urteilen, angemessene dynamische Eigenschaften. Schliesslich ist das Modell in der Lage, den historischen Verlauf wichtiger Variablen der Schweizer Volkswirtschaft recht genau zu replizieren.

Résumé

Ce document présente un modèle DSGE – d'équilibre général intertemporel et stochastique – de l'économie suisse utilisé depuis 2007 dans le processus de décision de politique monétaire de la Banque nationale suisse. Le modèle DSGE-CH sert non seulement aux prévisions, sous

certains scenarios, des variables macroéconomiques importantes pour l'économie suisse, mais aussi de laboratoire pour étudier les cycles conjoncturels et examiner les effets de politiques monétaires existantes ou hypothétiques. DSGE-CH est un modèle microfondé qui représente la Suisse comme une petite économie ouverte peuplée d'agents économiques optimisant et faisant face à une multitude de rigidités réelles et nominales ainsi qu'à des chocs exogènes étrangers et domestiques. La comparaison, à l'aune de tests traditionnels, entre les implications du modèle et la réalité indique que DSGE-CH fonctionne bien. Il reproduit la structure stochastique de l'économie suisse, comme le montrent les moments des principales variables macroéconomiques, et il fait preuve d'un comportement dynamique satisfaisant, comme l'attestent ses fonctions de réponse aux chocs. Enfin, le modèle rend assez fidèlement le tracé historique de la plupart des variables macroéconomiques se rapportant à la Suisse.

1. Introduction

This paper introduces a new model used since 2007 in the regular monetary policy process at the Swiss National Bank (SNB). In what follows, we go over the main modelling assumptions and shed light on the principal building blocks of the model. The aim is to provide an intuitive understanding of the mechanisms involved and to document the empirical properties of the model.

This model is the outcome of a two-year project whose aim was to develop a dynamic, stochastic, general equilibrium (DSGE) model of the Swiss economy. The model is expected to serve as a laboratory for a) studying business cycles in Switzerland, b) examining the effects of actual and hypothetical monetary policies, and c) projecting (forecasting) the likely course of events – under various scenarios – for the Swiss economy in the short to medium term.[1] The new model differs from the existing models used so far by the SNB in interesting and useful ways and, as such, has the potential to contribute to the conduct of monetary policy.

We proceed by first discussing the key features of the modelling approach and then establishing that these features have many desired theoretical properties and, at the same time, do not seem to compromise success at the empirical front. In section 2, we focus on the Swiss model and try to convey the main intuitions behind its specification. Section 3 provides a more detailed but technical description of the model and can be skipped without loss of continuity by uninterested readers. We then summarise the main empirical properties of the model in section 4 and assess its performance in terms of its ability to mimic the stochastic behaviour of the variables of interest to policymakers and its ability to forecast them. Finally, section 5 concludes and describes the planned future stages of the project.

1.1 Key modelling features: microfoundations, general equilibrium and rational expectations

Spurred by innovations in macroeconomic theory and computational techniques, a number of central banks have, during the last few years, expended considerable resources in developing DSGE models.[2] The key property of DSGE models is that they rely on explicit microfoundations and a rational treatment of expectations in a general equilibrium context. They thus provide a coherent and compelling theoretical framework for macroeconomic analysis.

The models start by carefully specifying the types and numbers of economic agents present in the economy (firms, consumers, fiscal and monetary authorities, etc.), the objectives of these agents (profit or utility maximisation) as well as the various constraints they face

---

1 As an illustration of point b), note that a simplified version of the model has been used in Cuche-Curti et al. (2008, 2009).

2 For example, the Bank of Canada (TOTEM: Murchison and Rennison, 2006), the Bank of England (BEQM: Harrison et al., 2005), the European Central Bank (Smets and Wouters, 2004), the Sveriges Riksbank (RAMSES: Adolfson et al., 2007c), the Czech National Bank (new model: Benes et al., 2005) and the Norges Bank (NEMO: Brubakk et al., 2006).

(budget, informational, institutional, technological, etc.). These constraints include explicit restrictions on nominal price and wage setting opportunities of the individuals (the degree of nominal rigidity), define the degree of competition prevailing in individual markets, and include the various real costs that the agents face when pursuing their activities (the cost of changing financial positions, the capital stock, etc.). The economic decisions of the agents are derived under the assumption that they act in order to maximise their objectives in a rational, forward looking manner. Individual decisions are then aggregated into total demand and supply curves and the solution to the model is obtained via numerical methods.

The model solution represents the equilibrium of the economy and typically takes the form of a system of linearised, stochastic difference equations that relate the macroeconomic variables of interest (output, employment, inflation, trade balance, etc.) to exogenous shocks (world oil price, interest rate in the euro zone, fiscal policy, etc.) and to other predetermined variables (capital stock, foreign asset position, etc.). The solution reflects how the economy evolves over time as a function of past, present and future expected economic and policy decisions as well as uncontrollable (exogenous) changes in the economic environment.

1.2 Advantages of the DSGE approach

This approach has three distinct advantages in comparison to other modelling strategies. First and foremost, its microfoundations should allow it to escape the Lucas (1976) critique. In traditional ‘structural’ models, the estimated parameters are a function of the policies that were pursued during the estimation period. This implies that they should not be used to analyse the effects of radically different policies or that they are not valid during periods of structural change in the economy. The estimated (or calibrated) parameters of DSGE models, in contrast, represent deep parameters (preference, production, technology) and are thus independent of the conduct of policy. Unlike the traditional models, DSGE models can then be used to evaluate alternative monetary policies (a switch from a fixed to a flexible exchange rate regime, dollarisation, a change in the relative weights of the pillars of monetary policy, etc.).

Second, its reliance on deep structural parameters enables researchers to interpret economic outcomes through the lens of well-understood economic behaviours at the individual level. This contributes to gaining a clearer intuition on key issues, such as the transmission mechanism of well-defined shocks (productivity, fiscal expenses, oil price, etc.).

Third, it represents a flexible modelling approach as it can accommodate modifications and extensions along many dimensions. For instance, a detailed sectorial structure may be implemented, different sets of frictions can be contemplated, or different types of shocks can be introduced. The advances in computer sciences have made possible the production of richer and more complex models. Moreover, given the incremental nature of innovation in economic modelling, existing DSGE models can be adapted to accommodate future changes in economic thinking.

1.3 Weaknesses of DSGE models

In general, purely microfounded models have difficulty generating plausible inflation dynamics (persistence).[3] One way to deal with this weakness is to find mechanisms that allow the model to contain a backward looking component in the otherwise purely forward looking Phillips curve. This is typically accomplished with the help of rather ad hoc assumptions such as the existence of myopic agents (Galí and Gertler, 1999) or price or wage indexation schemes (Christiano et al., 2005 and Smets and Wouters, 2003), which is the approach taken here. Neither approach is entirely satisfactory because it introduces arbitrary, non-structural elements in a very sensitive part of the model (the pricing behaviour). This drawback is reminiscent of the earlier era of non-microfounded macro modelling. Alternative, rational inertial mechanisms either fail empirically (the sticky information model of Mankiw and Reis, 2002) or have not yet been introduced into policy-oriented models (the signal extraction mechanism of Collard and Dellas, 2004).

1.4 Has theoretical coherence come at the expense of empirical success?

We have described above the key features of DSGE models as well as their theoretical potential for improving upon alternative approaches, in particular with regard to coping with the Lucas (1976) critique and to offering a better understanding of the structure of the economy. Nonetheless, the conventional wisdom (mostly among econometricians) is that there is a trade-off between theoretical and empirical coherence in DSGE and VAR models (Pagan, 2003) and that the latter are more empirically useful.

Recent work seems to contradict this view. Not only have the new-generation models proved quite successful in fitting the data (Christiano et al., 2005), but some evidence exists that DSGE models may outperform less theoretically oriented forecasting models such as VAR and BVAR (Smets and Wouters, 2004 and Adolfson et al., 2007a) in the medium to long run because of their efficient use of constraints on parameters based on economic theory. Moreover, this applies in terms of both point forecast accuracy and predictive intervals.

---

[3] The dynamics of asset prices and exchange rates also remain difficult to replicate in current DSGE models. However, the inclusion of habit formation in consumption-based asset pricing models yields predictions consistent with the average difference of returns between equities and bonds found in the data. But such models still generate bond yields that are too volatile relative to the data (Campbell and Cochrane, 1999).

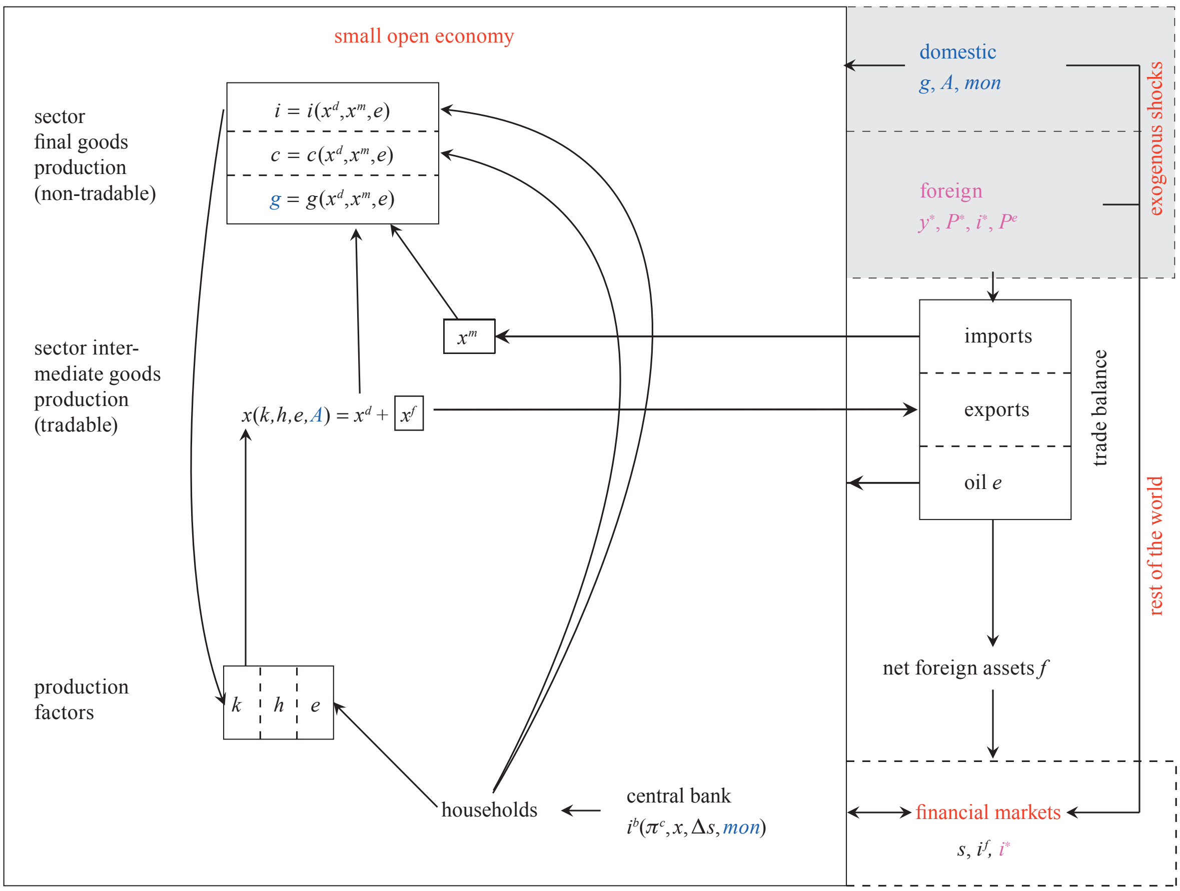
2. DSGE-CH overview

DSGE-CH is designed to adequately capture the basic structure of the Swiss economy, but it is also open to modifications and extensions that could further improve its realism and its performance.

The model entails more than 40 behavioural equations allowing us to simulate the dynamics of real output, investment and capital stock (equipment and housing), the rental rate of capital, real wages, the real marginal cost, consumption, exports, imports, employment, interest rates on foreign and domestic bonds, the nominal exchange rate, the accumulation of net foreign assets (current account), import and export prices, money demand and inflation. The solution of the model is obtained numerically using Dynare (Juillard, 1996).

The structure of the Swiss model, inspired by the work of Backus et al. (1995) on the modelling of open economies, is illustrated in Chart 1.

Chart 1: Main agents and mechanisms in DSGE-CH



Note: This diagram shows only the main channels built in DSGE-CH; acronyms are described in the text.

Sectors. The economy consists of two sectors. In the first sector, tradable intermediate goods (x) are produced using capital (k), labour (h), and imported oil (e). These intermediate goods are either used as inputs (x^d) into the domestic production of final goods or exported (x^f).

In the second sector, which can be seen as a retail sector, domestic (x^d) and foreign imported intermediate goods (x^m) are combined with oil (e) in order to produce final non-tradable domestic consumption (both private (c) and public (g)) and investment (i) goods.

Households. The economy is populated by infinitively lived households which care about consumption (c) and investment (i), leisure (or labour h) and real money balances. Households do not care directly about investment but – via their saving decisions – decide on the amount of investment that allows them to maximise their lifetime utility out of consumption and leisure.[4]

Households draw satisfaction from their level of consumption and dissatisfaction from its rate of change; i.e. they do not like their consumption basket to vary too quickly over time (habit formation in consumption following Abel, 1990 or Fuhrer, 2000). Moreover, in order to improve the empirical performance we also assume that some consumers do not have access to capital markets and, as such, consume their current labour income (Galí et al., 2007).

Market structure. We assume that imperfect competition prevails in the labour market (Erceg et al., 2000) as well as in the markets for intermediate goods (whether domestic, exported or imported; Kollmann, 2002). For simplicity, the markets for final goods are assumed perfectly competitive.

Trade. International trade involves intermediate goods only (x^f, x^m). Final goods are non-tradable. This assumption aims at capturing the fact that no foreign good can be sold directly at home without the intervention of some domestic input (labour, retail space, etc.).

We also make the assumption that Switzerland is a 'semi-small' economy, meaning that it enjoys some monopoly power in its export markets. As such, Swiss exporting firms are able to set prices in the currency of the buyer (local currency prices (LCP)) and then face a downward sloping demand curve for their products (Devereux and Engle, 1998). Similarly, importing (Swiss) firms are able to set prices in Swiss francs and maximise their profit accordingly. Exchange rate pass-through is thus incomplete in the short to medium run and the law of one price (LOP) does not hold.

Financial markets. In our model of a small open economy (SOE) we assume incomplete asset markets: international asset trade relies on foreign currency bonds only. As consumers want to smooth consumption over time and cannot insure themselves perfectly against idiosyncratic shocks – as they would with complete asset markets – they tend to accumulate net foreign assets. In the absence of a feedback mechanism linking consumption to the accu-

---

4 An equivalent but more cumbersome alternative would be to assign the investment decision to the firms. We could have assumed (Christiano et al., 2005) that firms decide each period how much to invest in order to maximise the value of their shares (the present value of future profits). As these shares enter the households' budget constraint, this approach is equivalent to the one in DSGE-CH. Optimal investment decisions by firms maximise the wealth of the representative agent, who spends it in a way that maximises his overall welfare. In our context, it is then irrelevant to know who makes the investment decision. Once we allow for different types of agents, i.e. household and entrepreneur, as in Bernanke et al. (1999), investment decisions is made by the entrepreneur and there is a role for financial intermediation to channel households' savings.

mulation of net foreign assets (a state variable of the system), temporary *i.i.d.* shocks have permanent effects: the solution to the dynamic system has a unit root.[5]

To ensure a unique steady state equilibrium in DSGE-CH, we let domestic households' borrowing conditions (i.e. the interest rate on foreign bonds, i^f) depend on the interest rate in the international financial markets (i^*) and on the accumulated stock of net foreign assets (f), thereby effectively creating a feedback mechanism between consumption and foreign assets through the evolution of i^f (Kollmann, 2002). This technical assumption has an intuitive interpretation. The interest rate at which domestic households can borrow from the international financial markets is assumed to depend negatively on the country's stock of net foreign assets via a debt-elastic risk premium. The more the country is indebted, the costlier it is to borrow further (e.g. Schmitt-Grohé and Uribe, 2003 for equivalent ways to ensure stationarity in SOE models).[6]

Rigidities. There exist two types of rigidities, nominal and real. On the nominal side, the prices of intermediate goods (domestic, imported or exported) as well as the price of labour (wages) exhibit rigidity according to the well-known scheme suggested by Calvo (1983). Firms (workers) know in advance that they will not be able to change their price (wage) in each and every period, and therefore have to take into account this element of uncertainty when setting their price (wage) in the current period. Note that prices are set in the currency of the buyer (LCP). By including nominal rigidities in the importing and exporting sectors we then allow for short-run, incomplete exchange rate pass-through to both import and export prices.

On the real side, there exist adjustment costs pertaining to investment, consumption changes as well as to the scale of operation (variable capacity utilisation).

Monetary policy. A standard interest rate (i^b) rule is postulated. The monetary authorities respond to movements in consumer price index (CPI) inflation (π^c), output (x) and the appreciation rate of the exchange rate (Δs).[7] A partial adjustment scheme is also assumed, as suggested by most of the literature on interest rate feedback rules.

Exogenous shocks. The model contains several shocks, domestic and foreign. The domestic shocks are to total factor productivity (TFP, $\mathcal{A}$), to fiscal expenditures (g), and to monetary policy (*mon*). The external shocks are to foreign interest rates (i^*), output (y^*), prices (P^*), and the price of oil (P^e).

---

5 This is a well-known property of standard models of SOE with incomplete asset markets (Schmitt-Grohé and Uribe, 2003).

6 The empirical properties of the model at business cycles frequencies are not affected by alternative specifications/calibrations to ensure a unique steady state equilibrium, as documented in Schmitt-Grohé and Uribe (2003). Thus, although arguably ad hoc, our assumption concerning i^f is rather innocuous for the type of fluctuations we are interested in.

7 The rule does not imply that a particular level of the exchange rate is targeted. It only implies that monetary authorities may react to large changes in the external value of the currency.

Oil. In DSGE-CH, oil enters production at two different levels: in the production of intermediate goods together with capital and labour, and in the production of final goods. Consequently, oil price changes affect Swiss inflation both directly, as it has a direct impact on the CPI, and indirectly via changes in the price of domestic intermediate goods, which are a function of changes in marginal costs.

Besides, Chart 1 shows how foreign and domestic shocks drive the dynamics of the system, and how the net external position (f) – the consequence of successive current account imbalances – affects the rate of interest on foreign assets (i^f) and, through the uncovered interest rate parity (UIP) relating foreign (i^f) and domestic (i^b) interest rates, the nominal exchange rate (s). Given the general equilibrium nature of the model, variations in nominal exchange rates affect simultaneously the terms of trade, the trade balance, and, as a result, the domestic production of intermediate goods. This has natural consequences for factor demands and prices, and feeds back into real marginal costs and the Phillips curves. This leads to inflation/deflation pressures and an appropriate monetary policy reaction from the central bank (i^b), which again affects the exchange rate via UIP. A more detailed (and quantitative) description of the dynamic responses of the model (in particular the impulse response functions, IRF) to productivity, fiscal expenditures, oil price and monetary policy shocks is given in section 4.

In the next section, we describe in a more detailed but technical way the specification of DSGE-CH. A summary of the model equations can be found at the end of the paper.

3 DSGE-CH

DSGE-CH is a model of a SOE linked to the rest of the world (the euro zone in the current calibration). It is based on the infinitely lived representative agent paradigm, and includes both fiscal and monetary authorities. It is assumed that fiscal authorities balance their budget each and every period,[8] while the central bank follows an interest rate feedback rule, reacting to deviations in inflation, the output gap and the change in nominal exchange rate from their respective steady state values.

There are different types of firms operating in this SOE. One type produces non-tradable final goods using as inputs imported and domestically produced intermediate goods as well as oil in a perfectly competitive environment. A second type produces tradable intermediate goods with capital, labour and oil. A third type imports the foreign intermediate goods that are used together with domestically produced intermediate goods and oil to produce the final goods. Intermediate goods are imperfectly substitutable, which gives producers/importers a certain market power and allows them to set prices at a markup above marginal costs.[9] We assume (like in most of the related literature) monopolistic competition in the markets for intermediate goods.

The households that have access to capital markets maximise their lifetime utility by selecting their desired level of consumption, investment and labour. They also attribute some value to holding money for transaction purposes. Because markets are assumed incomplete, households temporarily accumulate foreign debt (or foreign assets), which, to a certain extent, helps them smooth consumption over time. The remaining households simply consume their current labour income.

In the following subsections we present in details the optimal behavior of the various firms, the households. The monetary and fiscal policies, the market clearing conditions, the different shocks and the model calibration are presented.

3.1 Final goods firms

Following Backus et al. (1995) and Murchison and Rennison (2006), we assume that domestic final goods are produced by perfectly competitive firms which combine domestic (x^d) and imported (x^m) intermediate goods bundles with oil (e). We distinguish three types of final goods: private consumption goods (c), private investment goods (i) and public consumption goods (g), produced via nested CES technologies.

Starting with the production of the consumption goods bundle, we postulate

---

8 Given the very decentralised nature of the Swiss fiscal system, assuming a balanced budget is probably a good approximation for many cantons and communities (about two thirds of the total public sector). Nevertheless, the fiscal side of the model remains fairly underdeveloped and further work along this particular dimension is warranted.

9 Following Erceg et al., 2000 we assume that the same applies to the labour market. Labour inputs are assumed to be imperfectly substitutable. Households are then able to set their wage at a markup above the marginal rate of substitution between leisure and consumption.

$$c_t = \left[\omega_{e,c}^{1-\rho_{e,c}} \mathfrak{g}(x_t^c)^{\rho_{e,c}} + (1-\omega_{e,c})^{1-\rho_{e,c}} (e_t^c)^{\rho_{e,c}}\right]^{1/\rho_{e,c}} \tag{1}$$

for

$$\mathfrak{g}(x_t^c) = \left[\omega_c^{1-\rho_c} (x_t^{d,c})^{\rho_c} + (1-\omega_c)^{1-\rho_c} (x_t^{m,c})^{\rho_c}\right]^{1/\rho_c}$$

where c_t is the total domestic consumption basket which aggregates the core consumption bundle $(\mathfrak{g}(x_t^c))$ with oil (e_t^c) using weights $\omega_{e,c} \in [0,1]$. $\mathfrak{g}(x_t^c)$ is itself a CES composite which aggregates domestic $(x_t^{d,c})$ and imported $(x_t^{m,c})$ intermediate goods to produce the core consumption bundle, with $\omega_c \in [0,1]$ the weight of domestic intermediate goods in the production of $\mathfrak{g}(x_t^c)$. Also, $\rho_c \in]-\infty,1]$ determines the elasticity of substitution $(-1/(\rho_c-1))$ between intermediate goods (domestic and imported) in the production of $\mathfrak{g}(x_t^c)$. By analogy, $\rho_{e,c} \in]-\infty,1]$ determines the elasticity of substitution $(-1/(\rho_{e,c}-1))$ between the core consumption composite and oil.

The nested structure allows different elasticities of substitution between intermediate goods and between $\mathfrak{g}(x_t^c)$ and e_t^c. This is a very important feature of the model when it comes to matching the CPI dynamics in Switzerland. Moreover, when setting $\omega_{e,c}$ to one, we can compute a model consistent core CPI.

By analogy, the production of private investment (*i*) and public consumption goods (*g*) can be written as

$$i_t = \left[\omega_{e,i}^{1-\rho_{e,i}} \mathfrak{g}(x_t^i)^{\rho_{e,i}} + (1-\omega_{e,i})^{1-\rho_{e,i}} (e_t^i)^{\rho_{e,i}}\right]^{1/\rho_{e,i}} \tag{2}$$

and

$$g_t = \left[\omega_{e,g}^{1-\rho_{e,g}} \mathfrak{g}(x_t^g)^{\rho_{e,g}} + (1-\omega_{e,g})^{1-\rho_{e,g}} (e_t^g)^{\rho_{e,g}}\right]^{1/\rho_{e,g}} \tag{3}$$

for

$$\mathfrak{g}(x_t^i) = \left[\omega_i^{1-\rho_i} (x_t^{d,i})^{\rho_i} + (1-\omega_i)^{1-\rho_i} (x_t^{m,i})^{\rho_i}\right]^{1/\rho_i}$$

and

$$\mathfrak{g}(x_t^g) = \left[\omega_g^{1-\rho_g} (x_t^{d,g})^{\rho_g} + (1-\omega_g)^{1-\rho_g} (x_t^{m,g})^{\rho_g}\right]^{1/\rho_g}.$$

Final goods are non-tradable in our model. This assumption allows us to interpret the final goods firms as a retail sector layer. Each final good is produced with domestic intermediate goods, conveying the idea that no good can be sold at home without the intervention of some domestic input (labour, retail space, etc.).

Final goods producers want to minimise total expenditures $(P^{d,\mathrm{a}} x^{d,\mathrm{a}} + P^{m,\mathrm{a}} x^{m,\mathrm{a}} + P^{e,\mathrm{a}} e^{\mathrm{a}})$, for $\mathrm{a} \in \{c,i,g\}$, in acquiring the required quantity of intermediate goods to produce a given amount of final goods. We assume that the price charged by an intermediate firm $i \in [0,1]$ for the good it produces is invariant to its final use (*c*, *i* or *g*): $P_t^{\mathrm{k},c}(i) = P_t^{\mathrm{k},i}(i) = P_t^{\mathrm{k},g}(i) = P_t^{\mathrm{k}}(i)$ (for $\mathrm{k} \in \{d,m\}$). We also make the same assumption for the price of oil $(P_t^{e,c} = P_t^{e,i} = P_t^{e,g} = P_t^e)$. Therefore, intermediate goods producing/importing firms and oil producers face the following demand functions

$$(p_t^{d,\mathrm{a}})^{\frac{1}{\rho_\mathrm{a}-1}} \frac{\omega_\mathrm{a}\mathfrak{g}(x_t^\mathrm{a})^{\frac{\rho_{e,\mathrm{a}}-\rho_\mathrm{a}}{1-\rho_\mathrm{a}}}}{(\omega_{e,\mathrm{a}}\mathrm{a}_t)^{\frac{\rho_{e,\mathrm{a}}-1}{1-\rho_\mathrm{a}}}} = x_t^{d,\mathrm{a}}, \tag{4}$$

$$(p_t^{m,\mathrm{a}})^{\frac{1}{\rho_\mathrm{a}-1}} \frac{(1-\omega_\mathrm{a})\mathfrak{g}(x_t^\mathrm{a})^{\frac{\rho_{e,\mathrm{a}}-\rho_\mathrm{a}}{1-\rho_\mathrm{a}}}}{(\omega_{e,\mathrm{a}}\mathrm{a}_t)^{\frac{\rho_{e,\mathrm{a}}-1}{1-\rho_\mathrm{a}}}} = x_t^{m,\mathrm{a}} \tag{5}$$

and

$$(p_t^{e,\mathrm{a}})^{\frac{1}{\rho_{e,\mathrm{a}}-1}}(1-\omega_{e,\mathrm{a}})\mathrm{a}_t = e_t^\mathrm{a} \tag{6}$$

for $p_t^{\mathrm{k,a}} = P_t^\mathrm{k} / P_t^\mathrm{a}$ and $p_t^{e,\mathrm{a}} = P_t^e / P_t^\mathrm{a}$.[10] The determination of prices in the intermediate goods sector for domestic producers and importers $(P_t^d(i),\ P_t^m(i))$ are reviewed in section 3.3.1.

To obtain final goods price levels, we substitute demand functions for inputs (4) to (6) into the production of final goods (1) to (3) to end up with the following prices for investment and consumption (public and private) goods[11]

$$P_t^\mathrm{a} = \left\{ \begin{array}{c} \omega_{e,\mathrm{a}}\left[(\omega_\mathrm{a}(P_t^d)^{\frac{\rho_\mathrm{a}}{\rho_\mathrm{a}-1}} + (1-\omega_\mathrm{a})(P_t^m)^{\frac{\rho_\mathrm{a}}{\rho_\mathrm{a}-1}})\right]^{\frac{\rho_{e,\mathrm{a}}}{1-\rho_{e,\mathrm{a}}}\left(\frac{1-\rho_\mathrm{a}}{\rho_\mathrm{a}}\right)} \\ +(1-\omega_{e,\mathrm{a}})(P_t^e)^{\frac{\rho_{e,\mathrm{a}}}{\rho_{e,\mathrm{a}}-1}} \end{array} \right\}^{\frac{\rho_{e,\mathrm{a}}-1}{\rho_{e,\mathrm{a}}}}. \tag{7}$$

The bundles of goods $x^{d,\mathrm{a}}$ and $x^{m,\mathrm{a}}$ are themselves combinations of, respectively, domestic and foreign intermediate goods, each produced/imported by a firm *i*, according to

$$x_t^{d,\mathrm{a}} = \left(\int_0^1 x_t^{d,\mathrm{a}}(i)^\theta \mathrm{d}i\right)^{\frac{1}{\theta}} \text{ and } x_t^{m,\mathrm{a}} = \left(\int_0^1 x_t^{m,\mathrm{a}}(i)^{\theta_m} \mathrm{d}i\right)^{\frac{1}{\theta_m}}$$

where $\theta(\theta_m) \in\ ]-\infty,1]$ determines the elasticity of substitution $(-1/(\theta-1), -1/(\theta_m-1))$ between individual goods in the domestic (imported) intermediate goods bundle. Accordingly, demand functions faced by individual firms are

$$x_t^{d,\mathrm{a}}(i) = \left(\frac{P_t^d(i)}{P_t^d}\right)^{\frac{1}{\theta-1}} x_t^{d,\mathrm{a}} \tag{8}$$

---

10 Note that $\mathfrak{g}(x_t^\mathrm{a})$ can be rewritten as $\omega_{e,\mathrm{a}}\mathrm{a}_t\left[\omega_\mathrm{a}(p_t^{d,\mathrm{a}})^{\frac{\rho_\mathrm{a}}{\rho_\mathrm{a}-1}} + (1-\omega_\mathrm{a})(p_t^{m,\mathrm{a}})^{\frac{\rho_\mathrm{a}}{\rho_\mathrm{a}-1}}\right]^{\frac{1}{\rho_\mathrm{a}}\left(\frac{1-\rho_\mathrm{a}}{1-\rho_{e,\mathrm{a}}}\right)}$.

11 Inflation in the model is CPI inflation $(\Pi_t^c = P_t^c / P_{t-1}^c)$ and all nominal variables are deflated by CPI to conform with the households' problem in section 3.4.

and

$$x_t^{m,\mathrm{a}}(i) = \left(\frac{P_t^m(i)}{P_t^m} \right)^{\frac{1}{\theta_m - 1}} x_t^{m,\mathrm{a}} \tag{9}$$

and the corresponding price indices for intermediate goods bundles are given by

$$P_t^d = \left(\int_0^1 P_t^d(i)^{\frac{\theta}{\theta-1}} \mathrm{d}i \right)^{\frac{\theta-1}{\theta}} \text{ and } P_t^m = \left(\int_0^1 P_t^m(i)^{\frac{\theta_m}{\theta_m-1}} \mathrm{d}i \right)^{\frac{\theta_m-1}{\theta_m}} .$$

3.2 Domestic intermediate goods firms

3.2.1 Technology

Each domestic intermediate good firm *i* produces an intermediate good $x(i)$ – using capital (k), labour (h), and oil (e^x) – which is either used domestically in the production of the domestic final goods or exported $(x(i) = x^{d,\mathrm{a}}(i) + x^f(i))$.

Because capital utilisation (u) is variable, the effective capital used in production is given by $\tilde{k}_t(i) = u_t(i)k_{t-1}(i)$. k_{t-1} is the homogeneous predetermined stock of capital available for production in t. Technological progress is assumed of the labour-augmenting type, because technological progress has to be consistent with a balanced growth path. The level of technology is given by Γ and goods are produced according to the following CES production function

$$\tilde{x}_t(i) = \mathcal{A}_t \left\{ \alpha_c^{\frac{1}{\sigma_e}} \left(x_t^{kl} \left(\Gamma_t h_t(i), \tilde{k}_t(i) \right) \right)^{\frac{\sigma_e - 1}{\sigma_e}} + (1-\alpha_c)^{\frac{1}{\sigma_e}} \left(\tilde{e}_t^x(i) \right)^{\frac{\sigma_e - 1}{\sigma_e}} \right\}^{\frac{\sigma_e}{\sigma_e - 1}} \tag{10}$$

with

$$x_t^{kl} \left(\Gamma_t h_t(i), \tilde{k}_t(i) \right) = \left(\alpha_l^{\frac{1}{\sigma_{kl}}} \left(\Gamma_t h_t(i) \right)^{\frac{\sigma_{kl}-1}{\sigma_{kl}}} + (1-\alpha_l)^{\frac{1}{\sigma_{kl}}} \left(\tilde{k}_t(i) \right)^{\frac{\sigma_{kl}-1}{\sigma_{kl}}} \right)^{\frac{\sigma_{kl}}{\sigma_{kl}-1}}$$

where $\mathcal{A}$ is an exogenous stationary stochastic technological shock and σ_{kl} and σ_e are the elasticities of substitution between factors.

To be consistent with the 'Hodrick-Prescott-filtered' data describing the cyclical properties of the Swiss economy (see section 4), the cyclical component of the production function is obtained by detrending (10) with the secular productivity level Γ_t. We write the stationary production function $x_t(i)$

$$x_t(i) = \mathcal{A}_t \left\{ \alpha_c^{\frac{1}{\sigma_e}} \left(x_t^{kl} \left(h_t(i), \breve{k}_t(i) \right) \right)^{\frac{\sigma_e - 1}{\sigma_e}} + \left(1-\alpha_c\right)^{\frac{1}{\sigma_e}} \left(e_t^x(i) \right)^{\frac{\sigma_e - 1}{\sigma_e}} \right\}^{\frac{\sigma_e}{\sigma_e - 1}} \tag{11}$$

using $x_t(i) = \dfrac{\tilde{x}_t(i)}{\Gamma_t}$, $e_t^x(i) = \dfrac{\tilde{e}_t^x(i)}{\Gamma_t}$ and $\breve{k}_t(i) = \dfrac{\tilde{k}_t(i)}{\Gamma_t} = u_t(i) \dfrac{k_{t-1}(i)}{\Gamma_t}$.

3.2.2 Factor demand (capital, oil, labour)

Each period, the domestic intermediate goods firms solve a total cost minimisation problem and choose the optimal amount of each production factor given their respective prices. In other words, they minimise total costs $(TC_t(i) = z_t\breve{k}_t(i) + p_t^{e,c}e_t^x(i) + w_t h_t(i))$ under the constraint of the desired production level ($x_t(i)$) for given real prices of capital services, oil and labour (respectively, in units of the consumption good, z, $p^{e,c} = P^e / P^c$ and w).[12]

Dropping indices, the demands for production factors on the part of firms producing intermediate goods can be written as functions of relative prices, and the desired production level x_t

$$e_t^x = \left(\frac{\psi_t}{p_t^{e,c}}\right)^{\sigma_e} \mathcal{A}_t^{\sigma_e - 1}(1-\alpha_c)x_t, \tag{12}$$

$$h_t = \left(\frac{\psi_t}{w_t}\right)^{\sigma_{kl}} \mathcal{A}_t^{\sigma_{kl}\left(\frac{\sigma_e - 1}{\sigma_e}\right)} x_t^{kl}(h_t, \breve{k}_t)^{\frac{\sigma_e - \sigma_{kl}}{\sigma_e}} \alpha_l(\alpha_c x_t)^{\frac{\sigma_{kl}}{\sigma_e}} \tag{13}$$

and

$$\breve{k}_t = \left(\frac{\psi_t}{z_t}\right)^{\sigma_{kl}} \mathcal{A}_t^{\sigma_{kl}\left(\frac{\sigma_e - 1}{\sigma_e}\right)} x_t^{kl}(h_t, \breve{k}_t)^{\frac{\sigma_e - \sigma_{kl}}{\sigma_e}} (1-\alpha_l)(\alpha_c x_t)^{\frac{\sigma_{kl}}{\sigma_e}} \tag{14}$$

for

$$x_t^{kl}(h_t, \breve{k}_t) = \psi_t^{\sigma_e} \mathcal{A}_t^{\sigma_e - 1}\left[(\alpha_l w_t^{1-\sigma_{kl}} + (1-\alpha_l)z_t^{1-\sigma_{kl}})\right]^{\frac{\sigma_e}{\sigma_{kl}-1}} \alpha_c x_t$$

and the real marginal cost ψ_t

$$\psi_t = \frac{1}{\mathcal{A}_t}\left[\alpha_c\left[\alpha_l w_t^{1-\sigma_{kl}} + (1-\alpha_l)z_t^{1-\sigma_{kl}}\right]^{\frac{\sigma_e - 1}{\sigma_{kl}-1}} + (1-\alpha_c)(p_t^e)^{1-\sigma_e}\right]^{\frac{1}{1-\sigma_e}}.$$

The real marginal cost is increasing in the real prices of factors, decreasing in the TFP shifter $(\mathcal{A})$ and independent of the level of production due to the assumption of constant returns to scale.

Finally, recall that the labour market is characterised by imperfect competition in DSGE-CH. Households $j \in [0,1]$ are supplying differentiated types of labour inputs, which enables them to set their wages in a monopolistic competitive fashion (see section 3.4). Firms must choose an optimal (in terms of cost minimisation) bundle of differentiated labour inputs given their production needs. They minimise

$$\int_0^1 w_t(j)h_t(i,j)\mathrm{d}j \;\text{ s.t. }\; h_t(i) = \left(\int_0^1 h_t(i,j)^{\vartheta}\,\mathrm{d}j\right)^{\frac{1}{\vartheta}}$$

---

12 The factor prices (z, $p^{e,c}$ and w) are themselves the result of the general equilibrium solution of the model.

where $h_t(i)$ is the labour aggregate used by firm i in production, which gives rise to the following demand for labour of type j by firm i

$$h_t(i,j) = \left(\frac{w_t(j)}{w_t}\right)^{\frac{1}{\vartheta-1}} h_t(i). \tag{15}$$

Raising (15) to the power ϑ and integrating over j labour types, we obtain an expression for the overall wage index

$$w_t = \left(\int_0^1 w_t(j)^{\frac{\vartheta}{\vartheta-1}} \mathrm{d}j\right)^{\frac{\vartheta-1}{\vartheta}}. \tag{16}$$

Aggregating $h_t(i,j)$ over i and making use of the definition of overall labour demand $(h_t = \int_0^1 h_t(i)\mathrm{d}i)$, we obtain the total labour demand faced by household j

$$h_t(j) = \left(\frac{w_t(j)}{w_t}\right)^{\frac{1}{\vartheta-1}} h_t \tag{17}$$

with $\vartheta \in]-\infty,1]$ and the elasticity of substitution between labour types given by $-1/(\vartheta-1)$.

3.3 Price setting in the intermediate goods sectors

3.3.1 Domestic goods

In this section we describe price setting decisions in the sector of intermediate goods produced for domestic use.[13] Price setting decisions for exported goods are reviewed in the next section.

Real profits for domestic producers $(\mathcal{F}_t(i))$ can be written as the sum of profits realised by producing intermediate goods for domestic use $(\mathcal{F}_t^d(i))$ and for the export market $(\mathcal{F}_t^f(i))$. Because total costs are $(TC_t(i) = \psi_t x_t(i))$ for ψ_t the real marginal cost expressed in consumption units, real profits $(\mathcal{F}_t(i) = \mathcal{F}_t^d(i) + \mathcal{F}_t^f(i))$ can be expressed as

$$\mathcal{F}_t(i) = \left(\frac{P_t^d(i)}{P_t^c} - \psi_t\right) x_t^d(i) + \left(\frac{s_t P_t^f(i)}{P_t^c} - \psi_t\right) x_t^f(i) \tag{18}$$

where s_t is the exchange rate and $P_t^f(i)$ the price of domestic exports in foreign currency.

Domestic intermediate goods producers are monopolistic competitors. Therefore, they set their price $P_t^d(i)$ in order to maximise the real profit function $\mathcal{F}_t^d(i)$ (due to the separability of the profit function (18) and the assumption of LCP for exported goods).

---

[13] Recall that we assume that intermediate goods firms set their price considering the total demand for intermediate goods $(x_t^d(i) = x_t^{d,c}(i) + x_t^{d,i}(i) + x_t^{d,g}(i))$ regardless of their final use. Also, we assume that real quantities in our economy are defined in terms of consumption units. These two assumptions greatly reduce the size of the model: as firms try to maximise their real profit in terms of consumption units, they update their prices according to the same indexation scheme, meaning that, in the linearised version, $\hat{p}_t^{d,c} = \hat{p}_t^{d,i} = \hat{p}_t^{d,g} = \hat{p}_t^d$.

Assuming a staggered price setting scheme à la Calvo (1983)[14] where the firm knows that it may not be able to reset its price next period, even if it needs to. Each domestic producer has to solve the following dynamic program in order to set its price in time *t* optimally. Relying on asset pricing theory in a general equilibrium environment, we can write the date *t* value of a profit maximising firm $(v_{t,t}^{d}(i))$ as

$$v_{t,t}^{d}(i) = \max_{P_t^d(i)} \left[\mathcal{F}_t^d(i) + \tau\beta E_t \left(\frac{\lambda_{1,t+1}}{\lambda_{1,t}} v_{t+1,t}^{d}(i) \right) + (1-\tau)\beta E_t \left(\frac{\lambda_{1,t+1}}{\lambda_{1,t}} v_{t+1,t+1}^{d}(i) \right) \right] \tag{19}$$

where τ is the probability of no price change at time $t+1$, β reflects the rate of time preference, $v_{t+1,t}^{d}(i)$ is the value of the firm in $t+1$ when the price set in t is not allowed to be reset, $\lambda_{1,t+1}/\lambda_{1,t}$ is the ratio from future to current marginal utility of income ($\lambda_{1,t}$ is the Lagrange multiplier of the households' budget constraint, see section 3.4) and $\mathcal{F}_t^d(i)$ is the real profit realised at time *t*.

The right-hand side of this equation shows the two things that can happen to the firm in $t+1$. With probability τ it will be stuck with the price set in period *t* and its discounted expected value will be $\beta E_t(\lambda_{1,t+1}/\lambda_{1,t})v_{t+1,t}^{d}$, or it will be allowed to reset its price with probability $1-\tau$ and its discounted expected value will be $\beta E_t(\lambda_{1,t+1}/\lambda_{1,t})v_{t+1,t+1}^{d}$. Despite a somewhat cumbersome notation, the intuition is straightforward. Knowing that there is a probability that it will not be able to reset its price next period, the profit maximising firm has to take into account all expected future profit flows, assuming that the future prices may remain unchanged at $P_t^d(i)$, when setting its optimal price today.

Unlike Calvo (1983) we assume partial indexation (Del Negro et al., 2007 and Christiano et al., 2005)

$$P_{t+1,t}^{d}(i) = (\Pi^c)^{1-\gamma}(\Pi_t^c)^{\gamma} P_t^d(i), \tag{20}$$

for the non-adjusting firms. This implies that a fraction γ of the non-adjusting firms sets their price according to last period's CPI gross inflation rate $(\Pi_t^c = P_t^c / P_{t-1}^c)$ and the other part $1-\gamma$ follows the steady state inflation rate (Π^c). This scheme allows more persistence in the inflation process than a purely forward looking approach like Calvo's.[15]

Solving the dynamic program (19) forward for $P_t^d(i)$ using value function iteration, we can derive the optimal pricing formula

14 This means that price stickiness is introduced by assuming that in each period only a subset of firms is allowed to reset prices. As firms know that there is a certain probability that they will not be allowed to reset their prices in the next periods, they must be forward looking when setting their profit maximising price in *t*. This particular feature is at the core of the intertemporal behaviour in price settings assumed by Calvo (1983).

15 Note that assuming $\gamma = 0$ would bring back to the same purely forward looking Phillips curve.

$$P_t^d(i)=\frac{1}{\theta}\frac{\sum_{l=0}^{\infty}\left[\tau(\Pi^c)^{\frac{1-\gamma}{\theta-1}}\right]^l E_t\left[\rho_{t,t+l}x_{t+l}^d\left(\frac{P_{t+l-1}^c}{P_{t-1}^c}\right)^{\frac{\gamma}{\theta-1}}\psi_{t+l}(P_{t+l}^d)^{\frac{1}{1-\theta}}\right]}{\sum_{l=0}^{\infty}\left[\tau(\Pi^c)^{\frac{\theta(1-\gamma)}{\theta-1}}\right]^l E_t\left[\rho_{t,t+l}x_{t+l}^d\left(\frac{P_{t+l-1}^c}{P_{t-1}^c}\right)^{\frac{\gamma\theta}{\theta-1}}\frac{1}{P_{t+l}^c}(P_{t+l}^d)^{\frac{1}{1-\theta}}\right]} \tag{21}$$

where the stochastic discount factor $(\rho_{t,t+l}=\beta^l(\lambda_{1,t+l}/\lambda_{1,t}))$ is the time t discounted marginal rate of intertemporal substitution between consumption at t and at $t+l$ and serves as a stochastic discount device for valuing payoffs at $t+l$ in a general equilibrium environment. Equation (21) states that the optimal price set in t $(P_t^d(i))$ is a markup $1/\theta$ over a function of the discounted sum of all the (present and) future marginal costs (ψ_{t+l}).

Aggregating adjusting and non-adjusting prices and log-linearising, we derive a Phillips curve for the domestic intermediate goods inflation

$$\hat{\pi}_t^d=\beta E_t\hat{\pi}_{t+1}^d+\gamma(\hat{\pi}_{t-1}^c-\beta\hat{\pi}_t^c)+\frac{(1-\tau)(1-\tau\beta)}{\tau}(\hat{\psi}_t-\hat{p}_t^d) \tag{22}$$

for

$$\hat{\pi}_t^d=\hat{p}_t^d-\hat{p}_{t-1}^d+\hat{\pi}_t^c. \tag{23}$$

A 'hat' on a variable indicates that we refer to its percentage deviation with respect to its steady state value. Domestic goods inflation $(\hat{\pi}_t^d)$ is a function of expected future domestic inflation $(E_t\hat{\pi}_{t+1}^d)$, of the real marginal cost in domestic intermediate goods terms $(\hat{\psi}_t-\hat{p}_t^d)$, and of present and past CPI $(\hat{\pi}_{t-1}^c-\beta\hat{\pi}_t^c)$, where $\pi_t^c=(P_t^c/P_{t-1}^c)-1$ is the net inflation rate.

Substituting (23) into (22) we obtain the following equation for $\hat{p}_t^d$

$$\hat{p}_t^d=\frac{\tau\left(\beta E_t\hat{p}_{t+1}^d+\hat{p}_{t-1}^d+\beta E_t\hat{\pi}_{t+1}^c+\gamma\hat{\pi}_{t-1}^c-(1+\gamma\beta)\hat{\pi}_t^c+\frac{(1-\tau)(1-\tau\beta)}{\tau}\hat{\psi}_t\right)}{1+\tau^2\beta} \tag{24}$$

which implies, assuming homogeneous pricing and indexing by intermediate firms across final goods sectors, $\hat{p}_t^d=\hat{p}_t^{d,c}=\hat{p}_t^{d,i}=\hat{p}_t^{d,g}$ and $\hat{\pi}_t^d=\hat{\pi}_t^{d,c}=\hat{\pi}_t^{d,i}=\hat{\pi}_t^{d,g}$.

Finally, log-linearising the final goods price equations (7), we obtain the final goods inflation equations

$$\hat{\pi}_t^{\text{a}}=\omega_{e,\text{a}}(\omega_{\text{a}}\hat{\pi}_t^{d,\text{a}}+(1-\omega_{\text{a}})\hat{\pi}_t^{m,\text{a}})+(1-\omega_{e,\text{a}})\hat{\pi}_t^{e,\text{a}} \tag{25}$$

for $\text{a}\in\{c,i,g\}$.

3.3.2 Exported goods

We assume that Switzerland is a 'semi-small' economy in its export markets. Exporters are able to set the price for their product at a markup over their marginal cost. Moreover, Swiss exporters are assumed to price their product in the currency of the customer (LCP).[16] Because of (18), we solve separately the exporters' problem as setting the price $P_t^f(i)$ that would maximise real profit $\mathcal{F}_t^f(i)$.[17]

By analogy with the previous section on optimal pricing for domestic goods, we obtain the following Phillips curve for export prices

$$\hat{\pi}_t^f = \beta E_t \hat{\pi}_{t+1}^f + \gamma_f(\hat{\pi}_{t-1}^* - \beta\hat{\pi}_t^*) + \frac{(1-\tau_f)(1-\tau_f\beta)}{\tau_f}(\hat{\psi}_t - \widehat{rer}_t - \hat{p}_t^f) \tag{26}$$

with

$$\hat{\pi}_t^f = \hat{p}_t^f - \hat{p}_{t-1}^f + \hat{\pi}_t^* \tag{27}$$

where $\hat{\pi}_t^f$ is the exported goods price inflation rate (in foreign currency units), $\hat{\pi}_t^*$ is the euro area inflation rate and $\widehat{rer}_t$ is the percent change from steady state of the real exchange rate defined as $rer_t = (s_t P_t^*)/P_t^c$. τ_f is the Calvo probability and γ_f the indexation parameter. Intuitively, the rate of change of prices charged for exported goods in foreign currency unit increases with the real marginal cost and depends on the strength of domestic currency in real terms (an increase in rer_t is a real depreciation).

3.3.3 Imported goods

By analogy, the same logic applies to importers, which are domestic firms setting prices in Swiss francs as a markup over the import price of intermediate goods produced abroad. The profit function for importers is

$$\mathcal{F}_t^m(i) = \left(\frac{P_t^m(i)}{P_t^c} - rer_t\right) x_t^m(i) \tag{28}$$

and leads to the following Phillips curve

$$\hat{\pi}_t^m = \beta E \hat{\pi}_{t+1}^m + \gamma_m(\hat{\pi}_{t-1}^c - \beta\hat{\pi}_t^c) + \frac{(1-\tau_m)(1-\beta\tau_m)}{\tau_m}(\widehat{rer}_t - \hat{p}_t^m) \tag{29}$$

---

16 The law of one price does not hold for domestically produced intermediate goods, as domestic producers set prices independently for the domestic and export markets.

17 On aggregate, exporters face a downward sloping demand curve

$$x_t^f = \left(\frac{P_t^f}{P_t^*}\right)^{1/(\rho-1)} (1-\omega^*) y_t^*$$

where $1-\omega^*$ is the share of foreign final good production relying on domestic intermediate goods exports, P_t^f the price of domestic exports in foreign currency and P_t^* is the international price level.

describing the change in prices charged to final goods producers in domestic currency units for imported intermediate goods as an increasing function of real exchange rate expressed in units of imported goods $\widehat{rer}_t - \hat{p}_t^m$.

3.4 Households

DSGE-CH assumes the existence of two types of consumers[18] who differ according to their access to asset and credit markets (Galí et al., 2007). The first type, which we refer to as 'permanent income' consumers (*PI* consumers hereafter), faces a lifetime budget constraint and can freely borrow and lend to smooth consumption over time. These agents are assumed to own the domestic firms (and receive dividends accordingly) and are able to save using foreign and domestic bonds.[19]

The other type of consumers, so-called 'rule of thumb' consumers (*ROT* consumers hereafter), faces a period-by-period budget constraint. They do not own any assets and just consume their current labour income flow. Different interpretations for this behaviour have been put forward in the literature: myopia, ignorance of intertemporal trading opportunities, lack of or constrained access to capital markets, etc. (Campbell and Mankiw, 1989 for empirical evidence). In DSGE-CH, *ROT* consumers simply take the (average) wage rate negotiated by *PI* consumers as given and supply the amount of labour requested by domestic intermediate firms at this particular wage (Bjørnland et al., 2008).

We assume that our economy is populated by infinitively lived households. There exists a unit mass continuum of them indexed by $j \in [0,1]$ and *PI* consumers are indexed by $j \in [0, s_{PI}]$ for $s_{PI} \in [0,1]$. Total per capita consumption (in nominal terms) is then a weighted average of consumption by *PI* and *ROT* consumers

$$C_t = s_{PI} C_t^{PI} + (1 - s_{PI}) C_t^{ROT} \tag{30}$$

and *ROT* consumption (per *ROT* consumer) is given by the simple no saving/dissaving condition

$$C_t^{ROT} = W_t h_t. \tag{31}$$

3.4.1 Optimality conditions for 'permanent income' consumers

Because they have access to saving and investment vehicles, *PI* consumers solve a much more complex problem than *ROT* consumers. Mathematically, they maximise

---

18 This feature allows a better replication of consumption dynamics in Switzerland, as compared to previous versions of the model without this extension.

19 For simplicity, we assume (like in Bjørnland et al., 2008) that only *PI* consumers pay (lump-sum) taxes on their income.

$$U_{j,0} = E_{j,0}\left[\sum\nolimits_{t=0}^{\infty}\beta^t \mathcal{U}_t\left(\frac{C_t^{PI}(j)}{P_t^c}, \ell_t(j), \frac{M_t(j)}{P_t^c}\right)\right]$$

where the instantaneous utility function of agent *j* is given by

$$\mathcal{U}_t(c_t^{PI}(j), \ell_t(j), m_t(j)) = \frac{(c_t^{PI}(j) - J_t(j))^{1-\sigma}}{1-\sigma} + \frac{\ell_t(j)^{1-\nu}}{1-\nu} + \frac{m_t(j)^{1-\eta}}{1-\eta}$$

meaning that he cares about consumption $(c_t^{PI}(j) = C_t^{PI}(j)/P_t^c)$, leisure $(\ell(j))$ and real money balances $(m_t(j) = M_t(j)/P_t^c)$.

We assume 'habit-formation-in-consumption' for *PI* consumers. This hypothesis means that they try to smooth out both the level and the rate of change of consumption overtime. Their current utility is actually determined by current consumption relative to a reference level of consumption that could be either their own (internal habit, $J_t(j) = \varrho c_{t-1}^{PI}(j)$) or the overall level (external habit, $J_t(j) = \varrho c_{t-1}^{PI}$) of consumption. With internal habit[20] (Fuhrer, 2000), the consumption-related term in the utility function can be equivalently rewritten as $c_t^{PI}(j) - c_{t-1}^{PI}(j) + (1-\varrho)\, c_{t-1}^{PI}(j)$.[21] The parameter ϱ determines the importance of habit formation. If $\varrho = 1$ (the upper limit), consumption matters only as deviation from last period's consumption.

In each period, the household *j* faces a nominal budget constraint (dropping the *j* indices)

$$\begin{aligned} DIV_t - T_t + Z_t u_t k_{t-1} + R_{t-1}^b B_{t-1} + s_t R_{t-1}^f F_{t-1} + M_{t-1} + W_t h_t + N_t \\ = M_t + C_t^{PI} + I_t + P_t^c a(u_t) k_{t-1} + B_t + s_t F_t \end{aligned} \tag{32}$$

where B_t and F_t are respectively the amounts of domestic and foreign (in foreign currency) one-period bonds maturing in period $t+1$ that are held at the end of period *t*, R_{t-1}^b and R_{t-1}^f the gross nominal yield on these assets and M_t is the amount of money that the agent chooses to hold in *t*. Given its current income out of capital and labour $(Z_t u_t k_{t-1} + W_t h_t)$, its financial wealth inherited from last period $(R_{t-1}^b B_{t-1} + s_t R_{t-1}^f F_{t-1} + M_{t-1})$, transfers from firms ($DIV_t$, dividends) or the government (N_t, seigniorage revenues; T_t, lump-sum tax/transfer), the household *j* allocates its resources between consumption (C_t^{PI}), gross investment in physical capital (I_t), and the holding of money balances (M_t) and bonds $(B_t + s_t F_t)$. Note that increasing the rate of capital utilisation implies additional costs (e.g. accelerated depreciation of the capital stock) that directly enter the budget constraint as an additional element $P_t^c a(u_t) k_{t-1}$. The function $a(u_t)$ is increasing in the rate of capital utilisation u_t and has the properties described in Christiano et al. (2005) and Collard and Dellas (2005).[22]

---

20 The chosen specification in this paper.

21 We have also experimented with an alternative 'catching up with the Jones' (external habit) assumption. The results, in terms of the dynamics of the model, were not dependent on this assumption.

22 The cost to capital utilisation rate enters directly as a convex function $a(u_t) = a_1 e^{a_2(u_t-1)} - a_1$ with properties $a'(u_t) = a_1 a_2 e^{a_2(u_t-1)}$ and $a''(u_t) = a_1 a_2^2 e^{a_2(u_t-1)}$, which implies $a(1) = 0$, $a'(1) = a_1 a_2$, $a''(1) = a_1 a_2^2$ and $a''(1)/a'(1) = a_2$.

In real (total) consumption unit terms, the flow budget constraint can be rewritten as

$$
\begin{aligned}
& div_t - t_t + z_t u_t k_{t-1} + \frac{R^b_{t-1} b_{t-1}}{(1+\pi^c_t)} + \frac{R^f_{t-1} s_t f_{t-1}}{(1+\pi^c_t)} + \frac{m_{t-1}}{(1+\pi^c_t)} + w_t h_t + n_t \\
& = m_t + c^{PI}_t + \frac{P^i_t}{P^c_t} i_t + a(u_t) k_{t-1} + b_t + s_t f_t
\end{aligned} \tag{33}
$$

where $q_t = Q_t / P^c_t$ for $Q_t \in \{B_t, F_t, M_t, DIV_t, N_t, C^{PI}_t, T_t, W_t, Z_t\}$.

Households also face a time constraint

$$
\ell_t = 1 - h_t \tag{34}
$$

where the total time available is normalised to 1, and h_t is the proportion dedicated to labour.

Finally, capital accumulates according to the following law of motion

$$
k_t = \Theta(i_t, i_{t-1}, k_{t-1}) + (1-\delta) k_{t-1} \tag{35}
$$

where k_{t-1} is the predetermined level of capital available for production in *t* and δ is the depreciation rate of capital. The function Θ, with $\mathfrak{I}_t = i_t / i_{t-1}$,

$$
\Theta(i_t, i_{t-1}, k_{t-1}) = i_t - \varpi S(\mathfrak{I}_t) i_t - (1-\varpi) \frac{\varphi}{2} \left(\frac{i_t}{k_{t-1}} - \delta \right)^2 k_{t-1}
$$

is an overall adjustment cost function that allows two nested installation cost schemes.[23] When $\varpi = 0$ the adjustment cost relates to the change in the capital stock. When $\varpi = 1$ the adjustment cost relates to the change in the flow of investment (as assumed by Christiano et al., 2005).[24]

Recall that our specification assumes that the capital stock is owned by *PI* consumers and leased to firms. As a consequence, households choose consumption, leisure, money balances, but also the amount of gross investment that maximise their lifetime utility. They choose the sequences $\{b_t, f_t, m_t, c^{PI}_t, w_t, i_t, u_t, \ell_t\}_{t=0}^{\infty}$ in order to maximise their lifetime expected utility subject to budget, time and accumulation constraints.

In other words, they solve the following dynamic program under budget, time and accumulation constraints (equations (33), (34) and (35)),

$$
V\underbrace{(b_{t-1}, f_{t-1}, k_{t-1}, m_{t-1})}_{\mathcal{W}_t} = \max_{c^{PI}_t, i_t, w_t, u_t, \mathcal{W}_{t+1}} \{\mathcal{U}_t(c^{PI}_t, \ell_t, m_t) + \beta E_t V(\mathcal{W}_{t+1})\}
$$

---

23 $S(\mathfrak{I}_t) = O_3 (\mathrm{e}^{O_1(\mathfrak{I}_t - 1)} + (O_1/O_2)\mathrm{e}^{-O_2(\mathfrak{I}_t - 1)} - (1 + (O_1/O_2))$ as in Adolfson et al. (2007b).

24 We chose $\varpi = 1$ because it allows us to take into account implicitly a time-to-build dimension to the capital accumulation process.

where $\mathcal{W}_t$ stands for wealth inherited from the choices of the previous period.[25]

Assuming the usual solvability constraints leads to the following set of optimality conditions (first order necessary conditions, FONC) which form the core of the model's structure.

FONC $w_t(j)$

The optimal setting of wages by *PI* consumers is very similar to the optimal setting of prices in the intermediate goods sector. Because of assumed market power, the differentiated labour *j* is able to extract a rent over the optimal wage rate that would prevail in a perfectly competitive labour market. With perfect competition, the real wage rate (the ratio of relative prices of leisure and consumption) would be equal to the ratio of marginal utilities of leisure ($\mathcal{U}^{\ell}_{t+l,t} = \partial \mathcal{U}_{t+l,t} / \partial \ell_{t+l}$) and consumption ($\mathcal{U}^{c}_{t+l,t} = \partial \mathcal{U}_{t+l,t} / \partial c_{t+l}$) at the optimum (the marginal rate of substitution (MRS) between leisure and consumption). Here, instead, households are setting wages in a monopolistically competitive labour market according to a Calvo process. Household j sets its wage as a markup over the marginal rate of substitution between leisure and consumption, taking into account the possibility that it might not be allowed to change it again in the coming periods. Solving the problem in a similar fashion as for intermediate goods prices, we obtain the optimal wage setting equation

$$w_t(j) = \frac{1}{\vartheta} \frac{\sum_{l=0}^{\infty} \left(\beta \tau_w (\Pi^c)^{\frac{1-\gamma_w}{\vartheta-1}} \right)^l E_t \left(\mathcal{U}^{\ell}_{t+l,t}(j) \Xi_{t+l}^{\frac{1}{\vartheta-1}} w_{t+l}^{\frac{1}{1-\vartheta}} h_{t+l} \right)}{\sum_{l=0}^{\infty} \left(\beta \tau_w (\Pi^c)^{\frac{\vartheta(1-\gamma_w)}{\vartheta-1}} \right)^l E_t \left(\mathcal{U}^{c}_{t+l,t}(j) \Xi_{t+l}^{\frac{\vartheta}{\vartheta-1}} w_{t+l}^{\frac{1}{1-\vartheta}} h_{t+l} \right)} \tag{36}$$

for

$$\Xi_{t+l} = \frac{P_t^c \left(\frac{P^c_{t+l-1}}{P^c_{t-1}} \right)^{\gamma_w}}{P^c_{t+l}}.$$

Log-linearising this equation around its steady state, we derive an equation for the aggregate (average) real wage level w_t

$$\begin{aligned} \hat{w}_t = {} & \frac{1}{\Upsilon(1+\beta)} \hat{w}_{t-1} + \frac{\beta}{\Upsilon(1+\beta)} E_t \hat{w}_{t+1} + \frac{1}{\Upsilon(1+\beta)} (\beta(E_t \hat{\pi}^c_{t+1} - \gamma_w \hat{\pi}^c_t) - (\hat{\pi}^c_t - \gamma_w \hat{\pi}^c_{t-1})) \\ & + \frac{(1-\tau_w)(1-\tau_w \beta)}{(1-\tau_w)(1-\tau_w \beta) + \tau_w (1+\beta) \left(1 - \nu \frac{h}{1-h} \frac{1}{\vartheta - 1} \right)} \widehat{MRS}_t \end{aligned} \tag{37}$$

---

[25] Mathematically, this is done by maximising the following Lagrangian.

$$\begin{aligned} \mathcal{L} = {} & \mathcal{U}_t(c_t^{PI}, \ell_t, m_t) + \beta E_t V(\mathcal{W}_{t+1}) \\ & + \lambda_{1,t} \begin{pmatrix} div_t - t_t + z_t u_t k_{t-1} + R^b_{t-1}(1+\pi^c_t)^{-1} b_{t-1} + R^f_{t-1}(1+\pi^c_t)^{-1} s_t f_{t-1} \\ + m_{t-1}(1+\pi^c_t)^{-1} + w_t h_t + n_t - m_t - c_t^{PI} - \frac{P^i_t}{P^c_t} i_t - a(u_t) k_{t-1} - b_t - s_t f_t \end{pmatrix} \\ & + \lambda_{2,t} \left(\Theta(i_t, i_{t-1}, k_{t-1}) + (1-\delta) k_{t-1} - k_t \right) \end{aligned}$$

for

$$\widehat{MRS}_t = \widehat{\mathcal{U}}_t^{\ell} - \widehat{\mathcal{U}}_t^{c} \text{ and } \Upsilon = \frac{(1-\tau_w)(1-\tau_w\beta) + \tau_w(1+\beta)\left(1-\nu\frac{h}{1-h}\frac{1}{\vartheta-1}\right)}{\tau_w(1+\beta)\left(1-\nu\frac{h}{1-h}\frac{1}{\vartheta-1}\right)},$$

which describes the percentage change in the real wage as an increasing function of the marginal rate of substitution between leisure and consumption.

FONC $c_t^{PI}(j)$

$$\lambda_{1,t}(j) = (c_t^{PI}(j) - \varrho c_{t-1}^{PI}(j))^{-\sigma} - \varrho\beta E_t(c_{t+1}^{PI}(j) - \varrho c_t^{PI}(j))^{-\sigma} \tag{38}$$

This optimality condition states that the marginal utility of consumption at time t should be equal to its shadow price $\lambda_{1,t}$ (the marginal utility of income).

FONC $u_t(j)$

$$z_t = a'(u_t(j)) \tag{39}$$

This condition ensures that z_t, the marginal benefit of increasing the rate of capital utilisation – which is also the rental rate of an additional unit of capital service $\breve{k}_{t-1}(j)$ – is equal in equilibrium to the marginal cost of increasing the utilisation rate of capital, $a'(u_t(j))$.

FONC $b_t(j)$

$$\frac{1}{R_{t-1}^b} = E_t\left(\rho_{t,t+1}(j)\frac{1}{1+\pi_{t+1}^c}\right) \tag{40}$$

This means that the (inverse of the) ex-ante real interest rate on domestic bonds is equal, to a first order approximation, to the marginal rate of intertemporal substitution in consumption between t and $t+1$, the stochastic discount factor $\rho_{t,t+1}(j)$.

FONC $f_t(j)$

$$\frac{1}{R_{t-1}^f} = E_t\left(\rho_{t,t+1}(j)\frac{1}{1+\pi_{t+1}^c}\frac{s_{t+1}}{s_t}\right) \tag{41}$$

This means that the ex-ante real interest rate on foreign bonds expressed in domestic currency units is equal, up to a first order approximation, to the marginal rate of intertemporal substitution in consumption between t and $t+1$, $\rho_{t,t+1}(j)$.

Up to a (log-)linear approximation, equations (40) and (41) imply the following UIP condition

$$E_t \ln\left(\frac{s_{t+1}}{s_t}\right) \simeq i_t^b - i_t^f \tag{42}$$

where we define the net nominal interest rates on domestic and foreign one period bonds $i_t^b = R_t^b - 1$ and $i_t^f = R_t^f - 1$, such that exchange rate movements are driven by the classical UIP condition in DSGE-CH.

However, persistent and frequent deviations from UIP are well documented in the empirical literature. In order to be able to simulate policy-relevant scenarios implying large deviations of the nominal exchange rate from the model-implied behaviour, we allow for an exogenous 'portfolio' shock to the foreign currency bond Euler equation (41), ε_t^{port}, whose unconditional mean is $E_t \varepsilon_t^{port} = 1$ and whose stochastic process is given in section 3.7.

Portfolio shocks[26] alternatively interpreted as 'fads' (Jeanne and Rose, 2002) or risk premium shocks (McCallum and Nelson, 1999, 2000) drive a wedge between $i_t^b - i_t^f$ and $E_t \ln(s_{t+1}/s_t)$ in the otherwise traditional UIP condition

$$E_t \ln\left(\frac{s_{t+1}}{s_t}\right) \simeq i_t^b - i_t^f - (\varepsilon_t^{port} - 1). \tag{43}$$

FONC *kt*(*j*)

For $\varpi = 0$ (capital adjustment cost specification), the FONC is

$$\lambda_{2,t}(j) = \beta E_t \left(\begin{array}{c} \lambda_{1,t+1}(j)\left(z_{t+1}u_{t+1}(j) - a\left(u_{t+1}(j)\right)\right) \\ +\lambda_{2,t+1}(j)\left(1 - \delta + \frac{\varphi}{2}\left(\left(\frac{i_{t+1}(j)}{k_t(j)}\right)^2 - \delta^2\right)\right) \end{array} \right)$$

where the shadow cost $\lambda_{2,t}(j)$ of giving up one unit of time *t* consumption to increase the stock of installed capital in $t+1$ is equal at the optimum to the return of doing so – expressed here as the marginal benefit (in utility units) to the household *j* of having one more unit of capital in $t+1$, plus the marginal benefit of not having to invest in $t+1$ to reach the same level of capital, net of depreciation.

Alternatively, for $\varpi = 1$ (investment adjustment cost specification), our chosen specification[27], the FONC is

$$\lambda_{2,t}(j) = \beta E_t \left(\begin{array}{c} \lambda_{1,t+1}(j)\left(z_{t+1}u_{t+1}(j) - a(u_{t+1}(j))\right) \\ +\lambda_{2,t+1}(j)\left(\Theta_3\left(i_{t+1}(j), i_t(j), k_t(j)\right) + 1 - \delta\right) \end{array} \right). \tag{44}$$

---

26 None of our empirical validation exercises in section 4 rely on ε_t^{port} shocks which have been shut off in all simulations of the model.

27 Derivatives with respect to the elements of the function Θ are indexed by a subscript number referring to its elements.

FONC $i_t(j)$

Assuming $\varpi = 0$, optimal investment requires that the marginal value (in utility units) of additional installed capital in $t+1$ be equal to the shadow price of investment $\lambda_{1,t}(j)$, the FONC is

$$\lambda_{1,t}(j) = \lambda_{2,t}(j)\left(1-\varphi\left(\frac{i_t(j)}{k_{t-1}(j)}-\delta\right)\right),$$

or assuming $\varpi = 1$

$$\begin{aligned}\lambda_{1,t}(j) &= \lambda_{2,t}(j)\Theta_1\left(i_t(j), i_{t-1}(j), k_t(j)\right)\\ &\quad +\beta E_t\left(\lambda_{2,t+1}(j)\Theta_2\left(i_{t+1}(j), i_t(j), k_{t+1}(j)\right)\right).\end{aligned} \tag{45}$$

3.5 Market clearing conditions

Market clearing for labour, rental capital and oil requires that

$$h_t = \int_0^1 h_t(i)\mathrm{d}i,\ k_t = \int_0^1 k_t(i)\mathrm{d}i \ \text{ and } \ e_t = \int_0^1 e_t^x(i)\mathrm{d}i + e_t^c + e_t^i + e_t^g$$

where $\int_0^1 h_t(i)\mathrm{d}i,\ \int_0^1 k_t(i)\mathrm{d}i,$ and $\int_0^1 e_t^x(i)\mathrm{d}i$ represent total demand for labour, capital and energy from intermediate goods producers and $e_t^c + e_t^i + e_t^g$ the total demand for oil by final goods producers.

Similarly the supply of domestic and imported intermediate goods bundles must be equal to the total demand for them from final goods producers

$$x_t^d = x_t^{d,c} + x_t^{d,i} + x_t^{d,g} \text{ and } x_t^m = x_t^{m,c} + x_t^{m,i} + x_t^{m,g}.$$

Because it is assumed that foreigners do not hold domestic bonds, in equilibrium

$$B_t = 0 \ \forall t.$$

The overall resource constraint implies that in equilibrium the current account balance finances the net purchasing of foreign assets and the cost of increasing the utilization rate of capital

$$s_t f_t + a(u_t)k_{t-1} = R_{t-1}^f(1+\pi_t^c)^{-1}s_t f_{t-1} + s_t p_t^f x_t^f - rer_t x_t^m - p_t^e e_t, \tag{46}$$

for $s_t p_t^f x_t^f - rer_t x_t^m - p_t^e e_t$ the trade balance and $R_{t-1}^f(1+\pi_t^c)^{-1}s_t f_{t-1}$ the gross return on net foreign asset position in domestic private consumption units.

To close the model, thereby imposing a unique stationary equilibrium, the interest rate on foreign assets/liabilities is endogenised. We assume that the nominal interest rate on foreign liabilities (R^f) carries a risk premium over the world (gross) interest rate (R^*) which is decreasing in the level of the country's net foreign assets (Kollmann, 2002). This can be expressed as

$$\frac{R_t^f}{1+\pi^*} = \frac{R_t^*}{1+\pi^*} - \iota \frac{F_t}{\mathcal{X} P_t^*} \tag{47}$$

where ι measures the degree of capital mobility (perfect for $\iota = 0$) and $\mathcal{X}$ is a scaling factor measuring the steady state value of exports in units of foreign output. If $F < 0$, then $R^f > R^*$. This specification, which has some intuitive interpretation, ensures the existence of a unique stationary equilibrium in a SOE provided capital mobility is not completely perfect. Otherwise $\iota = 0$ and $R_t^f = R_t^* \;\; \forall t$, meaning that R_t^f is exogenous at the level of the foreign interest rate.

3.6 Monetary and fiscal policy

We postulate an interest rate feedback rule where the central bank is allowed to respond to deviations of output, inflation and changes in the nominal exchange rate (CHF/EUR)

$$\hat{i}_t^b = \rho_R \hat{i}_{t-1}^b + (1-\rho_R)(k_\pi \hat{\pi}_t^c + k_x \hat{x}_t + k_s \widehat{\Delta s_t}) + \varepsilon_t^{mon}. \tag{48}$$

Note that this specification does not imply that the central bank is targeting any particular level of exchange rate, but that large variations in exchange rates may trigger a policy reaction.

Fiscal policy is very basic in this model, and there is considerable room for improvement along this dimension. We simply assume that fiscal expenditures are determined by an exogenous stochastic process, and that the fiscal authorities run a balanced budget each period using lump-sum taxation. Also, seigniorage (N_t) is transferred to the households every period in a lump-sum fashion. In nominal terms, the fiscal rule is then

$$T_t = G_t + N_t - (M_t - M_{t-1}).$$

3.7 Calibration and shocks

The most important parameters of the model are listed in Tables 1–2. Because of the lack of microeconomic studies on Switzerland, some parameters have been 'borrowed' from related work using comparable models for a SOE (e.g. Adolfson et al., 2007b), while other parameters – those determining the exogenous driving processes[28] (Table 2) and those defining the fundamental steady state ratios of the Swiss economy (the so-called great ratios) – have been estimated/calibrated using aggregate Swiss data on the largest sample available.[29] The remaining parameters are functions of the solution of the model at the steady state.

---

[28] Note that the chosen order of the autoregressive data generating processes for productivity, fiscal and foreign interest rate is larger than 1. This specification was necessary to ensure white noise residuals, a crucial assumption of the RBC literature.

[29] Usually from 1975 Q1 to 2006 Q2 (results not reported).

Table 1: Calibration of DSGE-CH

Technology	shares	$\omega_{e,c}\ \omega_{e,i}\ \omega_{e,g}$	0.99
		$\omega_c\ \omega_i\ \omega_g$	0.85; 0.28; 0.85
		$\alpha_c\ \alpha_l$	0.98; 0.68
	elasticities	$\rho_{e,c}\ \rho_{e,i}\ \rho_{e,g}$	–10
		$\rho_c\ \rho_i\ \rho_g\ \rho_f$	0.01; 0.01; 0.01; 0.1
		$\theta\ \theta_m\ \theta_f$	0.7; 0.7; 0.9
		$\sigma_e\ \sigma_{kl}$	0.15; 0.9999
	capital acc.	$\delta\ O_1\ O_2\ O_3$	0.019; 1; 1; 1
Household preference	labour	$\vartheta\ \nu$	0.8; 1
	consumption	$\sigma\ s_{PI}\ \beta\ \varrho$	1.1; 0.3; 0.996; 0.7
Nominal friction	stickiness	$\tau\ \tau_m\ \tau_f\ \tau_w$	0.9; 0.7; 0.6; 0.7
	indexation	$\gamma\ \gamma_m\ \gamma_f\ \gamma_w$	0.6
Monetary policy	rule	$\rho_R\ k_\pi\ k_x\ k_s$	0.8; 1.5; 0.1; 0:1

Table 2: Calibration of shocks in DSGE-CH

Stoch. driving process of shocks		std. dev. ε^i	ρ_j for $j \in [0;4]$
Productivity	ε^a	0.0052	0.66; 0.15; –0.30; 0.053
Fiscal	ε^g	0.0053	1.29; –0.37; 0; 0
Foreign output	ε^{y*}	0.00817	0.6; 0; 0; 0
Foreign interest rate	ε^{R*}	0.001	1.29; –0.45; 0; 0
Foreign inflation	ε^{p*}	0.0039	0.13; 0; 0; 0
Oil price	ε^{p^e}	0.14	0.69; 0; 0; 0
Portfolio	ε^{port}	0	0.8; 0; 0; 0
Monetary policy	ε^{mon}	0.0015	0; 0; 0; 0

Note: The following driving process $\varepsilon_t^i = \left[1-\sum_{j=1}^{4}\rho_j\right]\varepsilon_{ss}^i + \sum_{j=1}^{4}\rho_j\varepsilon_{t-j}^i + \varepsilon_t^i$ applies to the shocks, and is estimated via OLS; *ss* is the steady state value.

4 Main findings

In order for the model to be taken seriously as a laboratory for the analysis of the Swiss economy, it needs to exhibit satisfactory empirical performance. The model should generate a theoretical probability density function (pdf) for its key variables that is not 'too' distant from the one observed in reality. While there are many elements of the pdf that can be used to validate the model, the literature has focused mostly on two: unconditional second moments (variances, cross-variances and autocorrelations) and model dynamics as captured by IRF and historical simulations.[30] How does our model perform along these dimensions?

4.1 Moments

Tables 3-5 report real world moments as well as the corresponding model-implied moments.[31] The model does quite a good overall job in capturing both the relative and absolute levels of volatility in the economy as well as the cyclical positive comovements across real variables and the persistence at the business cycle frequency.

Table 3a: Standard deviation

	Actual	Model
GDP	0.0116	0.0108
Consumption	0.0066	0.0069
Investments	0.0307	0.0320
Exports	0.0307	0.0277
Imports	0.0316	0.0187
Employment	0.0087	0.0080
Inflation	0.0033	0.0023
Interest rate	0.0026	0.0024
Exchange rate	0.0114	0.0101
RER	0.0230	0.0122

Table 3b: Relative standard deviation

	Actual	Model
Consumption./GDP	0.57	0.64
Investment/GDP	2.65	2.96
Exports/GDP	2.65	2.57
Imports/GDP	2.72	1.73
Employment/GDP	0.75	0.74
Inflation/GDP	0.28	0.22
Interest rate/GDP	0.22	0.22
Exchange rate/GDP	0.98	0.93
RER/GDP	1.98	1.13

Table 4: Correlation with GDP

	Actual	Model
Consumption	0.59	0.74
Investment	0.66	0.56
Exports	0.78	0.87
Imports	0.69	0.51
Employment	0.77	0.65
RER	0.04	0.37

30 Conditional second moments as well as variance decompositions may be additional criteria.

31 All variables are HP-detrended.

Table 5: AUTOCORRELATIONS

	Lag 1		Lag 2	
	Actual	Model	Actual	Model
GDP	0.81	0.90	0.63	0.73
Consumption	0.66	0.95	0.54	0.85
Investments	0.78	0.97	0.59	0.89
Exports	0.76	0.83	0.52	0.60
Imports	0.67	0.97	0.46	0.90
Employment	0.84	0.89	0.66	0.73
Inflation	0.59	0.58	0.19	0.27
Exchange rate	0.17	–0.05	0.08	–0.07
RER	0.73	0.75	0.43	0.53

Naturally, some weaknesses remain. The most important concerns the behaviour of the real exchange rate (RER). Although the model-generated RER is as persistent as in the data and also more volatile than GDP, its standard deviation remains lower than the actual one. The correlation of RER with GDP in the model is also much larger than in the data.[32] Also, inflation is slightly less volatile in the model than in the data. We attribute this feature to the fact that before 1995, the SNB's monetary policy rule may have been significantly different than the one postulated in the model.[33]

4.2 Impulse response functions

The dynamic properties of the model in the form of IRF are depicted in Charts 2 to 5.

4.2.1 Monetary policy shock

Chart 2 shows the response of the key macroeconomic variables to a contractionary monetary policy shock (an increase in the short-term nominal interest rate). The predicted patterns are plausible, with monetary tightening leading to a reduction in economic activity across the board (a reduction in output, investment, employment, exports, imports and consumption), a reduction in the rate of inflation and a currency appreciation.

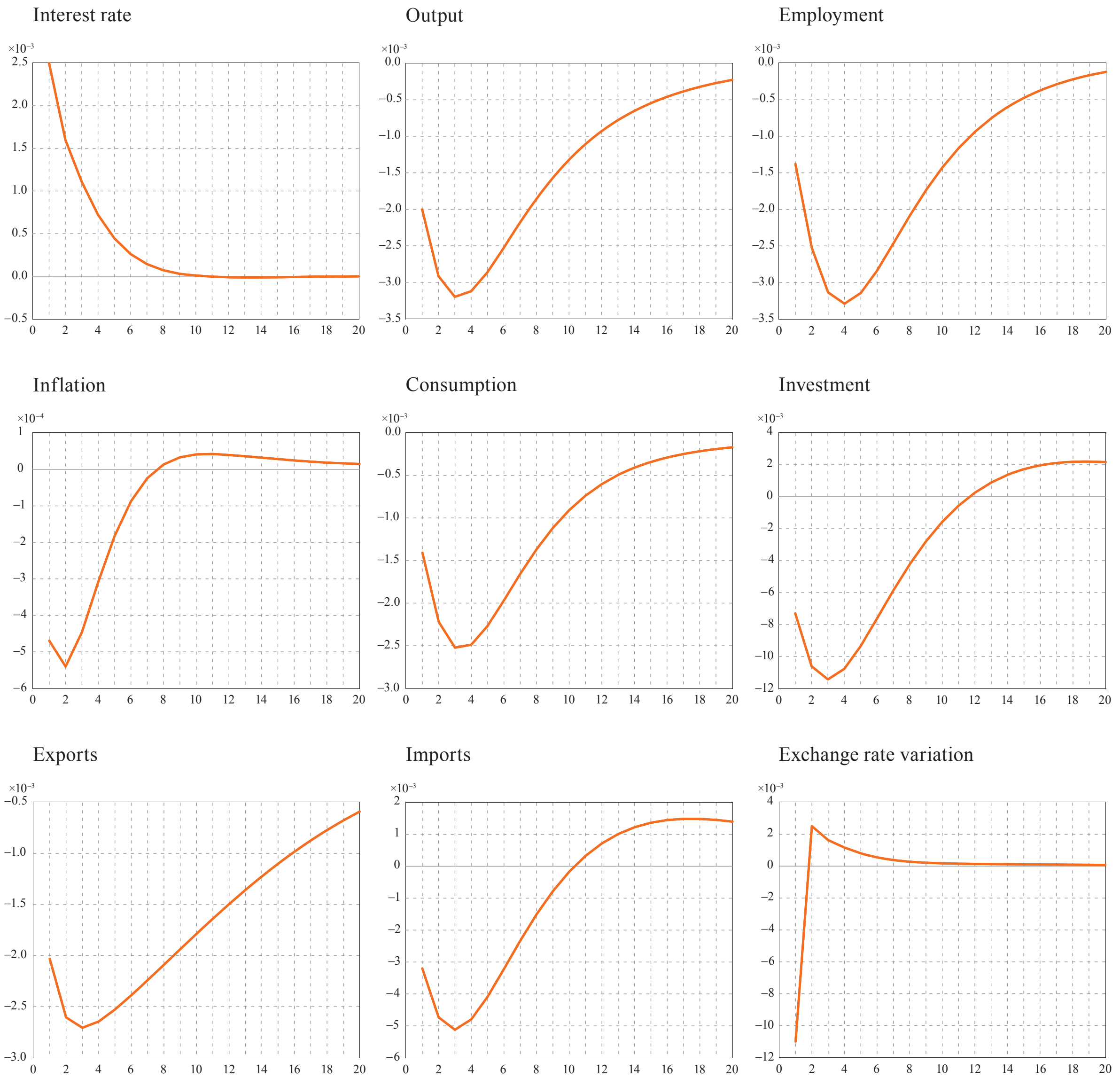
What do these IRF tell us about the transmission mechanism of monetary policy in a SOE? There are basically two mechanisms. The first one is the standard open economy channel that operates through the effects of a change in the nominal interest rate on the exchange rate. With price rigidity, an increase in the nominal interest rate translates into an increase in the real interest rate, which makes foreign assets less attractive. This leads to domestic currency appreciation and the usual expenditure switching effect, detrimental to the demand for domestically produced goods.

---

32 See Chari et al. (2002) for a thorough analysis of the so-called exchange rate disconnect puzzle in the context of an open economy DSGE model with sticky prices.

33 Computing the standard deviation of inflation since 1995 Q1, we get a much lower estimate at 0.0016.

Chart 2: IRF TO A MONETARY POLICY SHOCK



Note: These charts show the reaction (20 quarters) of different variables after a 25-basis-point positive shock to the nominal short-term interest rate. The reaction eventually converges to zero because variables are expressed in deviation from their steady state.

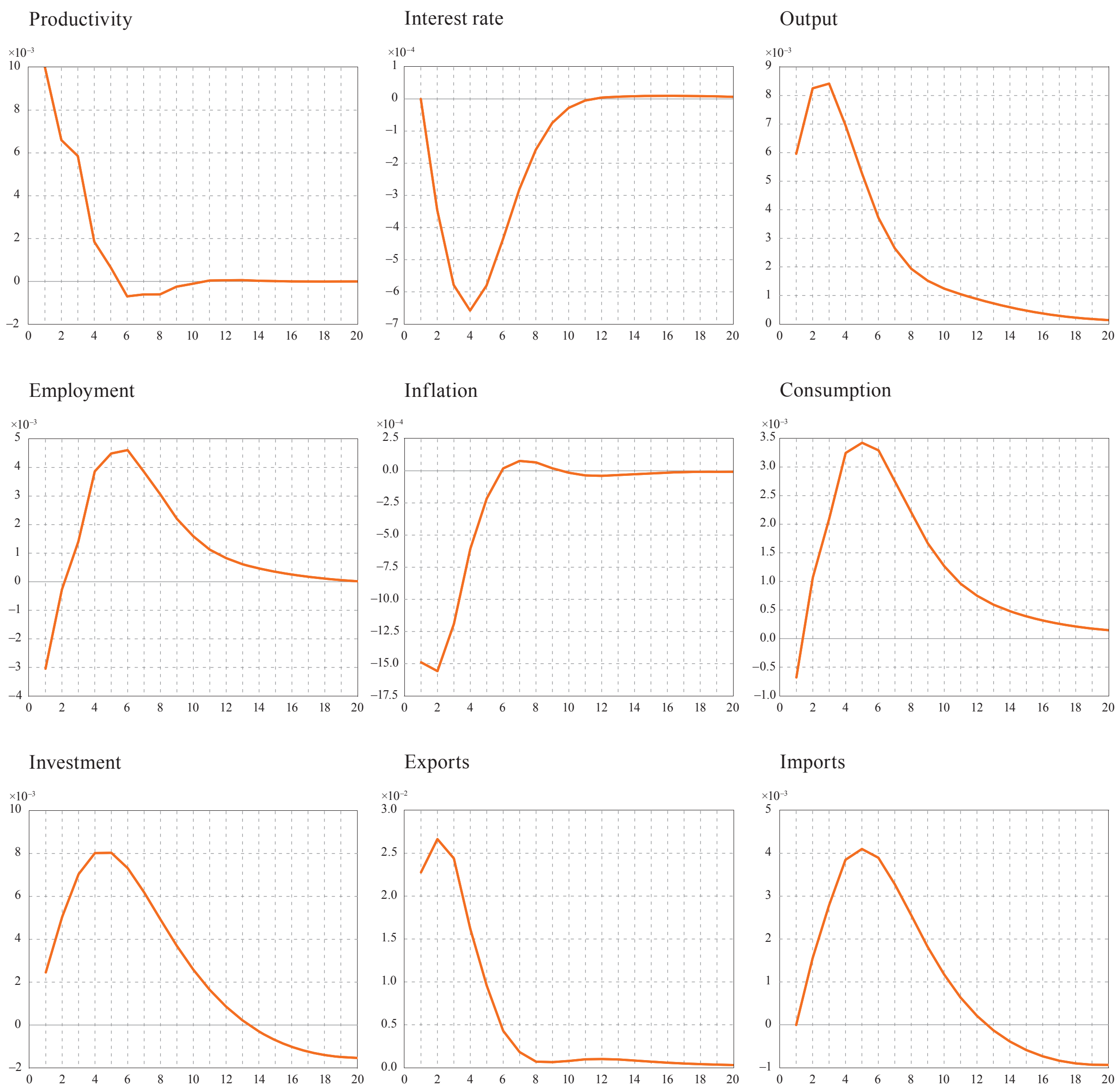
The second channel operates through the effect of higher real interest rates on investment and consumption. Both are discouraged. The flip side of this contraction is obviously the reduction in inflation which results from both the currency appreciation (which leads to a drop in import prices) and a cutback in marginal costs.

4.2.2 Productivity shock

A positive, temporary, but persistent supply shock (Chart 3) leads to lower inflation and to higher domestic output, consumption and investment. It also weakens the domestic currency on impact (not reported) stimulating exports. Responding to a lower inflation rate, the central bank lowers interest rates according to the interest rate feedback rule postulated. In the process, the Swiss franc appreciates against the euro to the extent implied by the UIP when the domestic interest rate is expected to be below the foreign one for a while.

As the effect of the productivity shock slowly wears off, the nominal interest rate has to be raised in order to contain nascent inflationary pressures and all the variables converge towards their long-run equilibrium. In the very short run (up to about six months), the increase in productivity leads to a decrease in hours worked. With nominal rigidities, the increase in production capacity cannot be absorbed completely by consumers in the short run (Galí and Gertler, 1999) and the increase in productivity temporarily crowds out employment.[34] But while productivity recedes towards its steady state equilibrium, aggregate demand is stimulated by the depreciation of the exchange rate and the lower interest rate, so that hours eventually increase.

Chart 3: IRF TO A PRODUCTIVITY SHOCK



Note: These charts show the reaction (20 quarters) of different variables after a 100-basis-point positive shock to productivity in the intermediate goods sector. The reaction eventually converges to zero because variables are expressed in deviation from their steady state.

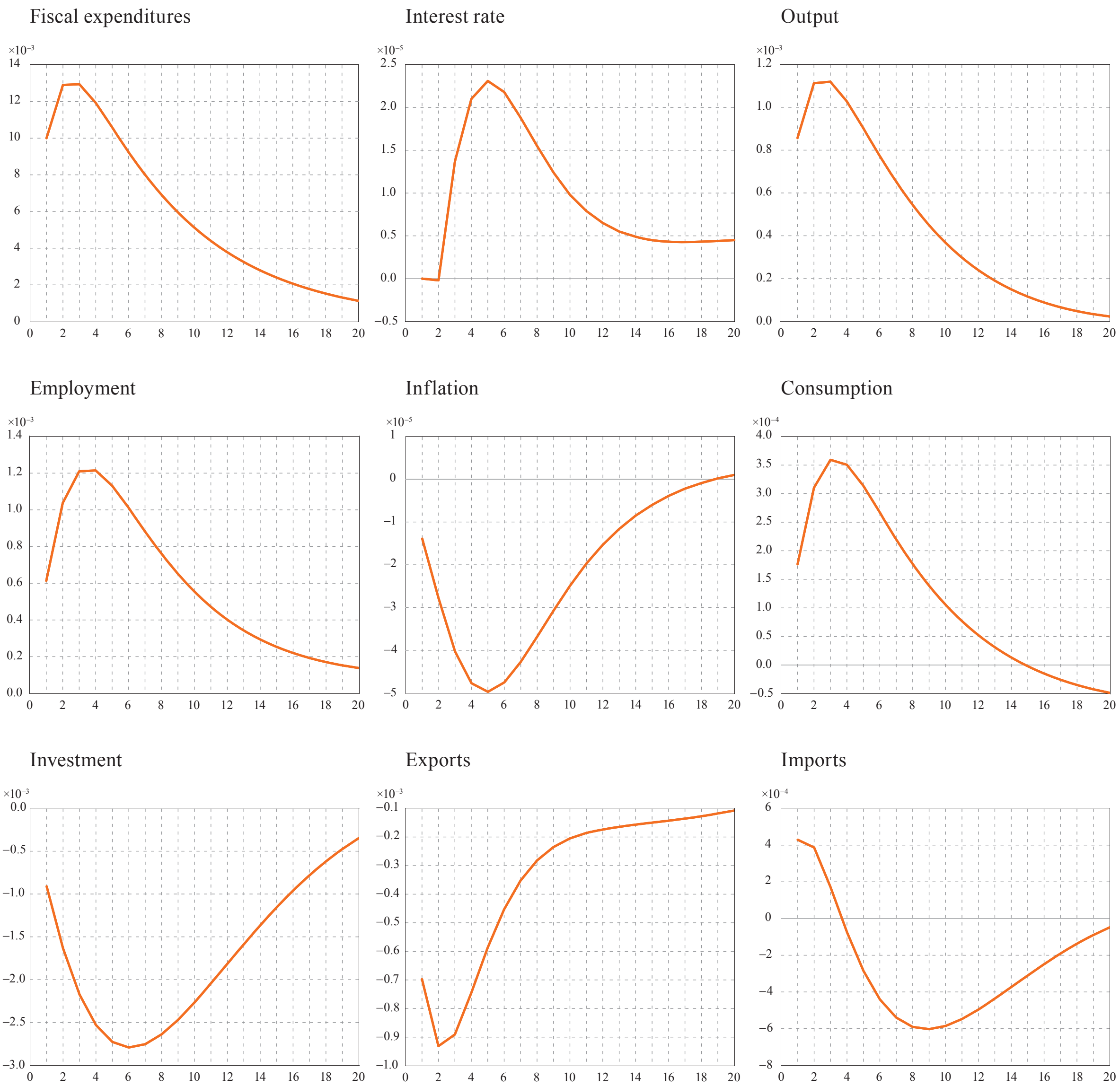
34 This is in contrast to the standard prediction of RBC models without nominal rigidities.

4.2.3 Fiscal policy shock

A positive, temporary, but persistent fiscal shock (Chart 4) increases output and consumption (as expected in the presence of *ROT* consumers,[35] whose consumption is a direct function of their labour income) but crowds out private investment. The currency appreciates on impact (not shown) and leads to a slight decrease in inflation which displays a hump-shaped pattern.

The scale of the responses, however, is very small. A one-percent persistent increase in public expenditures leads to an increase of about 0.1% in output (almost everything is crowded out despite the increase in labour input) and to a decrease of 0.005% in quarterly inflation. Because they are financed by lump-sum taxation, public expenditures in our model hardly contribute at all to the overall dynamics of output and inflation.

Chart 4: IRF TO A FISCAL EXPENDITURE SHOCK



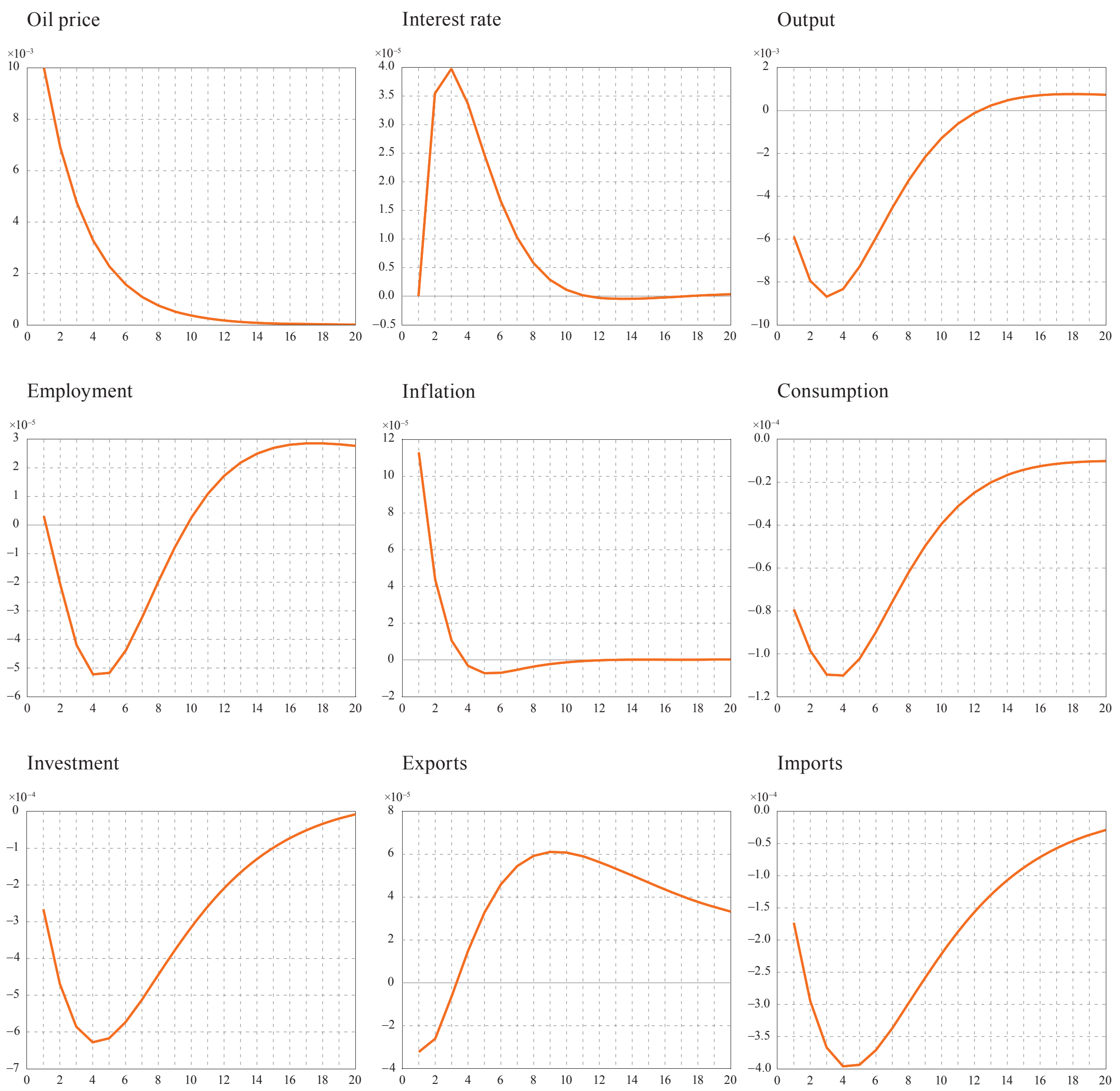
Note: These charts show the reaction (20 quarters) of different variables after a 100-basis-point positive shock to fiscal expenditures. The reaction eventually converges to zero because variables are expressed in deviation from their steady state.

[35] This result, however, depends on the assumed proportion of *ROT* consumers.

4.2.4 Oil price shock

Finally, Chart 5 sheds light on the response of the economy to a temporary, but persistent oil price shock of one-percent point. Although the pass-through to inflation is incomplete due to price rigidities at the production level, the effect of an oil price increase on headline inflation is immediate. A one-percent increase in oil price leads to a rise in quarterly inflation of 0.012%. The negative supply-side effect on output is more muted on impact (0.006%), but reaches a comparable magnitude about one year after the shock. Monetary authorities raise interest rates progressively, weighting the negative effect on output with the positive effect on inflation: the short-term nominal interest rate peaks three quarters after the shock.

Chart 5: IRF to an oil price shock



Note: These charts show the reaction (20 quarters) of different variables after a 100-basis-point positive shock to the oil price. The reaction eventually converges to zero because variables are expressed in deviation from their steady state.

4.3 Historical simulations

In order to judge the fit of the model from its ability to reproduce the historical pattern of a variable of great interest to the central bank, such as the inflation rate or output growth, we engage in historical simulations. We use the solution of the model together with the realised values of the exogenous driving processes (foreign output, interest rate and inflation, as well as the price of oil, domestic productivity and fiscal expenditures) to trace out the paths of the most important variables and compare them to the actual data over the same period.[36] This is the most demanding exercise for a theoretical model as it goes beyond simply capturing average statistics, such as the variance-covariances of variables of interest. We believe, however, that DSGE-CH should be able to replicate the past behaviour of the most important variables when conditioned on the realised external environment, if it is to be successful in forecasting the behaviour of the Swiss economy in the short to medium run.

Charts 6 to 14 show the actual and simulated paths of the national accounts components, inflation, the nominal interest rate and money demand over the period 1987 Q1–2008 Q2, conditioned on the historical values of the exogenous shocks. The model does a reasonable job in capturing most turning points in real GDP, consumption and investment growth. The magnitude of fluctuations is also well reproduced over the whole period. The model is also able to reproduce rather well the pattern of inflation over these years. Inflation drops in 1986 and picks up in 1990 and 1991 to reach almost 6% and retreats thereafter as nominal interest rates are raised. From 1995, it hovers around 1%.

However, DSGE-CH is not able to account for the drop in consumption and investment (housing and equipment) following the housing market crash of 1992 and the bursting of the equity bubble in 2001, suggesting a role for the financial sector in the analysis of business cycles in Switzerland.

---

36 The information set available to DSGE-CH is limited to the values of the state variables in 1987 Q1 and the realised values of the exogenous driving processes. No use is made of the realised values of the endogenous variable in the simulations. In technical terms, we feed the model with the realised values of the shocks to the exogenous driving processes starting in 1986. The shocks are the one-step-ahead AR forecast errors for the exogenous driving variables.

Chart 6: Historical simulation

Output

-- Past — Observed — Simulated

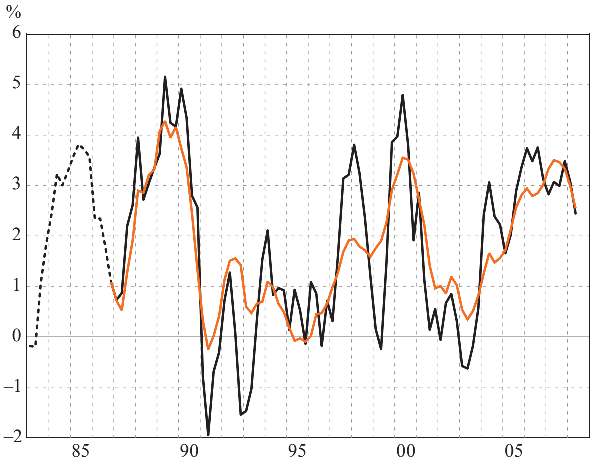


Chart 7: Historical simulation

Consumption

-- Past — Observed — Simulated

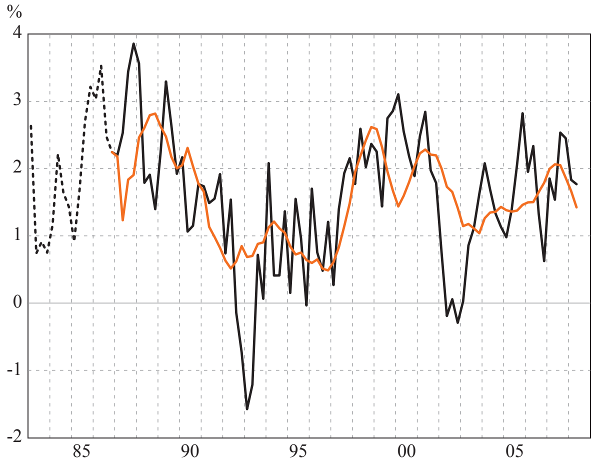


Chart 8: Historical simulation

Investment

-- Past — Observed — Simulated

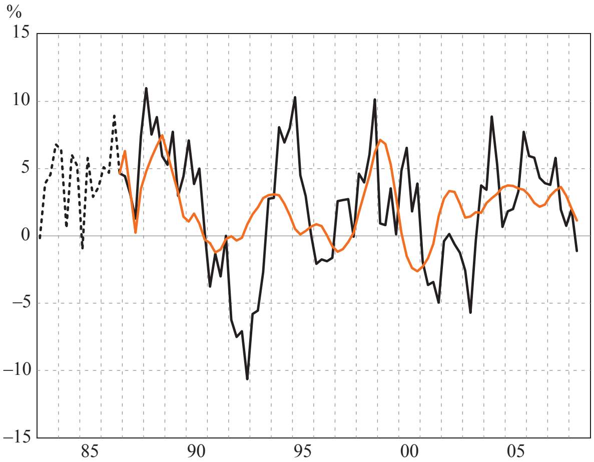


Chart 9: Historical simulation

Employment

-- Past — Observed — Simulated

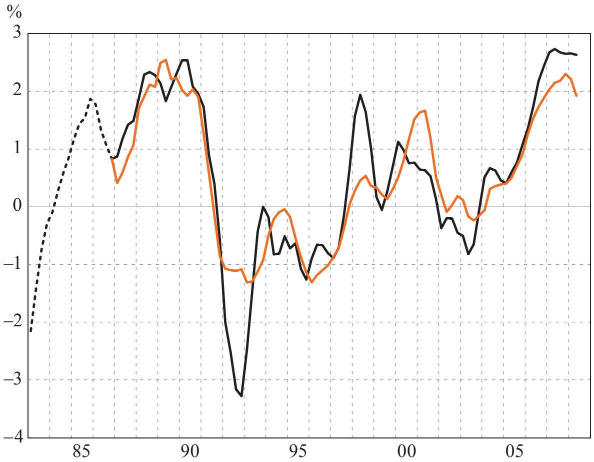


Chart 10: Historical simulation

Exports

-- Past — Observed — Simulated

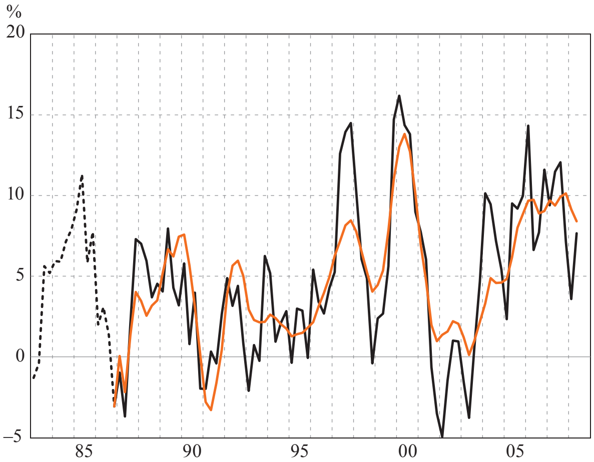
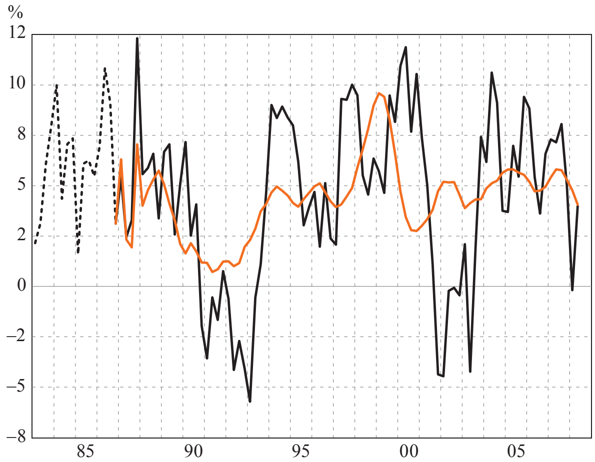


Chart 11: Historical simulation

Imports

-- Past — Observed — Simulated



Note to Charts 6 to 11: These historical simulations cover the period 1987 Q1 to 2008 Q2.

Chart 12: Historical simulation

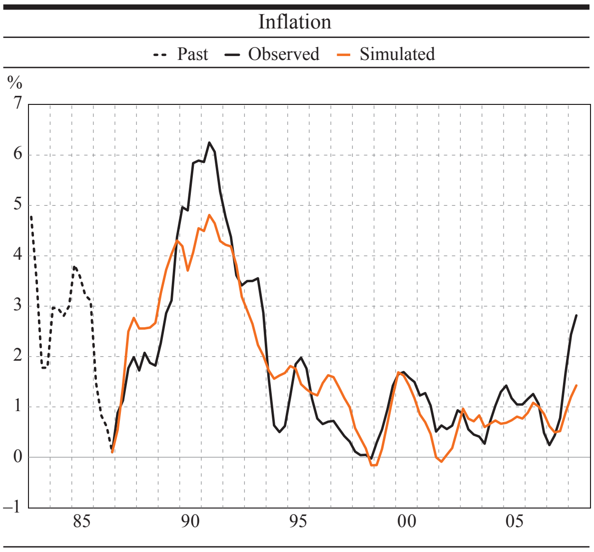


Chart 13: Historical simulation

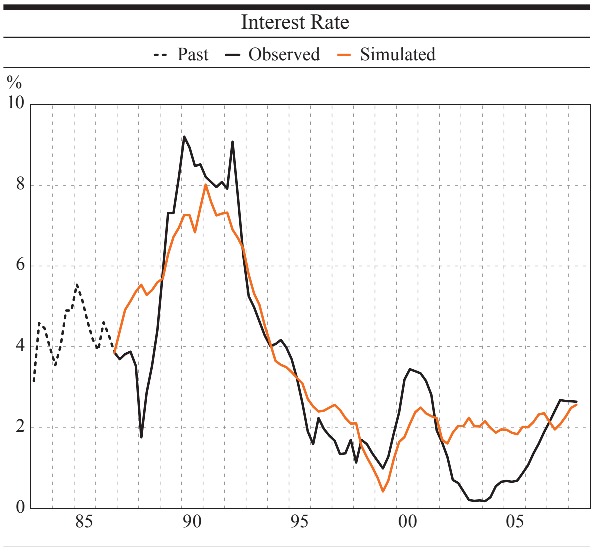
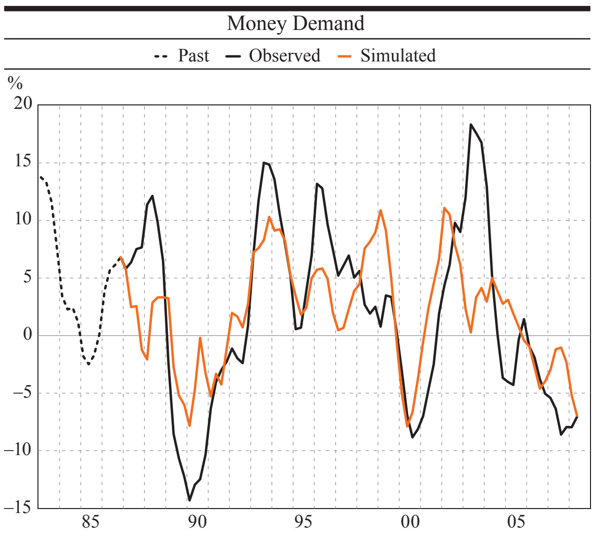


Chart 14: Historical simulation



Note to Charts 12 to 14: These historical simulations cover the period 1987 Q1 to 2008 Q2.

5. Conclusions

As a result of recent advances in macroeconomic theory and computational techniques, it has become feasible to construct richly structured dynamic stochastic general equilibrium models and use them as laboratories for the study of business cycles and for the formulation and analysis of monetary policy. One after the other, the central banks of the industrial countries have been undertaking this task.

DSGE-CH is a medium-sized model which contains most macroeconomic variables of interest and several domestic and foreign shocks. The comparison of the implications of the model to the real world indicates that the model, while being far from perfect, performs rather well along standard dimensions. In particular, it captures well the overall stochastic structure of the Swiss economy (as represented by the unconditional moments of its key macroeconomic variables). It has sensible dynamic properties, as judged by its implied IRF, and finally, it can replicate the historical path of major Swiss variables rather accurately, and as such has been used since 2007 in the quarterly monetary policy decision process at the SNB.

DSGE-CH constitutes a promising first step in the construction of a DSGE model for the Swiss economy that can be relied upon to plot the short to medium term path of the Swiss economy as a result of known, expected or hypothetical events. Also, such a model can help in the formulation of the appropriate monetary policy reactions to such events, and as such has been used since 2007 in the quarterly monetary policy decision process at the SNB.

However, the model is not yet fully developed. First, DSGE-CH does not yet include a banking/financial nexus, a sector that is potentially of greater importance for Switzerland than for other countries. Second, it relies on a rather ad hoc monetary policy rule that, by and large, replicates the past behaviour of the monetary authorities. How far this policy rule is from a model-based optimal policy remains an open question. Extensions along these two dimensions are work in progress.

References

Abel, Andrew B. 1990. Asset prices under habit formation and catching up with the Joneses. *American Economic Review* 80(2): 38–42.

Adolfson, Malin, Michael K. Andersson, Jesper Lindé, Mattias Villani and Anders Vredin. 2007a. Modern forecasting models in action: improving macroeconomic analyses at central banks. *International Journal of Central Banking* 3(4): 111–144.

Adolfson, Malin, Stefan Laséen, Jesper Lindé and Mattias Villani. 2007b. Bayesian estimation of an open economy DSGE model with incomplete pass-through. *Journal of International Economics* 72(2): 481–511.

Adolfson, Malin, Stefan Laséen, Jesper Lindé and Mattias Villani. 2007c. RAMSES – a new general equilibrium model for monetary policy analysis. Sveriges Riksbank *Economic Review* (2): 5–39.

Backus, David, Patrick J. Kehoe and Finn E. Kydland. 1995. International business cycles: theory and evidence. In *Frontiers of Business Cycle Research*, Thomas F. Cooley, ed., 331–356. Princeton University Press.

Benes, Jaromir, Tibor Hledik, Michael Kumhof and David Vavra. 2005. An economy in transition and DSGE: what the Czech National Bank's new projection model needs. Czech National Bank Working Paper 12.

Bernanke, Ben S., Mark L. Gertler and Simon Gilchrist. 1999. The financial accelerator in a quantitative business cycle framework. In *Handbook of Macroeconomics* 1, John B. Taylor and Michael Woodford, eds., 1341–1393. Elsevier.

Bjørnland, Hilde, Leif Brubakk and Anne S. Jore. 2008. Forecasting inflation with an uncertain output gap. *Empirical Economics* 35(3): 413–436.

Brubakk, Leif, Tore A. Husebø, Junior Maih, Kjetil Olsen and Magne Østnor. 2006. Finding NEMO: documentation of the Norwegian economy model. Norges Bank Staff Memo 2006/6.

Calvo, Guillermo A. 1983. Staggered prices in a utility maximising framework. *Journal of Monetary Economics* 12(3): 383–398.

Campbell, John Y. and John H. Cochrane. 1999. By force of habit: a consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy* 107(1): 205–251.

Campbell, John Y. and N. Gregory Mankiw. 1989. International evidence on the persistence of economic fluctuations. *Journal of Monetary Economics* 23(2): 319–333

Chari, Varadarajan V., Patrick J. Kehoe and Ellen R. McGrattan. 2002. Can sticky prices explain the persistence and volatility of real exchange rates? *Review of Economics Studies* 69(3): 533–563.

Christiano, Lawrence J., Martin Eichenbaum and Charles L. Evans. 2005. Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy* 113(1): 1–45.

Collard, Fabrice and Harris Dellas. 2005. Imperfect information and inflation dynamics, mimeo.

Collard, Fabrice and Harris Dellas. 2004. The new Keynesian model with imperfect information and learning. IDEI Working Paper 273.

Cuche-Curti, Nicolas A., Harris Dellas and Jean-Marc Natal. 2009. Price stability and the case for flexible exchange rates, forthcoming in *Open Economies Review.*

Cuche-Curti, Nicolas A., Harris Dellas and Jean-Marc Natal. 2008. Inflation targeting in a small open economy. *International Finance* 11: 1–18.

Del Negro, Marco, Frank Schorfheide, Frank Smets and Rafael Wouters. 2007. On the fit of new-Keynesian models. *Journal of Business and Economic Statistics* 25(2): 123–143.

Devereux, Michael B. and Charles Engel. 1998. Fixed versus floating exchange rates: how price setting affects the optimal choice of exchange rate regime. NBER Working Paper 6897.

Erceg, Christopher J., Dale W. Henderson and Andrew T. Levin. 2000. Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics* 46(2): 281–313.

Fuhrer, Jeffrey C. 2000. Habit formation in consumption and its implications for monetary policy models. *American Economic Review* 90(3): 367–390.

Galí, Jordi and Mark L. Gertler. 1999. Inflation dynamics: a structural econometric analysis. *Journal of Monetary Economics* 44(2): 195–222.

Galí, Jordi, J. David López-Salido and Javier Vallés. 2007. Understanding the effects of government spending on consumption. *Journal of the European Economic Association* 5(1): 227–270.

Harrison Richard, Kalin Nikolov, Meghan Quinn, Gareth Ramsay, Alasdair Scott and Ryland Thomas. 2005. *The Bank of England quarterly model*. Bank of England.

Jeanne, Olivier and Andrew K. Rose. 2002. Noise trading and exchange rate regimes. *Quarterly Journal of Economics* 117(2): 537–569

Julliard, Michel. 1996. Dynare: a program for the resolution and simulation of dynamic models with forward variables through the use of a relaxation algorithm. Cepremap Working Paper 9602.

Kollmann, Robert. 2002. Monetary policy rules in the open economy: effects on welfare and business cycles. *Journal of Monetary Economics* 49(5): 989–1015.

Mankiw, N. Gregory and Ricardo Reis. 2002. Sticky information versus sticky prices: a proposal to replace the new Keynesian Phillips curve. *Quarterly Journal of Economics* 117(4): 1295–1328.

McCallum, Bennett T. and Edward Nelson. 1999. Nominal income targeting in an open-economy optimizing model. *Journal of Monetary Economics* 43(3): 553–578.

McCallum, Bennett T. and Edward Nelson. 2000. Monetary policy for an open economy: an alternative framework with optimizing agents and sticky prices. *Oxford Review of Economic Policy* 16(1): 74–91.

Murchison, Stephen and Andrew Rennison. 2006. TOTEM: the Bank of Canada's new projection model. Bank of Canada Technical Report 97.

Lucas, Robert E. Jr. 1976. Econometric policy evaluation: a critique. *Carnegie-Rochester Conference Series on Public Policy* 1: 19–46.

Pagan, Adrian. 2003. Report on modelling and forecasting at the Bank of England. Bank of England *Quarterly Bulletin* 43(1): 60–88.

Schmitt-Grohé, Stephanie and Martin Uribe. 2003. Closing small open economy models. *Journal of International Economics* 61(1): 163–185.

Smets, Frank and Rafael Wouters. 2004. Forecasting with a Bayesian DSGE model: an application to the euro area. *Journal of Common Market Studies* 42(4): 841–867.

Smets, Frank and Rafael Wouters. 2003. An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association* 1(5): 1123–1175.

Equation summary

Final goods

Eq. 1	$x_t^{d,c} = \omega_c\, \mathfrak{g}(x_t^c)^{\frac{\rho_{e,c}-\rho_c}{1-\rho_c}} (\omega_{e,c} c_t)^{\frac{1-\rho_{e,c}}{1-\rho_c}} (p_t^{d,c})^{\frac{1}{\rho_c - 1}}$
Eq. 2	$x_t^{m,c} = (1-\omega_c)\mathfrak{g}(x_t^c)^{\frac{\rho_{e,c}-\rho_c}{1-\rho_c}} (\omega_{e,c} c_t)^{\frac{1-\rho_{e,c}}{1-\rho_c}} (p_t^{m,c})^{\frac{1}{\rho_c - 1}}$
Eq. 3	$\mathfrak{g}(x_t^c) = \omega_{e,c} c_t \left[\omega_c (p_t^{d,c})^{\frac{\rho_c}{\rho_c-1}} + (1-\omega_c)(p_t^{m,c})^{\frac{\rho_c}{\rho_c-1}}\right]^{\frac{1}{\rho_c}\left(\frac{1-\rho_c}{1-\rho_{e,c}}\right)}$
Eq. 4	$\frac{P_t^i}{P_t^c} = \left\{ \omega_{e,i} \left[\left(\omega_i (p_t^{d,c})^{\frac{\rho_i}{\rho_i-1}} + (1-\omega_i)(p_t^{m,c})^{\frac{\rho_i}{\rho_i-1}}\right)\right]^{\left(\frac{\rho_{e,i}}{1-\rho_{e,i}}\right)\frac{1-\rho_i}{\rho_i}} + (1-\omega_{e,i})(p_t^{e,c})^{\frac{\rho_{e,i}}{\rho_{e,i}-1}} \right\}^{\frac{\rho_{e,i}-1}{\rho_{e,i}}}$
Eq. 5	$x_t^{d,i} = \omega_i \mathfrak{g}(x_t^i)^{\frac{\rho_{e,i}-\rho_i}{1-\rho_i}} (\omega_{e,i} i_t)^{\frac{1-\rho_{e,i}}{1-\rho_i}} \left(\frac{P_t^i}{P_t^c}\right)^{\frac{1}{1-\rho_i}} (p_t^{d,c})^{\frac{1}{\rho_i-1}}$
Eq. 6	$x_t^{m,i} = (1-\omega_i)\mathfrak{g}(x_t^i)^{\frac{\rho_{e,i}-\rho_i}{1-\rho_i}} (\omega_{e,i} i_t)^{\frac{1-\rho_{e,i}}{1-\rho_i}} \left(\frac{P_t^i}{P_t^c}\right)^{\frac{1}{1-\rho_i}} (p_t^{m,c})^{\frac{1}{\rho_i-1}}$
Eq. 7	$\mathfrak{g}(x_t^i) = \omega_{e,i} i_t \left(\frac{P_t^i}{P_t^c}\right)^{\frac{1}{1-\rho_{e,i}}} \left[\left(\omega_i (p_t^{d,c})^{\frac{\rho_i}{\rho_i-1}} + (1-\omega_i)(p_t^{m,c})^{\frac{\rho_i}{\rho_i-1}}\right)\right]^{\frac{1}{\rho_i}\left(\frac{1-\rho_i}{1-\rho_{e,i}}\right)}$
Eq. 8	$\frac{P_t^g}{P_t^c} = \left\{ \omega_{e,g} \left[\left(\omega_g (p_t^{d,c})^{\frac{\rho_g}{\rho_g-1}} + (1-\omega_g)(p_t^{m,c})^{\frac{\rho_g}{\rho_g-1}}\right)\right]^{\left(\frac{\rho_{e,g}}{1-\rho_{e,g}}\right)\frac{1-\rho_g}{\rho_g}} + (1-\omega_{e,g})(p_t^{e,c})^{\frac{\rho_{e,g}}{\rho_{e,g}-1}} \right\}^{\frac{\rho_{e,g}-1}{\rho_{e,g}}}$
Eq. 9	$x_t^{d,g} = \omega_g \mathfrak{g}(x_t^g)^{\frac{\rho_{e,g}-\rho_g}{1-\rho_g}} (\omega_{e,g} g_t)^{\frac{1-\rho_{e,g}}{1-\rho_g}} \left(\frac{P_t^g}{P_t^c}\right)^{\frac{1}{1-\rho_g}} (p_t^{d,c})^{\frac{1}{\rho_g-1}}$
Eq. 10	$x_t^{m,g} = (1-\omega_g)\mathfrak{g}(x_t^g)^{\frac{\rho_{e,g}-\rho_g}{1-\rho_g}} (\omega_{e,g} g_t)^{\frac{1-\rho_{e,g}}{1-\rho_g}} \left(\frac{P_t^g}{P_t^c}\right)^{\frac{1}{1-\rho_g}} (p_t^{m,c})^{\frac{1}{\rho_g-1}}$

Eq. 11	$\mathfrak{g}(x_t^g) = \omega_{e,g} g_t \left(\frac{P_t^g}{P_t^c}\right)^{\frac{1}{1-\rho_{e,g}}} \left[\omega_g (p_t^{d,c})^{\frac{\rho_g}{\rho_g - 1}} + (1-\omega_g)(p_t^{m,c})^{\frac{\rho_g}{\rho_g - 1}}\right]^{\frac{1}{\rho_g}\left(\frac{1-\rho_g}{1-\rho_{e,g}}\right)}$
Eq. 12	$e_t^c = (p_t^{e,c})^{\frac{1}{\rho_{e,c}-1}} (1-\omega_{e,c}) c_t$
Eq. 13	$e_t^g = (p_t^{e,c})^{\frac{1}{\rho_{e,g}-1}} (1-\omega_{e,g}) g_t \left(\frac{P_t^g}{P_t^c}\right)^{\frac{1}{1-\rho_{e,g}}}$
Eq. 14	$e_t^i = (p_t^{e,c})^{\frac{1}{\rho_{e,i}-1}} (1-\omega_{e,i}) i_t \left(\frac{P_t^i}{P_t^c}\right)^{\frac{1}{1-\rho_{e,i}}}$

Intermediate goods

Eq. 15	$x_t = \mathcal{A}_t \left\{ \alpha_c^{\frac{1}{\sigma_e}} \left(x_t^{kl}(h_t, \breve{k}_t)\right)^{\frac{\sigma_e - 1}{\sigma_e}} + (1-\alpha_c)^{\frac{1}{\sigma_e}} (e_t^x)^{\frac{\sigma_e - 1}{\sigma_e}} \right\}^{\frac{\sigma_e}{\sigma_e - 1}}$
Eq. 16	$x_t^{kl}(h_t, \breve{k}_t) = \left(\alpha_l^{\frac{1}{\sigma_{kl}}} h_t^{\frac{\sigma_{kl}-1}{\sigma_{kl}}} + (1-\alpha_l)^{\frac{1}{\sigma_{kl}}} \breve{k}_t^{\frac{\sigma_{kl}-1}{\sigma_{kl}}} \right)^{\frac{\sigma_{kl}}{\sigma_{kl}-1}}$ with $\breve{k}_t = u_t \frac{k_{t-1}}{\Gamma_t}$
Eq. 17	$e_t^x = \left(\frac{\psi_t}{p_t^{e,c}}\right)^{\sigma_e} \mathcal{A}_t^{\sigma_e - 1} (1-\alpha_c) x_t$
	with $\psi_t = \frac{1}{\mathcal{A}_t} \left[\alpha_c (\alpha_l w_t^{1-\sigma_{kl}} + (1-\alpha_l) z_t^{1-\sigma_{kl}})^{\frac{\sigma_e - 1}{\sigma_{kl} - 1}} + (1-\alpha_c)(p_t^e)^{1-\sigma_e} \right]^{\frac{1}{1-\sigma_e}}$
Eq. 18	$h_t = \left(\frac{\psi_t}{w_t}\right)^{\sigma_{kl}} \mathcal{A}_t^{\sigma_{kl}\left(\frac{\sigma_e - 1}{\sigma_e}\right)} x_t^{kl}(h_t, \breve{k}_t)^{\frac{\sigma_e - \sigma_{kl}}{\sigma_e}} \alpha_l (\alpha_c x_t)^{\frac{\sigma_{kl}}{\sigma_e}}$
Eq. 19	$\breve{k}_t = \left(\frac{\psi_t}{z_t}\right)^{\sigma_{kl}} \mathcal{A}_t^{\sigma_{kl}\left(\frac{\sigma_e - 1}{\sigma_e}\right)} x_t^{kl}(h_t, \breve{k}_t)^{\frac{\sigma_e - \sigma_{kl}}{\sigma_e}} (1-\alpha_l)(\alpha_c x_t)^{\frac{\sigma_{kl}}{\sigma_e}}$
Eq. 20	$x_t^f = (P_t^f / P_t^*)^{\frac{1}{\rho - 1}} (1-\omega^*) y_t^*$

Consumption

Eq. 21	$\lambda_{1,t} = (c_t^{PI} - \varrho c_{t-1}^{PI})^{-\sigma} - \varrho \beta E_t (c_{t+1}^{PI} - \varrho c_t^{PI})^{-\sigma}$
Eq. 22	$c_t^{ROT} = w_t h_t$
Eq. 23	$c_t = s_{PI} c_t^{PI} + (1 - s_{PI}) c_t^{ROT}$

Capital accumulation

Eq. 24	$\lambda_{1,t} = \lambda_{2,t} \left[\begin{array}{l} 1 - O_3 \left(\mathrm{e}^{O_1(\Im_t - 1)} + \frac{O_1}{O_2} \mathrm{e}^{-O_2(\Im_t - 1)} - \left(1 + \frac{O_1}{O_2} \right) \right) \\ -\Im_t O_1 O_3 \left(\mathrm{e}^{O_1(\Im_t - 1)} - \mathrm{e}^{-O_2(\Im_t - 1)} \right) \end{array} \right]$ $+ \lambda_{2,t+1} \Im_{t+1}^2 O_1 O_3 (\mathrm{e}^{O_1(\Im_t - 1)} - \mathrm{e}^{-O_2(\Im_t - 1)})$ with $\Im_t = i_t / i_{t-1}$
Eq. 25	$\lambda_{2,t} = \beta E_t \left(\lambda_{1,t+1} (z_{t+1} u_{t+1} - a(u_{t+1})) + \lambda_{2,t+1} (1 - \delta) \right)$
Eq. 26	$z_t = a_1 a_2 \mathrm{e}^{a_2 (u_t - 1)}$

Prices, inflation and wages

Eq. 27	$\hat{p}_t^d = \frac{\tau}{1 + \tau^2 \beta} \left(\beta E_t \hat{p}_{t+1}^d + \hat{p}_{t-1}^d + \beta E_t \hat{\pi}_{t+1}^c + \gamma \hat{\pi}_{t-1}^c - (1 + \gamma \beta) \hat{\pi}_t^c + \frac{(1-\tau)(1-\tau\beta)}{\tau} \hat{\psi}_t \right)$
Eq. 28	$\hat{\pi}_t^d = \hat{p}_t^d - \hat{p}_{t-1}^d + \hat{\pi}_t^c$
Eq. 29	$\hat{p}_t^m = \frac{\tau_m}{1 + \tau_m^2 \beta} \left(\begin{array}{l} \beta E_t \hat{p}_{t+1}^m + \hat{p}_{t-1}^m + \beta E_t \hat{\pi}_{t+1}^c + \gamma_m \hat{\pi}_{t-1}^c \\ -(1 + \gamma_m \beta) \hat{\pi}_t^c + \frac{(1-\tau_m)(1-\tau_m \beta)}{\tau_m} \widehat{rer}_t \end{array} \right)$ with $rer_t = \frac{s_t P_t^*}{P_t^c} = \frac{\Delta s_t}{\Pi_t^c} p_t^*$ and $p_t^* = \frac{s_{t-1} P_t^*}{P_{t-1}}$ and $\hat{p}_t^* = \hat{p}_{t-1}^* + \widehat{\Delta s}_{t-1} + \hat{\pi}_t^* - \hat{\pi}_{t-1}^c$
Eq. 30	$\hat{\pi}_t^m = \hat{p}_t^m - \hat{p}_{t-1}^m + \hat{\pi}_t^c$
Eq. 31	$\hat{\pi}_t^e = \hat{p}_t^e - \hat{p}_{t-1}^e + \hat{\pi}_t^c$
Eq. 32	$\hat{\pi}_t^c = \omega_{e,c} (\omega_c \hat{\pi}_t^{d,c} + (1 - \omega_c) \hat{\pi}_t^{m,c}) + (1 - \omega_{e,c}) \hat{\pi}_t^{e,c}$

Eq. 33	$\hat{p}_t^f = \frac{\tau_f}{1+\tau_f^2\beta}\begin{pmatrix} \beta E_t \hat{p}_{t+1}^f + \hat{p}_{t-1}^f + \beta E_t \hat{\pi}_{t+1}^* + \gamma_f \hat{\pi}_{t-1}^* \\ -(1+\gamma_f\beta)\hat{\pi}_t^* + \frac{(1-\tau_f)(1-\tau_f\beta)}{\tau_f}(\hat{\psi} - \widehat{rer}_t) \end{pmatrix}$
Eq. 34	$\hat{w}_t = \frac{1}{\Upsilon(1+\beta)}\hat{w}_{t-1} + \frac{\beta}{\Upsilon(1+\beta)}E_t\hat{w}_{t+1} + \frac{1}{\Upsilon(1+\beta)}\begin{pmatrix} \beta(E_t\hat{\pi}_{t+1}^c - \gamma_w\hat{\pi}_t^c) \\ -(\hat{\pi}_t^c - \gamma_w\hat{\pi}_{t-1}^c) \end{pmatrix}$ $+\frac{(1-\tau_w)(1-\tau_w\beta)}{(1-\tau_w)(1-\tau_w\beta)+\tau_w(1+\beta)\left(1-\nu\frac{h}{1-h}\frac{1}{\vartheta-1}\right)}\widehat{MRS}_t$ with $\widehat{MRS}_t = \widehat{\mathcal{U}}_t^\ell - \widehat{\mathcal{U}}_t^c = \nu\hat{h}_t\frac{h}{1-h} + \frac{\sigma}{1-\varrho}(\hat{c}_t - \varrho\hat{c}_{t-1})$ and $\Upsilon = \frac{(1-\tau_w)(1-\tau_w\beta)+\tau_w(1+\beta)\left(1-\nu\frac{h}{1-h}\frac{1}{\vartheta-1}\right)}{\tau_w(1+\beta)\left(1-\nu\frac{h}{1-h}\frac{1}{\vartheta-1}\right)}$

Asset prices and yields

Eq. 35	$\lambda_{1,t} = R_t^b\beta E_t\left(\frac{\lambda_{1,t+1}}{1+\pi_{t+1}^c}\right)$
Eq. 36	$\lambda_{1,t} = R_t^f\beta E_t\left(\Delta s_{t+1}\frac{\lambda_{1,t+1}}{1+\pi_{t+1}^c}\right)$
Eq. 37	$R_t^f = R_t^* + port_t - \iota\frac{\Pi_t^c}{\Delta s_t p_t^*}b_t^f$ with $b_t^f = s_t F_t / P_t^c$
Eq. 38	$\hat{i}_t^b = \rho_R\hat{i}_{t-1}^b + (1-\rho_R)(k_\pi\hat{\pi}_t^c + k_x\hat{x}_t + k_s\widehat{\Delta s}) + \varepsilon_t^{mon}$

Money demand

Eq. 39	$m_t = \left(\lambda_{1,t}\frac{i_t^b}{1+i_t^b}\right)^{-\frac{1}{\eta}}$

Definitions, constraints

Eq. 40	$x_t^d = x_t^{d,c} + x_t^{d,i} + x_t^{d,g}$
Eq. 41	$x_t^m = x_t^{m,c} + x_t^{m,i} + x_t^{m,g}$

Eq. 42	$e_t = \int_0^1 e_t^x(i)\mathrm{d}i + e_t^c + e_t^i + e_t^g$
Eq. 43	$x_t = x_t^d + x_t^f$
Eq. 44	$k_t = \Theta(i_t, i_{t-1}, k_{t-1}) + (1-\delta)k_{t-1}$ with $\Theta(i_t, i_{t-1}, k_{t-1}) = i_t - \varpi S(\mathfrak{I}_t)i_t - (1-\varpi)\frac{\varphi}{2}\left(\frac{i_t}{k_{t-1}} - \delta\right)^2 k_{t-1}$ and $S(\mathfrak{I}_t) = O_3\left(\mathrm{e}^{O_1(\mathfrak{I}_t - 1)} + \frac{O_1}{O_2}\mathrm{e}^{-O_2(\mathfrak{I}_t - 1)} - \left(1 + \frac{O_1}{O_2}\right)\right)$
Eq. 45	$b_t^f = \frac{\Delta s_t}{\Pi_t^c} b_{t-1}^f R_{t-1}^f + \frac{\Delta s_t p_t^*}{\Pi_t^c} p_t^f x_t^f - \frac{\Delta s_t p_t^*}{\Pi_t^c} x_t^m - p_t^e e_t - a(u_t)k_{t-1}$

Exogenous shocks

Eq. 46	$\mathcal{A}_t = \rho_{\mathcal{A}1}\mathcal{A}_{t-1} + \rho_{\mathcal{A}2}\mathcal{A}_{t-2} + \rho_{\mathcal{A}3}\mathcal{A}_{t-3} + \rho_{\mathcal{A}4}\mathcal{A}_{t-4}$ $+(1 - \rho_{\mathcal{A}1} - \rho_{\mathcal{A}2} - \rho_{\mathcal{A}3} - \rho_{\mathcal{A}4})\mathcal{A} + \varepsilon_t^{\mathcal{A}}$
Eq. 47	$g_t = \rho_{g1}g_{t-1} + \rho_{g2}g_{t-2} + (1 - \rho_{g1} - \rho_{g2})g + \varepsilon_t^g$
Eq. 48	$R_t^* = \rho_{R1^*}R_{t-1}^* + \rho_{R2^*}R_{t-2}^* + (1 - \rho_{R1^*} - \rho_{R2^*})R^* + \varepsilon_t^{R^*}$
Eq. 49	$\hat{\pi}_t^* = \rho_{\hat{\pi}^*}\hat{\pi}_{t-1}^* + (1 - \rho_{\hat{\pi}^*})\hat{\pi}^* + \varepsilon_t^{\hat{\pi}^*}$
Eq. 50	$y_t^* = \rho_{y^*}y_{t-1}^* + (1 - \rho_{y^*})y^* + \varepsilon_t^{y^*}$
Eq. 51	$p_t^e = \rho_{p^e}p_{t-1}^e + (1 - \rho_{p^e})p^e + \varepsilon_t^{p^e}$
Eq. 52	$port_t = \rho_{port}port_{t-1} + (1 - \rho_{port})port + \varepsilon_t^{port}$