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Robert Oleschak

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Robert Oleschak
Swiss National Bank (SNB)
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robert.oleschak@snb.ch

Abstract

In this paper, I analyse first-price single-item auctions in case of a default of a clearing agent in a central counterparty (CCP). The bidding surviving clearing agents attach a private value to the item to be sold and share eventual losses with the CCP. The CCP as auctioneer can choose the time of auction and the loss allocation mechanism in order to minimize her own losses.

I show that incentives (e.g. juniorising default fund contributions) are irrelevant for the outcome of the auction but that the composition of bidders matters. Auctions with a subset of bidders have distributional effects, i.e. the invited bidders are better off than those who are not invited to the auction. Conversely, inviting additional bidders (i.e., clients) could lead to an inefficient auction, yet their participation leaves the CCP as well as all the losing bidders better off. Recovery measures increase the safety and soundness of CCPs but can adversely affect incentives of a CCP in an auction. I show that in cases of extreme losses a CCP would rather prefer to wait than to swiftly conduct an auction, thereby inflicting costs on the financial system. Finally, I show that tear-ups are not only more costly than other recovery measures but that they fail to coordinate the actions of bidders, leading to an inferior equilibrium for all.[1]

Keywords

Central Counterparty, Default Management, Auctions, Recovery

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1 Introduction

A CCP reduces the propagation of shocks in a financial system by guaranteeing the performance of the financial contracts (e.g., derivatives) between two parties, even when one party at some point is not able or willing to meet her obligations. Should a default occur, then the CCP re-allocates all the contracts of the defaulting agent to the surviving agents. There are different ways in which a CCP can re-allocate the defaulting agent's contracts, for example, by selling them in a central market. However, in cases where the position to be transferred is large in relation to market liquidity or where a central market does not exist, auctions with the surviving agents as bidders is the mechanism of choice. In addition, CCPs can use auctions to incentivise the surviving agents to provide higher bids than the current market price in the central market and, thus, avoid fire sale conditions, as demonstrated by Vuillemey (2019). Potential losses that could occur as a result of the auction will typically be covered by the defaulting agent's collateral. Should the losses exceed the collateral, the CCP and the surviving agents step in to cover the losses based on a loss allocation arrangement. An incentive that some CCPs use is to juniorize the default fund contributions of those bidders who provided low or no bids at all (see CPMI-IOSCO (2019)). In this paper, I will analyse a simple version of a juniorization scheme where the winner does not lose her default fund contribution (but the losers do) and combine it with the loss allocation mechanism of a CCP. In addition, I analyse the bidding strategy in case the default fund is exhausted by defining how the residual losses are distributed.

Research Question and Approach

What are the CCP's and the agents' incentives in an auction? Do the incentives depend on whether the default fund is exhausted or not? Can a CCP incentivise bidders to provide higher bids and how does this affect the losses carried by the CCP and the other non-defaulting agents? Are the bidders willing to participate in such auctions or continue meeting their obligations? This paper seeks to provide answers to these questions.

What makes auctions conducted by CCPs particularly interesting and different from the standard literature on auctions is its loss allocation; depending on the size of the loss, the bidders might have to carry some of the losses themselves, i.e., the payoff of an agent does not need to be zero if she fails to win the auction.

Analysing CCP auctions is important for two reasons. First, CCPs have gathered some knowledge over the years on how auctions work in less extreme cases[2] but have no or very little experience in situations where the default fund is entirely used up. Second, with the regulatory push to mandate clearing, a broader group of agents use CCPs.[3] Auctions have been designed towards a smaller and more homogeneous group of participants. This is currently being rethought, including inviting

[2]For example, LCH and CME auctioned off Lehman's positions in 2008, or the auction at Nasdaq OMX in 2018 triggered by a default of a member.

[3]To reduce systemic risks in the OTC derivatives markets, the G20 announced in 2009 that standardized OTC derivative contracts should be cleared through CCPs: "All standardised OTC derivative contracts should be traded on exchanges or electronic trading platforms, where appropriate, and cleared through central counterparties by end-2012 at the latest. OTC derivative contracts should be reported to trade repositories. Non-centrally cleared contracts should be subject to higher capital requirements. We ask the FSB and its relevant members to assess regularly implementation and whether it is sufficient to improve transparency in the derivatives markets, mitigate systemic risk, and protect against market abuse." As a result, regulators throughout the world mandate the use of CCPs for OTC derivative contracts that are deemed sufficiently standardized (Board (2018) provides the latest progress report on implementation). The CCP's market share in OTC derivatives markets has rapidly grown in the last few years, mainly due to the interest rate and credit default swaps, according to Board (2017), and there is ample scope for foreign exchange, commodity, and equity OTC derivatives to be cleared by CCPs in the future Board (2017).

clients to the auction or using the loss allocation to encourage bidding.[4] A consistent and clear framework supports the discussion.

The approach taken is best described by way of an example supported by Figure 1, which depicts time on the x-axis and the value of the financial contract of the defaulted agent on the y-axis. Assume that an agent defaults at time D. The earliest date a CCP can conduct an auction is at $D + \eta$. As long as the CCP has not transferred the contract to another agent, she is subject to be marked to market gains or losses. In the example provided, the value of the contract falls between default D and auction $A \geq D + \eta$, leading to a holding loss of $v_D - v_A$ at the time of the auction. In addition, during the auction, the winning bidder might provide a bid $\beta^* < v_A$, which is lower than the value of the contract leading to an additional auction loss. The overall loss $v_D - \beta^*$ will be allocated by the CCP according to some pre-defined rules. Typically, the losses are first allocated to the defaulting agent (by using initial margins and default fund contribution). If that is not sufficient, then the CCP's share of equity (or skin in the game) is used. Finally, the CCP can use the surviving agents' remaining default fund and by applying additional recovery measures.

In this paper, I analyse the optimal bidding strategy given a loss allocation that covers all possible sizes of losses, consider whether the CCP can choose an optimal loss allocation to incentivise bidding and minimize her expected loss in the auction, whether recovery measures (e.g., usage of initial margins, cash calls, variation margin haircuts and tear-ups) can cover any possible size of losses, and what the optimal time of auction A from the viewpoint of the CCP is. Finally, I analyse auctions with a subset of agents or where a tear-up of contracts looms if the auction fails to provide a minimum prize.

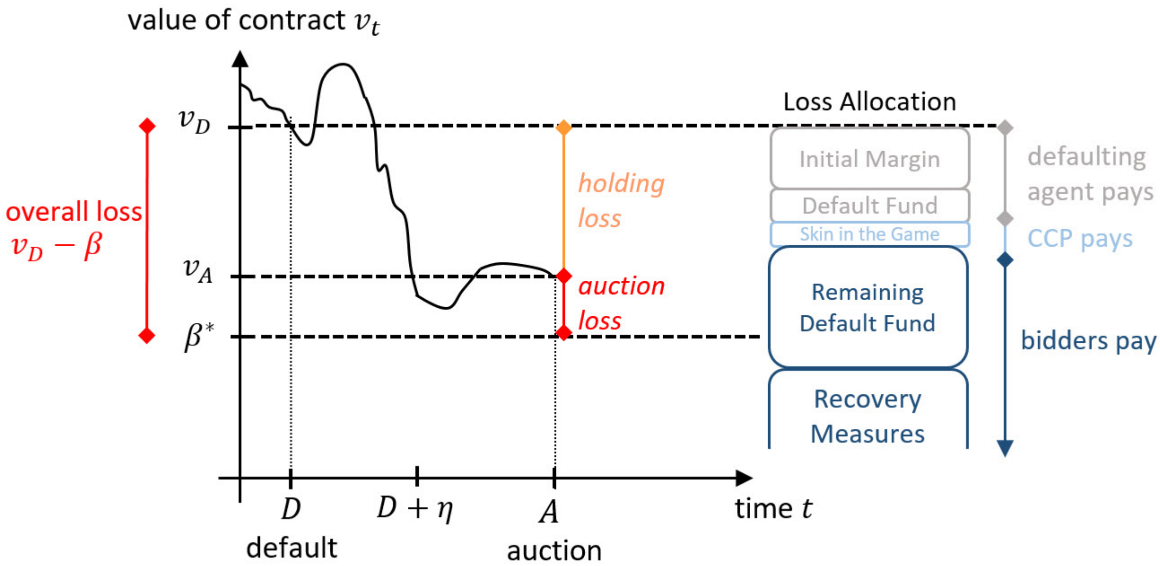

Figure 1: Auction Loss and Loss Allocation

Main Findings

First, I show that incentives do not affect the outcome of the auction. A CCP can increase the bids of the surviving agents by making the winner pay less for the auction loss. This incentive can be thought of as a simple juniorization of default fund contributions. The larger the loss that

[4] A recently published discussion paper CPMI-IOSCO (2019) aims at facilitating the sharing of existing practices and considerations that CCPs might want to take into account when designing an auction.

the winning agent can avoid, the higher the equilibrium price that the bidders submit. However, higher bidding does not translate into lower losses. This is true for both the CCP and the surviving agents because the gain of receiving higher bids is entirely consumed by the transfer to the winning bidder.

Second, I show that the composition of agents in an auction can have a material impact on the outcome of the auction. Conducting auctions with a subset of agents has distributional effects because the agents invited to the auction are always better off than those who are not. The result crucially depends on how good the CCPs are at picking agents with high private values. Conversely, inviting additional agents to the auction (e.g., clients) could be inefficient (in the sense that the bidder with the highest private value does not need to win) but the CCP and all losing bidders are better off when additional bidders are invited to the auction.

Third, I show that recovery measures can affect the CCP's incentives in an auction. A CCP can design a loss allocation arrangement where the maximal loss is restricted to a share of her equity (typically required by regulators and called skin-in-the game) and that any losses beyond that can be allocated to the surviving agents (by using exhaustive recovery measures). Importantly, the recovery measures can be designed in such a way that the surviving agents are not only willing to participate in an auction but, once the winning bid and losses are announced, are willing to meet all their obligations. Because the maximal loss of a CCP is capped, the CCP might in cases of high losses prefer to wait until the value of the contract has eventually rebounded rather than choosing to swiftly conduct an auction. In such extreme events, the governance arrangements of a CCP can be very crucial.

Finally, I show that tear-ups (partial or full) inflict higher costs on the financial system compared to other recovery measures, such as cash calls, variation margin haircuts or usage of initial margins. Thus, agents prefer to share the necessary costs in order to avoid a tear-up. However, the auction fails to coordinate the actions of the bidders, leading to an inferior equilibrium for all. Tear-ups should be avoided.

Relevant Literature

This paper draws on the well-established auction theory developed by many authors dating back to Vickrey (1961). The literature on all-pay auctions (for example Baye et al. (1993) on interest group lobbying) resembles the problem of a CCP auction in that the losers might face negative payoffs. However, there is a crucial difference from the CCP auction in that the losing bidder's payment depends on the bids of others and not on her own bid.

Ferrara and Li (2017) apply the auction theory to CCPs. In their paper, the authors analyse CCP auctions under static penalty schemes and note the inefficiencies that such an approach can have. Huang and Zhu (unpublished) study the incentives of bidders in a uniform-price CCP auction and find that juniorization of the guarantee fund contributions can elevate the equilibrium price.

The present paper extends the literature on CCP auctions in three important ways. First, I study not only the incentives of the bidders in an auction but also those of the CCP. This allows to identify potential moral hazard problems inherent in a CCP. I show that in case of extreme losses, a CCP might not act in the interest of her participants, which is discussed for example, by Bignon and Vuillemey (2017) too. Second, I consider auctions at all levels of losses, including where recovery measures become necessary. This allows to analyze the compatibility of different recovery measures with the auction format. An important new result presented in this paper is that tear-ups do not incentivise the bidders to provide higher bids, even though all bidders would be willing to pay to avoid this recovery measure. Third, I explicitly formulate the CCPs budget as well as the auction participants' participation and termination constraint thereby extracting additional insights, including the interaction between participating and non-participation agents as well as the losses incurred on the whole financial system. An important additional insight presented in this paper is the irrelevance of incentives, i.e. that incentives increase the size of the bids but do not affect the losses of the CCP or of the bidders. The notation used in this paper is based on Krishna (2010).

More generally, the literature on central clearing has grown rapidly following the financial crisis of 2008. One branch of studies focuses on the systemic aspect of CCP clearing by comparing counterparty risks in an environment where financial contracts are netted bilaterally between financial institutions (bilateral clearing) with the situation where CCPs allow for multilateral netting of traded financial contracts (central clearing). For example, Duffie and Zhu (2011),Cont and Kokholm (2014), and Lewandowska (2015) analyse whether central clearing can reduce counterparty risks compared to bilateral clearing. Another branch of papers focuses on the disincentives present in markets cleared by CCPs. Huang (2019), for example, assesses the appropriate size of collateral and equity from the viewpoint of a profit-oriented CCP subject to limited liability or Bignon and Vuillemey (2017) show, based on historical empirical evidence, that CCPs might have in the past delayed declaring an agent into default and in hopes of a resurrection. Other papers (for example Angelo et al. (2019) or Arora et al. (2012)) analyse the effects of mandating CCP clearing on interest rates or credit markets.

In the following chapter, I provide a short description of CCPs, with a focus on the loss allocation. In Chapter 3, I characterize the model and state the CCP's and bidders' maximization problem subject to the CCP's loss allocation and formulate the constraints. In Chapter 4, some examples of loss allocation are discussed and the optimal bidding strategy is formulated. The general results are presented in three separate chapters, as follows: the optimal loss allocation (chapter 5), the completeness of loss allocation and asymmetric auctions (chapter 6) and the optimal time of auction (chapter 7). Chapter 8 contains some additional considerations, including auctions with a subset of agents and auctions against the background of a tear-up of contracts if the auction fails to provide a minimum prize. Chapter 9 closes with conclusions. The appendix contains all relevant proofs.

2 A Primer on Central Counterparties

A CCP is "an entity that interposes itself between counterparties to contracts traded in one or more financial markets, becoming the buyer to every seller and the seller to every buyer and thereby ensuring the performance of open contracts." (CPSS (2016)). A CCP consists of a set of rules and procedures defining i) how the liabilities arising from these contracts are measured and covered with the agent's collateral, ii) how these contracts will be re-allocated if one agent is unable (or unwilling) to fulfil them and iii) how losses will be allocated that might arise during the transfer of the positions to a new agent.[5] I will briefly discuss each element in turn.

Measures liabilities and requires collateral: The CCP continuously measures each agent's current and future liability arising from all contracts submitted for clearing. To limit and manage its counterparty risk, a CCP requires from all trading agents variation margin, initial margin and

[5] These sets of rules and procedures have evolved over more than a hundred years and differ substantially (see for example Kroszner (1999)). Recent regulatory initiatives (mainly the Principles for Financial Market Infrastructure CPMI-IOSCO (2012)) have, however, contributed to a homogenization of the CCP's rules and procedures.

default fund contributions. The *variation margins* α is a cash payment from the agent with a positive liability to the agent with a negative liability. Variation margins, once paid, ensure that the current net liability of each agent is set to zero. The sizes of the variation margin payments cannot be predicted since they are a function of a stochastic market variable. In case an agent stops paying variation margins (and subsequently defaults), it might take time for the CCP to transfer the contract to another agent. Any losses resulting from changes in the valuation of the contract during the transfer period are covered by the other two types of collateral. The *initial margins* γ needs to be paid by both agents involved in the transaction once a contract is submitted to the CCP for clearing. The initial margins of a defaulting agent can be used to cover losses that might arise during the transfer of the positions to another agent.[6] Finally, the *default fund contribution* δ of each agent can be used to cover the losses arising from its own default but - as opposed to initial margins - from the default of another agent also. The default fund D is the sum of all default fund contributions $D = \sum_i \delta_i$.[7]

Re-allocates defaulter's positions: A CCP can re-allocate the defaulting agent's position by way of an auction to the highest bidder. To cover the losses that might arise as a result of the default and the following re-allocation, CCPs have defined rules regarding how losses will be allocated.

Order	**Who Pays What**	**Notation**	Auction View
1	Defaulting Agent's Initial Margins	γ	
2	Defaulting Agent's Default Fund Contribution	δ	Somebody else pays
3	CCP's Share of Equity (Skin in the Game)	ϵ	
4	Surviving Agents' Default Fund Contribution	$D - \delta$	
5	Surviving Agents' Committed Additional Resources (Recovery Measures) including Cash Calls, use of Initial Margins, and Variation Margin Haircuts	-	Bidders pay
6	CCP's Remaining Equity	$1 - \epsilon$	

Table 1: Loss Allocation of a CCP

Allocates Losses: There are three aspects to be considered. First, the CCP defines the order of who has to pay what (see Table 1), which is often coined as the default waterfall; typically the initial margin γ and default fund contribution δ of the defaulting agent and then the CCP's share of equity ϵ (or skin in the game) are used first. If that is not sufficient, then the surviving agents' default fund contributions and additional recovery measures are used. At the end of the waterfall, the CCP's remaining equity $1-\epsilon$ will be used. Second, in cases where the surviving agents have to pay, the CCP can set up rules regarding how the losses are to be spread across the surviving agents and connect it to the outcome of an auction. For example, the CCP can equally share all losses or juniorise the default fund contribution by making the winner of the auction pay less compared the losers. Third, if the losses are covered by the first three layers, then from the viewpoint of a bidder, it is an auction where "somebody else pays" for the losses. It is only when the losses

[6] To be precise and connect it to the auction, the initial margins aim to cover losses that might arise in the period between market price v_D, i.e., when the defaulting participant paid its last variation margin, and any market price that might arise at the time of the auction v_A plus any auction losses with a certain degree of confidence. The modelling of the initial margins of a CCP is based on highly sophisticated risk models, which are not discussed in detail here.

[7] CCPs usually define the overall size of the default fund based on the overall level of risks that they assumed. The size of the default fund is usually based on some stress tests.

exceed $\gamma + \delta + \epsilon$ that the "bidders pay" auction point of view occurs. In such a case, the bidders can either lose their default fund contribution or have to commit additional resources, including cash calls, use of initial margins, or variation margin haircuts. These so called recovery measures are treated in detail in chapter 6.

3 The Model

The model consists of one continuous-time period, where $t \in [0, 1]$ depicts time. The economy is populated by $N+1$ risk-neutral agents holding two types of illiquid risky assets and one risk-neutral CCP. On the date $t = 0$, the agents can agree on a financial contract mandatorily cleared by a CCP. On the date $t = D$, one trader unexpectedly defaults due to an external valuation shock. The CCP can conduct an auction at time $A \in [D + \eta, 1)$, i.e., at the earliest, $D + \eta$ after default or at the latest, right before the expiration $t = 1$ to re-allocate the contract of the defaulted party. The losses will be shared based on a pre-agreed loss allocation arrangement. On the date 1, the payoffs of all risky assets are determined and, thus, all uncertainty is resolved.

3.1 Endowment and Cleared Market

On the date $t = 0$, the CCP is endowed with one unit of equity owned by third-parties and each agent is endowed with cash m and a risky asset. Let $B(t) = B_t$ follow some continuous-time stochastic martingale process where $B_0 = 0$ and $E_t[B_1] = B_t$. One half of the agents' (type 1) risky asset yields $\pi + B_1$ at $t = 1$, where π is a fixed return and B_1 is the final value of the continuous-time stochastic process B_t. The other half of the agents' (type 2) risky asset yields $\pi - B_1$, i.e., there is no aggregate risk because the aggregate supply of risk is zero.[8] Both types cannot dispose of the asset even if $\pi + B_t < 0$ for type 1 and $\pi - B_t < 0$ for type 2.
The agents carry private cost c_i of holding the risky part B_t of the asset, which can be interpreted as the cost of managing the market risk or capital costs.
Each type 1 agent is randomly matched with another type 2 agent at a central market where they trade a contract and subsequently clear with a CCP.[9] I formalize this contract as follows: type 1 will pay B_1 to type 2 at time 1. This contract shields both agents involved in the trade from the market risk B_t and so the cost c_i can be avoided.
Since this contract is cleared, each agent i commits to the CCP to i) to pay the variation margin α_i throughout the lifetime of the contract,[10] ii) pay the initial margin γ and default fund contribution δ at $t = 0$, and iii) share any losses based on a pre-agreed loss allocation arrangement, as depicted in Table 1. For each cleared contract, the agents bear uniform clearing costs k, which can be interpreted as the likelihood of an agent losing her default fund contribution or having to meet other obligations. I take all parameters as given and do not provide for the optimal size of collateral or equity.[11]

[8] The risky asset can be interpreted as a project that the agent has invested in and cannot trade with a fixed return and a variable return, which depends on some stochastic market value, for example, the interest or exchange rate.
[9] The agents could trade without a CCP. I assume that trading these contracts is subject to mandatory clearing.
[10] From the viewpoint of type 2, her current liability is $-B_t$; therefore, the value of the contract at time t from her point of view is $v_t = B_t$. To keep the current liability at zero at all times, the variation margin (again from the viewpoint of type 2) must be defined as $\alpha_t = -B_t$.
[11] Note, that the size of the initial margin γ and default fund contribution δ required for each contract cleared, CCP's equity and recovery measures that the CCP has at hand, as well as the order in which these financial resources would be used are given. However, even though I do not model the optimal size of the initial margin and default fund, there is an implicit trade-off that the CCP needs to consider. By increasing the required financial resources from her clearing agents, the CCP reduces the likelihood of her share of equity ϵ being used but increases the clearing

An agent i will prefer to trade the contract on a centrally cleared market as long as $k \leq c_i$. To keep the analysis simple, I will set $k = 0$ and assume that the agent always has sufficient cash to meet any variation margin payment α_t at any time.

3.2 Default Management Auction Problem

At time $t = D$, a type 2 agent is hit by an exogenous negative valuation shock and, as a result, the agent stops paying the variation margins and subsequently defaults. The value of the contract at the time that the defaulting agent paid its last variation margin was $v_D = E_D[B_1] = B_D$. The CCP can conduct a single-item first-price auction at any time $A \in [D + \eta, 1)$ to re-allocate the contractual obligations of the defaulting party to one of the remaining non-defaulting N agents. The value of the auctioned contract $v_A = E_A[B_1] = B_A$ is publicly observable.
The private value $x_i = v_A - c_i$ that a bidder i attaches to the object at the time of the auction A is the value of the contract v_A minus the private costs c_i of holding the (additional) auctioned contractual obligation. The cost c_i re-appears because any agent holding an additional contract would not be optimally hedged any longer and would hold the risk B_t on her books. The distribution of the cost $c_i \sim U[0, 1]$ is i.i.d. across agents and is common knowledge. Therefore, from the viewpoint of the CCP, the private value of each bidder is uniformly distributed on the interval $x_i \sim U[v_A - 1, v_A]$. The corresponding cumulative distribution function is $F(x_i) = (x_i - (v_A - 1))$ with density $f(x_i) = 1$.

The CCP's Maximization Problem

Given that an agent has defaulted, the CCP conducts an auction to minimize the expected loss of equity L_{CCP} by choosing the optimal time of auction A, and loss allocation $\langle \mathcal{W}(\cdot), \mathcal{L}(\cdot) \rangle$, as defined in equation (2), as follows:[12]

$$L_{CCP} = \begin{cases} 0 & \text{if } v_D - \beta(v_A - 1) \leq \gamma + \delta \\ \epsilon & \text{if } v_D - \beta(v_A) \geq \gamma + \delta + \epsilon \\ (0, \epsilon) & \text{if } x_i \in [v_A, v_{A-1}] \text{ where } \gamma + \delta < v_D - \beta(x_i) < \gamma + \delta + \epsilon \end{cases} \tag{1}$$

where $\beta(y)$ is the equilibrium bidding strategy of the winning bidder with private value y. The above equation simply states that the expected loss of a CCP conducting an auction at time A can have three states, as follows: either her share of equity will not be lost (first line), will definitely be lost (second line), or some of the equity will be lost (third line).

The Agent's Maximization Problem

I will now define the equilibrium bidding strategy β of the bidders. The bidder i providing bid b_i faces the following payoffs where $b_{-i} = \max b_{j \neq i}$ is the best bid of the other $N - 1$ bidders:

$$u_i = \begin{cases} x_i - b_i - \mathcal{W}(b_i) & \text{if } b_i > b_{-i} \\ -\mathcal{L}(b_{-i}) & \text{if } b_i < b_{-i} \end{cases} \tag{2}$$

cost k. Therefore, some agents would no longer find it profitable to trade the contract with the CCP if $k > c_i$.

[12] To simplify the analysis, I will generally assume that all agents participate in the auction. Section 8.1 discusses the case where not all agents are invited to the auction.

Note that ties where $b_i = b_{-i}$ will occur with zero probability, so I will ignore it. The payoff to the bidder i is $x_i - b_i - \mathcal{W}(b_i)$ if she wins and is $-\mathcal{L}(b_{-i})$ if she loses.[13] $\mathcal{W}(b_i)$ defines the loss or profit a CCP will inflict on bidder i with the bid b_i given that she has won. $\mathcal{L}(b_{-i})$ defines the loss or profit a CCP will inflict on bidder i given that bidder $-i$ with bid b_{-i} has won. The loss allocation as described in section 2, including all possible recovery measures, can be analysed in this framework.

Given that the other bidders follow the symmetric, increasing, and differentiable equilibrium strategy β, bidder i maximizes the following expected profit by choosing the optimal bid b_i as follows:

$$\pi(x_i, b_i) = G(\beta^{-1}(b_i))(x_i - b_i - \mathcal{W}(b_i)) - \int_{\beta^{-1}(b_i)}^{v_A} \mathcal{L}(\beta(y))dG(y) - d(A)\xi_i \tag{3}$$

where $G(y) = F(y)^{N-1}$ denotes the distribution of the highest order statistics of the remaining $N-1$ bidders that bidder i is competing against.[14] If the bidder wins, then her expected profit is given by the first expression. If she loses, then her expected loss is given by the integral of the loss function and the corresponding distribution of the highest bid. The last expression expresses costs $\xi_i \leq c_i$, which reflects the agents indirect exposure towards market risks should the CCP decide not to hold the auction at the earliest point $\bar{A} = \eta + D$ but to delay it. Therefore $d(A = \bar{A}) = 0$ and $d(A > \bar{A}) = 1$.

Forming the first-order condition and noting that in equilibrium $b_i = \beta(x_i)$, I obtain the following:

$$G(x_i)\beta'(x_i)\Big(1 + \mathcal{W}'(\beta(x_i))\Big) + g(x_i)\Big(\beta(x_i) + \mathcal{W}(\beta(x_i)) - \mathcal{L}(\beta(x_i))\Big) = x_i g(x_i) \tag{4}$$

The equation above is the basis for calculating the equilibrium bidding strategy and the expected CCP's auction loss. Note that even though in (2) the payoff $\mathcal{L}(b_{-i})$ depends on the winning bid of somebody else, which cannot be known to the bidder in advance, the first-order condition only requires her own bid $\beta(x_i)$ as the input. Although no solution to this general form can be presented, I can note some general properties. Since β is increasing, and $G(x_i)$ as well as $g(x_i)$ are positive but fixed for agent i, there are two ways that the optimal bidding strategy can increase. First, if increasing the bid leads to a lower payment $\mathcal{W}'(\beta(x_i)) < 0$ and, second, if the losses if I lose are bigger than the losses if I win, i.e., $\mathcal{W}(\beta(x_i)) - \mathcal{L}(\beta(x_i)) < 0$.

Constraints

There are *three types of constraints* that must be imposed on loss allocation $\langle \mathcal{W}(\cdot), \mathcal{L}(\cdot) \rangle$.

First, the overall loss has to be accurately covered. In the case where the surviving agents are paying at the margin, the *budget constraint* can be written for any winning bid β^* as follows:

$$\underbrace{(v_D - \beta^*)}_{\text{loss}} - \underbrace{(\gamma + \delta + \epsilon)}_{\text{used collateral}} = \underbrace{(N-1)\mathcal{L}(\beta^*) + \mathcal{W}(\beta^*)}_{\text{loss allocation}} \tag{5}$$

Second, the CCP cannot force a bidder to participate in an auction that offers her less expected utility with the optimal bidding strategy β than when not participating. The *participation constraint* is satisfied if the expected profit of participating is at least as large as the expected profit

[13] In this paper, I use a special case of the loss allocation $\langle \mathcal{W}(\cdot), \mathcal{L}(\cdot) \rangle$, where only my bid b_i and the highest bid of all the other bids b_{-i} define the loss allocation. A more general function would require as input all bids; thus, $\mathcal{W} : R^N \to R$. Such a function would be more difficult to manage since it would require the handling of the 2nd, 3rd,..., Nth-highest order statistics.

[14] For a detailed discussion of the order statistics, see Krishna (2010), Appendix C.

of staying out of the auction, as follows:

$$\pi(x_i) \geq - \underbrace{\int_{v_A-1}^{v_A} \mathcal{L}(\beta(y))dG(y),}_{\text{expected loss when not participating}} \qquad \forall i \tag{6}$$

Third, the agent could - once the outcome of the auction has been determined - decide to leave the CCP for good to avoid sharing in the losses. A CCP does not allow agents to terminate contracts during a default and has contractual powers to inflict losses l on any leaving agent. The CCP could return the agent's collateral γ and δ only after subtracting any losses that the agent contractually agreed to share or she could refuse to pay the variation margins α to the leaving agent. In addition, the leaving agent would suffer private costs $c_i \geq 0$. Therefore, to guarantee that ex post no bidders prefer to leave the CCP after any winning bid $\beta^*(x_i)$ has been announced, the following *termination constraint* must be satisfied:

$$\mathcal{L}(\beta^*(x_i)) \leq c_{-i} + l, \text{ where } c_{-i} = \min c_{j \neq i} \tag{7}$$

The exact form of l will depend on the losses a CCP can inflict on a leaving agent and will be defined in chapter 6.

Definition 1 *A loss allocation $\langle \mathcal{W}(\cdot), \mathcal{L}(\cdot) \rangle$ that satisfies all three constraints in (5), (6), and (7) for all possible sizes of holding losses $v_D - v_A$ is complete.*

3.3 Efficient Allocation

Assuming that the planner's objective is to maximize the unweighted sum of agents' and the CCP's utilities, the obvious solution to the social planners problem is to allocate the position to the agent with the highest private value $x_h = \max c_i$ immediately after the default has occurred. Any delay in auctioning off the position inflicts costs on the financial system $\sum_i \xi_i$. An inefficient auction where the position is allocated to an agent with a lower private value x_l reduces aggregated utility by $x_h - x_l$.

4 Optimal Bidding: Examples

I now turn to the detailed analysis of the bidder's behaviour and consider the case where somebody else pays (section 4.1), as well as two examples of a loss allocation when the bidders pay at the margin (section 4.2). Additionally, I analyse situations where it is ex ante not clear whether bidders will have to pay or not (section 4.3). In the last section 4.4, I compare all loss allocation regimes. In all cases, a type 2 agent defaults. Finally, I will use the following definition for an optimal bidding strategy.

Definition 2 *The optimal bidding strategy $\beta(x_i)$ for a bidder with private value x_i is such that it maximizes (3) given the time of the auction A, the number of bidders N, and the loss allocation $\langle \mathcal{W}(\cdot), \mathcal{L}(\cdot) \rangle$ subject to budget condition (5).*

The participation and termination constraints will not be considered at this point. A full treatment of all conditions, including their completeness, is provided in chapter 6.

4.1 Somebody Else Pays

The auction starts with the observation that the collateral of the defaulting agent and the CCP's share of equity are sufficient to cover the losses resulting from even the lowest possible bid:

$$v_D - \beta(v_A - 1) \leq \gamma + \delta + \epsilon$$

This includes the scenario where the position of the defaulting agent carries a profit. Bidder i's payoffs are expressed in equation (2), where the bidders do not share any losses, so $\mathcal{W}(\cdot) = \mathcal{L}(\cdot) = 0$. The first-order condition

$$G(x)\beta'(x) = g(x)(x - \beta(x))$$

states that the expected marginal cost of increasing the bid (lhs) must equal the marginal profit (rhs). The optimal bidding strategy can be expressed as follows:

$$\beta(x_i) = \frac{N-1}{N}x_i + \frac{1}{N}(v_A - 1) \tag{8}$$

The bidders provide quotes that are lower than their private values. The bids approach their private value as the number of bidders N increases.
The expected profit of a bidder with private value x_i is as follows:

$$\pi(x_i) = \frac{(x_i - (v_A - 1))^N}{N} \geq 0$$

Inviting more agents to the auction increases optimal bidding and reduces the expected profit of the bidders. Finally, the expected loss of a CCP is zero $L_{CCP} = 0$.
The result is very standard. I will use this bidding strategy as a benchmark.

4.2 Losses Covered by Bidders

In case the value of the position v_A is such that the best possible bid exhausts the defaulter's collateral and the CCP's share of equity, as follows:

$$v_D - \beta(v_A) > \gamma + \delta + \epsilon \tag{9}$$

then the bidders will share the losses at the margin and the expected loss of a CCP is her full share of equity $L_{CCP} = \epsilon$. In the following, I will discuss two loss allocation arrangements subject to budget condition (5).

Example 1: Equal Loss Allocation

The equal loss allocation shares the losses equally among all surviving agents, i.e., for any winning bid β_e^*, I have that $\mathcal{W}_e(\beta_e^*) = \mathcal{W}(\beta_e^*) = \mathcal{L}(\beta_e^*) > 0$. Given the budget constraint in (5), the loss allocation function can be expressed as $\mathcal{W}_e(\beta_e^*) = \frac{(v_D - \beta_e^*) - (\gamma + \delta)}{N}$. The first-order condition, which is as follows:

$$\frac{N-1}{N}G(x)\beta_e'(x) = g(x)(x - \beta_e(x)) \tag{10}$$

and the optimal bidding strategy

$$\beta_e(x) = \frac{N}{N+1}x + \frac{1}{N+1}(v_A - 1) \tag{11}$$

leads to higher bidding compared to the benchmark, which can be best explained by the first-order condition in (10). While the expected cost of increasing the bid has been lowered by the factor $\frac{N-1}{N} < 1$, the expected marginal profit remains the same.
Finally, the expected profit of a bidder is lower compared to the benchmark model, as follows:

$$\pi_e(x_i) = \pi(x_i) - \frac{1}{N}\big((v_D - v_A) - (\gamma + \delta) + \frac{2}{N+1}\big)\big)$$

Example 2: Losers Loss Allocation

This loss allocation shares losses equally among the losing bidders - the winning bidder does not share any losses, i.e., for any winning bid β_w^*, I have $\mathcal{W}_w(\beta_w^*) = 0$, and $\mathcal{L}_w(\beta_w^*) > 0$. Given the budget constraint, the losing bidders pay $\mathcal{L}_w(\beta_w^*) = \frac{(v_D - \beta_w^*) - (\gamma+\delta+\epsilon)}{N-1}$. Inserting this into the first-order condition (4), I obtain the following:

$$G(x)\beta_w'(x) = g(x)(x - \beta_w(x) + \mathcal{L}(\beta_w(x))) \tag{12}$$

leading to the optimal bidding strategy, as follows:

$$\beta_w(x_i) = \underbrace{\frac{N-1}{N+1}x_i + \frac{N-1}{N(N+1)}(v_A - 1)}_{\frac{N-1}{N}\beta_e(x)} + \frac{v_D - (\gamma + \delta + \epsilon)}{N} \tag{13}$$

The expected profit of a bidder in the case that the losers' loss allocation has the same value as the profit of a bidder in the equal sharing function, i.e., $\pi_w(x_i) = \pi_e(x_i), \forall i$.

4.3 Overlapping Losses

Consider that there is a bid $\beta_e(\psi) = \sigma$ with $\psi \in (v_A - 1, v_A)$ where the defaulting agent's collateral as well as the CCP's equity share is used up, i.e.,

$$v_D - \sigma = \gamma + \delta + \epsilon$$

Given the equal loss allocation $\mathcal{W}_e(\cdot)$, bidders face the following payoff:

$$u_i = \begin{cases} x_i - b_i & \text{if } b_i > b_{-i} \wedge b_i \geq \sigma \\ 0 & \text{if } b_i < b_{-i} \wedge b_j \geq \sigma \\ x_i - b_i - \mathcal{W}_e(b_i) & \text{if } b_i > b_{-i} \wedge b_i < \sigma \\ -\mathcal{W}_e(b_{-i}) & \text{if } b_i < b_{-i} \wedge b_j < \sigma \end{cases} \tag{14}$$

which simply means that as long as the winning bid is equal to or higher than σ, the bidders do not pay (because the defaulter's collateral is sufficient) or else they pay $\mathcal{W}_e(\cdot)$. I will solve the problem by backward induction.
First, for those bidders whose optimal equilibrium bid lies below σ, only the last three lines in equation (14) apply. The optimal bid b maximizes the following expected profit (given that the other bidders follow some equilibrium strategy β_D):

$$\pi(x, b_i) = G(\beta_D^{-1}(b_i))(x_i - b_i - \mathcal{W}_e(b_i)) - \int_{\beta_D^{-1}(b_i)}^{\psi} \mathcal{W}_e(\beta_D(y))dG(y)$$

Since the bidder can only influence his own likelihood of winning and the lower bound of the integral, the first-order condition as well as the optimal bid are identical to equations (10) and (11).
Second, for any bidder whose bid is higher than or equal to σ, only the first two lines of the payoff function apply. Therefore, she needs to find the optimal bid to maximize the following profit:

$$\pi(x, b_i) = G(\beta_D^{-1}(b_i))(x - b_i) \tag{15}$$

To find the optimal bidding strategy, special attention needs to be paid to the transition from bids below and above σ. The following reasoning is supported by Figure (2); starting with the private value $v_A - 1$, the optimal bidding strategy follows $\beta_D = \beta_e$, as expressed in (11), until the private value $\psi = \beta_e^{-1}(\sigma)$ is reached, which constitutes the boundary between usage of the default fund or not. At this point, i.e., A in the figure, the bidder's loss allocation function drops to zero ($\mathcal{W}_e = 0$) and the optimal bidding-curve in such a case would be β, as expressed in (8). However, jumping to point B is not an option since the bidder would re-enter the range where the loss allocation function is not zero and she would immediately jump back to the bidding curve β_e and so forth. Moving horizontally to point C and then following the β bidding curve is not optimal either since any bidder on this vertical line could increase her likelihood of winning by increasing her bid by a small amount and be better off. The solution is, therefore, a convergence to the β bidding curve, as expressed by the following equation:

$$\beta_D(x_i) = \begin{cases} \frac{N-1}{N}x_i + \frac{1}{N}(v_A - 1) + \kappa & \text{if } \beta_D \geq \sigma \\ \beta_e(x_i) & \text{if } \beta_D < \sigma \end{cases} \tag{16}$$

where $\kappa = \frac{1}{N(N+1)} \frac{(\psi-(v_A-1))^N}{(x-(v_A-1))^{N-1}}$.
The implication is that all bidders, even those who provide bids that will not use the default fund, increase their bids.
Finally, it is easy to show that the bidding curve β_w must intersect the bidding curve β_e at point A, so that β_D for $\beta_D \geq \sigma$ follows the same path, independent of the loss allocation arrangement (as depicted on right panel in Figure 3). Take equation (13), where $\beta_w(\psi) = \frac{N-1}{N}\beta_e(\psi) + \frac{v_D-(\gamma+\delta+\epsilon)}{N}$, inserting $v_D - (\gamma + \delta + \epsilon) = \beta_e(\psi)$, I obtain $\beta_w(\psi) = \beta_e(\psi)$, which proves that bidding curves β_e and β_w intersect at $x_i = \psi$.

4.4 Comparison of the Loss Allocation Functions

A comparison of the optimal bidding strategies shows the following pattern.
First, the left panel of Figure 3 shows that for each private value $x_i \in [v_A - 1, v_A]$, I have that $\beta \leq \beta_e < \beta_w$, where β plots the optimal bidding curve in case the defaulting agent or the CCP covers the marginal costs (somebody else pays from the viewpoint of the bidders), β_e tracks the optimal bidding curve in case the default fund of the bidders is used but where the auction loss is shared equally among the bidders, and, finally, β_w is the optimal bidding where only the losing bidders cover the marginal auction losses. β_w leads to aggressive bidding of all parties, especially of those with low private values (or high costs c_i).
Second, the right panel of figure 3 shows that the above result is only valid as long as the bids lead to the bidders marginally sharing the loss, i.e., where $v_D - \beta > \gamma + \delta + \epsilon$. As soon as the bids enter the space where somebody else (the CCP) pays, then $\beta_w = \beta_e$, i.e., the bidders provide the same bid,

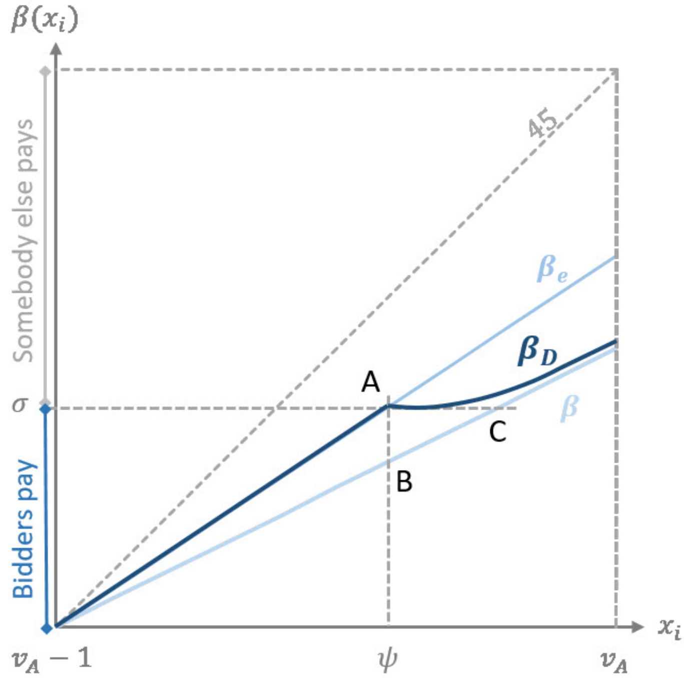

Figure 2: Optimal Bidding β_D when Bidders Might Have to Pay

independent of the loss allocation, which means that the CCP's expected $L_{CCP}(\beta_e) = L_{CCP}(\beta_w)$ in equation (1) is equal for both loss allocation arrangements.

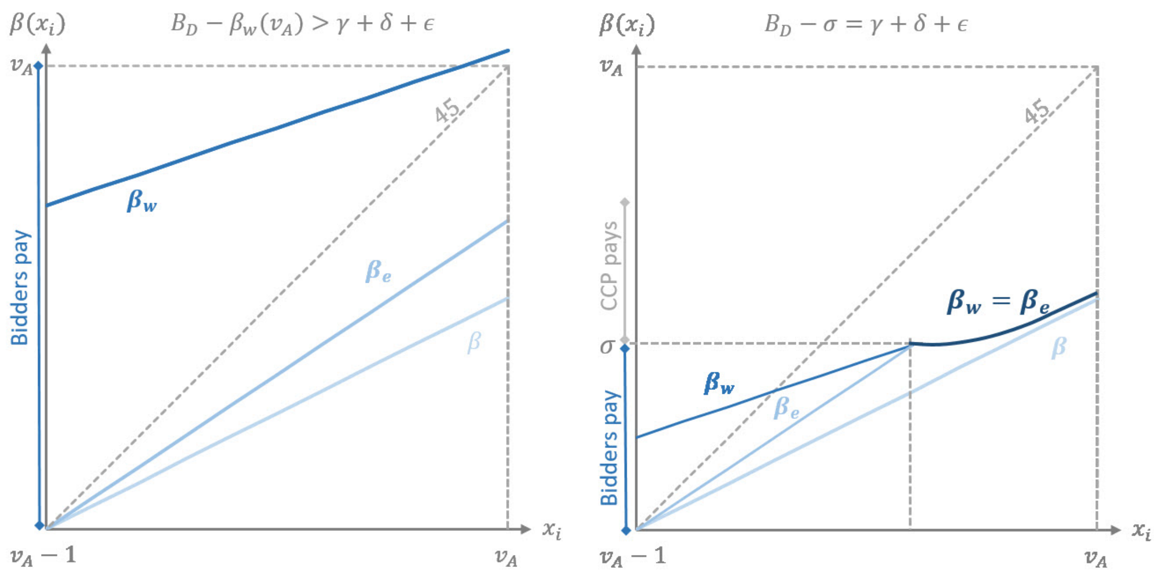

Figure 3: Equilibrium Bidding

Third, the expected profit of the bidders, as shown in the preceding sections, follows $\pi(x_i) > \pi_e(x_i) = \pi_w(x_i)$, i.e., the expected profit of the bidders is highest, in the case that somebody else pays. However, there is no difference in the expected profit for the equal and the losers' loss allocation.

5 Optimal Loss Allocation

In this chapter, I will analyse whether a CCP can minimize her expected loss by choosing an optimal loss allocation $\langle \mathcal{W}^*(\cdot), \mathcal{L}^*(\cdot)\rangle$, as defined below.

Definition 3 *The optimal loss allocation of a CCP $\langle \mathcal{W}^*(\cdot), \mathcal{L}^*(\cdot)\rangle$ when conducting an auction at time A with N number of participants, minimizes the expected loss of the CCP given by (1) subject to the budget condition (5).*

I can formulate the following two propositions (see proof in the appendix).

Proposition 1 *If the bidders might have to pay, then a bidder's as well as a CCP's expected (as well as ex ante) losses are independent of the CCP's loss allocation $\langle \mathcal{W}(\cdot), \mathcal{L}(\cdot)\rangle$, i.e., any loss allocation is optimal.*

Intuitively, an agent optimizes $Z(x_i) = \beta(x_i) + \mathcal{W}(\beta(x_i))$ - the payment she would need to make to the CCP if she wins - which depends on exogenous factors and is not affected by the loss allocation $\mathcal{W}(\cdot)$. Given the optimal $Z(x_i)$, then $\mathcal{L}(x_i)$ can be directly derived and the expected profit of a bidder is shown to be independent of any loss-sharing arrangement, as follows:

$$
\begin{aligned}
Z(x_i) &= \frac{N-1}{N+1}x_i + \frac{N-1}{N(N+1)}(v_A - 1) + \frac{v_D - (\gamma + \delta + \epsilon)}{N} \\
\mathcal{L}(x_i) &= -\frac{1}{N+1}x_i - \frac{1}{N(N+1)}(v_A - 1) + \frac{v_D - (\gamma + \delta + \epsilon)}{N} \\
\pi(x_i) &= \frac{(1-c_i)^N}{N} - \frac{1}{N}\big((v_D - v_A) - (\gamma + \delta + \epsilon) + \frac{2}{N+1}\big)
\end{aligned}
\tag{17}
$$

Proposition 2 *If the bidders might have to pay, then a CCP can incentivise higher bidding by reducing the loss allocation of the winner $\mathcal{W}(\cdot)$.*

The proof immediately follows from proposition 1. Since $Z(x_i)$ is independent of the loss allocation function, the optimal bidding strategy adjusts to the loss allocation function, as follows: $\beta(x_i) = Z(x_i) - \mathcal{W}(\beta(x_i))$.

It is clear from the two propositions that by designing loss allocation $\langle \mathcal{W}^*(\cdot), \mathcal{L}^*(\cdot)\rangle$, a CCP can affect the optimal bidding strategy β but not the expected loss of the CCP nor of the bidder.

6 Completeness of Loss Allocation

A CCP can use recovery measures to allocate uncovered losses (see CPMI-IOSCO (2017) for a discussion). In this chapter I will consider cash calls, use of initial margins, and variation margin haircuts and analyse whether they are complete in the sense of Definition 1.

Proposition 1 simplifies the analysis in the following way. First, it can be shown that the PC from equation (6) is always satisfied when using the generally valid expressions from equation (17), which means that the PC is always satisfied for all types of loss allocation arrangements as long as the non-participating agent is subject to the same loss allocation arrangement as the participating bidders. Therefore, to verify completeness, I need to check only for the BC and the TC (see the proof in the appendix for proposition 4).

Second, for a given holding loss, the TC can be violated or not depending on the private value of the winning bidder and the bidder with the second highest bid. It can be shown (again based on the

generally valid payments from equation (17)) that the TC is never violated whenever $\mathcal{L}(\beta_e(v_A)) \leq l$, i.e., I need to check only for the situation where the bidder i with private cost $c_i = 0$ wins and the second highest bidder j's private cost infinitesimally approaches zero, i.e., $\lim c_{-i} \to 0$ (see the proof in the appendix for proposition 4).
For ease of exposition, I will treat the default fund δ as already lost to the bidders. However, the CCP could organize an auction where she shields the winner from losing her default fund as well as from any additional recovery measures. As shown in proposition 2, this would increase the bidding dramatically, but based on proposition 1, would not affect the expected losses nor the ex post losses of the bidders or the CCP.

Cash Calls

This recovery measure covers additional losses by simply calling additional cash from the surviving agents so that given the winning bid β^*, the net loss is met by cash-calls $\mathcal{W}(\beta^*)$ from the winning agent and $\mathcal{L}(\beta^*)$ from the $N-1$ losing agents. In principle, the amount of cash that a CCP can call from the bidders is uncapped.[15] The budget constraint can be formulated as follows:

$$(v_D - \beta^*) - (\gamma + D + \epsilon) = \mathcal{W}(\beta^*) + (N-1)\mathcal{L}(\beta^*)$$

Since the default fund is already lost, the initial margin γ represents the maximum loss that a CCP can inflict on a losing bidder who wants to leave. For the TC to always be satisfied, I must have the following:

$$\mathcal{L}(\beta^*) \leq \gamma$$

The l.h.s depicts the cash call that a losing bidder would need to pay-in and the r.h.s depicts the maximal loss that a CCP can inflict on the leaving agent.

Use of Initial Margins

Additional losses are met simply by writing down the pre-paid initial margins, which the surviving agents would need to remargin again. It is obvious that the BC is violated whenever the loss is too big, i.e., as follows:

$$(v_D - \beta_e^*) - (\gamma + D + \epsilon) > N\gamma$$

The TC is met since prepaid resources are used to cover the losses.

Variation Margin Haircuts

This recovery tool achieves the budget balance by writing down the unrealized gains of in-the-money positions of the non-defaulting agents. Since I assume that a type 2 agent defaulted and losses occurred subsequently, all N_1 type 1 agents made gains and would be subject to vmgh. Since the loss-sharing applies only to a subset of agents, I first discuss the asymmetric payoffs, as follows:

$$u_i = \begin{cases} x_i - b_i - \delta - \mathcal{W}_1(b_i) & \text{if } b_i > b_{-i} \\ -\delta - \mathcal{L}_1(b_{-i}) & \text{if } b_i < b_{-i} \end{cases} \tag{18}$$

[15] It has, however, become practice for the CCP to limit cash calls by a multiple of each agent's default fund contribution δ.

where $\mathcal{W}_1(\cdot)$ and $\mathcal{L}_1(\cdot)$ refer to the vmgh applied to type 1 agents only. This asymmetric loss allocation leads to a situation where type 1 agents bid more aggressively than type 2 members, i.e., $\beta_1(x_i) > \beta_2(x_i)$, but where both types provide higher bids compared to where one type bids on its own. Two results immediately follow. First, asymmetric auctions can be inefficient since for a small enough $\varepsilon > 0$, it is the case that $\beta_1(x_i - \varepsilon) > \beta_2(x_i + \varepsilon)$, so that the bidder with the higher private costs might obtain the position. Second, not allowing type 2 agents to participate ensures efficiency but reduces the competition between bidders and increases the CCP's expected loss. The results can be generalized as follows:

Proposition 3 *Auctions with asymmetric loss allocation can be inefficient. Bidders who are subject to the loss allocation bid more aggressively than those who are not. Adding latter can still increase the overall bidding, reduce the CCPs expected loss, and the losing bidders ex post loss.*

Continuing with the analysis of completeness, I will analyse for ease of exposition the equal loss allocation function and note that the maximum vmgh a CCP can apply to any type 1 agent is $v_D - v_A$.[16] The BC is always met whenever the following holds:

$$v_D - \beta_e^* < (\gamma + D + \epsilon) + N_1(v_D - v_A) \iff v_A - \beta_e^* < (\gamma + D + \epsilon) + (N_1 - 1)(v_D - v_A)$$

The difference $v_A - \beta_e^*$ is driven by the number of auction participants N and the private cost c_i^* of the winning bidder. The equation shows that the private cost of the winning bidder would need to be unrealistically large to violate this inequality. Basically, the difference between the market price and the winning bid would need to be large enough to wipe out all prepaid collateral as well as the holding loss multiplied by $(N_1 - 1)$.

Finally, the TC is always met since agent type 1 cannot dispose of the risky asset and therefore would be subject to the valuation loss $v_D - v_A$ anyway. In the following, I want to explore the TC in the case that the agents would be able to dispose of the risky asset at any time. The agent would keep the risky asset as long as the expected profit of the asset and the cleared hedged position is above zero, as follows:

$$\pi + \underbrace{(v_A - v_D)}_{\text{Loss due to holding risky asset}} - \underbrace{(\alpha_D - \alpha_A)}_{\text{Variation Margin Payments}} - \underbrace{\mathcal{L}_1(\beta^*)}_{\text{vmgh}} \geq 0$$

Note that $\alpha_D = v_D$ and $\alpha_A = v_A$, so that the vmgh cannot exceed $\mathcal{L}_1(\beta^*) \leq \pi$. In addition, if the agent leaves, the CCP can write down the initial margins γ also so that the TC is satisfied whenever the following holds:

$$\mathcal{L}_{1,e}(\beta^*) \leq \pi + \gamma$$

Combination of the Recovery Measures

Since the losses are allocated immediately, the initial margins γ can be counted only once. The CCP can either call cash up to the size of the γ or write down the initial margins up to γ, but it cannot do both. Combining either the cash call or the initial margin write down with the vmgh, the CCP can achieve a complete loss allocation. I obtain the following result.

Proposition 4 *Cash calls and initial margin write-downs are non-complementary recovery measures. A combination of the variation margin haircut and the write-down of initial margins or cash*

[16] This requires that the CCP stopped paying the variation margin to type 1 members immediately after the default.

calls is complete as long as $v_D - \beta^*_e < D + \epsilon + (N+1)\gamma + N_1(v_D - v_A)$.

As the following table shows, bringing de facto the initial margins into the CCP's loss allocation adds significant amounts of additional financial resources.

	CME	**LCH**	**ICE**
D	7′416	10′205	2′472
ϵ	250	84	50
$(N+1)\gamma$	125′448	154′215	34′153

Table 2: Financial Resources of CCPs at Q42018, in USD Mio.

As shown, recovery measures enhance a CCP's ability to allocate losses considerably. I considered here only the willingness of the agents to continue meeting their obligations but not their ability to do so. The losses that the CCP allocates through the recovery measures described in this chapter affect the agents' capital and liquidity position almost immediately. The strength or vulnerability of a CCP is, therefore, her clearing agents, who, in one way or another, will have to carry the losses that go beyond the defaulting agent's collateral (and a small layer of the CCP's capital).

7 Optimal Time of Auction

I will analyse whether a CCP can minimize her expected loss by choosing the optimal time of the auction, as defined below.

Definition 4 *The optimal time of auction* $A^* \in [D + \eta, 1)$ *given a complete loss allocation* $\langle \mathcal{W}(\cdot), \mathcal{L}(\cdot) \rangle$ *minimizes the expected loss of the CCP in (1).*

I will analyse the three cases of CCP's expected loss depicted in equation (1) at $\bar{A} = D + \eta$, i.e., the earliest time that a CCP can conduct an auction, as follows:

$$L_{CCP} = \begin{cases} 0 & \text{if } v_D - \beta(v_{\bar{A}} - 1) \leq \gamma + \delta \\ \epsilon & \text{if } v_D - \beta(v_{\bar{A}}) \geq \gamma + \delta + \epsilon \\ (0, \epsilon) & \text{if } x_i \in [v_{\bar{A}}, v_{\bar{A}} - 1] \text{ where } \gamma + \delta < v_D - \beta(x_i) < \gamma + \delta + \epsilon \end{cases}$$

When at $\bar{A}$, the CCP finds that $v_D - \beta(v_{\bar{A}} - 1) \leq \gamma + \delta$, i.e., she does not expect to lose her equity (first line), then the CCP will conduct the auction immediately. In this case, $A^* = D + \eta$.
However, if the CCP finds at $\bar{A}$ that $v_D - \beta(v_{\bar{A}}) > \gamma + \delta$, i.e., she expects to lose her share of equity for sure when conducting an auction right away (second line), then by waiting, she cannot lose more if the price declines further but can gain if the price rebounds. I do not calculate the optimal time A^* explicitly but note that the corresponding value v_{A^*} must satisfy in any case the following two inequalities at the same time: $v_D - \beta(v_{A^*}) < \gamma + \delta + \epsilon$ as well as $v_D - \beta(v_{A^*} - 1) > \gamma + \delta$. Since it is not guaranteed that the price will rebound, the CCP will wait to hold the auction until the price rebounds to value v_{A^*} or hold the position until $t = 1$.
If the CCP finds at $\bar{A}$ that the expected loss of her equity is neither zero nor her full share ϵ, then she might decide to wait or conduct the auction right away. No clear answer can be given in such a case.

Proposition 5 *A CCP will conduct an auction at the earliest time possible $\bar{A} = D + \eta$ when the corresponding value of the auctioned contract $v_{\bar{A}}$ translates into no expected loss of equity. If the value of the contract at $\bar{A}$ is such that she will lose her entire share of equity ϵ for sure, then she prefers to wait until the price either rebounds or hold the position until maturity.*

The implication of a CCP holding the position for an extended period of time for the surviving members is that all agents are indirectly exposed to the market risk B_t and thus will carry cost ξ_i as expressed in equation (3). It can be argued that the type 1 members are more heavily exposed than type 2 members since the former would potentially be subject to $vmgh$ in addition to potentially losing the initial margin γ. The CCP will not internalize these costs because all participants still weakly prefer to stay with the CCP. Crucially, this holds only for markets subject to mandatory clearing. If mandatory clearing was not in place, then two agents of opposite type could meet and agree to trade out of a CCP and hedge their exposure towards B_t in a bilateral contract.[17]

8 Additional Consideration

8.1 Auctions with a Subset of Bidders

CCPs have in the past conducted auctions with only a subset of agents.[18] Therefore, it is of practical relevance to analyse how this affects the invited and not invited agents to the auction. I will analyse the situation where N_A is the number of agents invited to the auction but all agents $N > N_A$ are subject to the loss allocation of the CCP. It can be shown (see the proof in the appendix) that the payment of the winning bidder $Z_{N_A}(x_i)$ in such an auction is lower compared to an auction where all bidders are invited, which has direct implications for the non-invited agents who have to expect to share more losses. From this, the following proposition follows.

Proposition 6 *Inviting a subset of agents $N_A < N$ to the auction puts the non-invited agents at a comparative disadvantage.*

The benefits of increasing the number of agents participating in an auction crucially depend on how good the CCP is at picking agents with low private costs (or with a high private value). In this model, the CCP picks agents randomly. This is in contrast to the situation where the CCP does not know the exact private costs c_i but is able to divide all agents into two groups, as follows: agents with low private costs $c_i^l \sim U[0, x]$ and agents with high private costs $c_i^h \sim U[y, 1]$. In the simple case, where the support of the two groups does not intersect, i.e., $x \leq y$ (and where there are at least two low private cost agents), it can be shown that inviting the high cost agents to the auction will never improve the result and that the high cost agents will be indifferent regarding whether to participate in the auction or not.
The analysis can become very complicated when the support of the agents with low and high private costs intersect. Vickrey (1961) already noted that asymmetries among the bidders could lead to inefficient second-price auctions but noted that the mathematics of this problem might become intractable. Since then, many authors have tried to approach the problem from different angles. In addition to others, Maskin and Riley (2000) have studied the properties of first-price and second-price auctions in the case of two asymmetric bidders whose stochastic values do not

[17] This operation would require the agents to agree on two simultaneous transactions, as follows: first, an opposite trade to the one described in section 3.2 would be submitted for clearing to the CCP and, second, the same trade as described in section 3.2 would be agreed bilaterally.

[18] See, for example, a very well documented case of Lehman's Default at CME in Valukas (2010), p. 1841-1870.

necessarily share the same support. Only recently did Hubbard and Kirkegaard (2015) extend the analysis to more than two bidders who do not have the same value support. They show that the results deviate crucially from the case of two bidders and extend the application to many additional relevant cases. Clearly, if the assumption of the CCP not knowing the private values of the agent is relaxed a bit, the results can change dramatically. Generally, it cannot be excluded that CCPs have some insights into the private values of the agents. After all, CCPs can observe the trading behaviour of the agents, their turnover, positions and so forth.

8.2 Re-Establishing a Matched Book: Partial Tear-Up

Another recovery measure a CCP can take is to terminate some or all contracts to return to a matched book and stem further losses (see CPMI-IOSCO (2017)). The CCP would establish a (market) price to calculate payments due to the affected surviving agents. If the available collateral is not sufficient, then the payments due would be reduced pro rata. In this paper, I will consider the partial tear-up, where the termination affects only those contracts necessary to offset the position of the defaulting agent. The results can be immediately translated into the termination of all the contracts of the CCP (complete tear-up).

The starting point is that the holding loss of the CCP is such that the available financial resources are not sufficient, i.e., that $v_D - v_A \geq \gamma + D + \epsilon$, and that the CCP would tear-up the contracts of the surviving agents unless the auction provides a sufficiently high winning bid β^*, where $v_D - \beta^* = \gamma + D + \epsilon$. Contracts once torn cannot be re-traded with another agent and the affected agent carries costs c_i since her risk is not fully hedged any longer. Is it desirable for the bidders to avoid a tear-up? And if yes, can an auction achieve a desirable outcome?

A partial tear-up is economically equivalent to the variation margin haircut, i.e., the CCP would partially tear-up a fraction $\frac{1}{N_1}$ of each agent type 1 contract with the CCP and the compensation would be reduced pro-rata as follows: $\mathcal{L}_1 = \frac{(v_D - v_A) - (\gamma + D + \epsilon)}{N_1}$.[19] Given this, the CCP calls an auction and announces that positions will be partially torn and accordingly compensated if the winning bid β^* falls below $\beta^* < v_D - (\gamma + D + \epsilon)$. The bidders are faced with the following three possible outcomes and the respective payoffs (payoff in brackets apply to type 1 only):

$$u_i = \begin{cases} x_i - b_i - \delta & \text{if } b_i > b_{-i} \wedge v_D - b_i = \gamma + D + \epsilon \\ -\delta & \text{if } b_i < b_{-i} \wedge v_D - b_{-i} = \gamma + D + \epsilon \\ -\delta - (c_i + \mathcal{L}_1) & \text{if } v_D - b^* > \gamma + D + \epsilon \end{cases} \tag{19}$$

The first two payoffs describe the situation where the winning bid is sufficiently high to ensure that the CCP does not have to tear-up the positions. In the first line, the bidder i wins and in the second line, some other bidder $-i$ wins. The third line displays the payoffs in case the bids are not sufficiently high and the CCP would partially tear-up a fraction of each type 1's contract. Note that type 1 agent i still occurs costs c_i even though only a part of the contract was torn.

Do bidders have an incentive to provide bids as to avoid the tear-up?

To simplify the analysis, I consider the following reduced form in an example where two type 1 bidders i and j face an auction (bidders of type 2 are not considered here since they have no incentive to avoid a tear-up). They can either "pay" $b = v_D - (\gamma + D + \epsilon) = v_A + N_1 \mathcal{L}_1$ and avoid the tear-up or "not pay" (meaning that they offer any price lower than b) and have their contracts

[19] Each type 1 member would have a fraction $\frac{1}{N_1}$ of its contract torn and instead of receiving $\frac{v_A}{N_1}$ for the torn contract, it would receive only $\frac{v_D + (\gamma + D + \epsilon)}{N_1}$, so the loss would be $\frac{(v_D - v_A) - (\gamma + D + \epsilon)}{N_1}$.

torn. If both decide to pay, then the contract will be divided.

		Agent j	
		Pay	Not Pay
Agent i	Pay	$-c_i - \mathcal{L}_1,\, -c_j - h_1$	$-c_i - 2\mathcal{L}_1,\, 0$
	Not Pay	$0,\, -c_j - 2\mathcal{L}_1$	$-c_i - h_1,\, -c_j - \mathcal{L}_1$

Table 3: Payoff (δ not considered)

It is clear that all bidders prefer that someone else pays a high enough bid to avoid the tear- up. The payoff in Table 3 shows that there can be only one equilibrium, i.e., both do not pay. Both would be equally well off if both paid or if none paid, but in the situation where both pay, each bidder has an incentive to deviate and not pay. Therefore, the bidders do not provide higher bids even if the partial tear-up is at risk.

The result of this game can be understood better if the variables are given concrete values. Consider the case where each type 1 would receive $\mathcal{L}_1 = -1$ less compensation for the share of torn contract and where $c_i = 0$ and $c_j = 1$ in table 4. From an aggregated point of view, the losses would be lowest if i pays and j does not pay since agent i has the lowest cost of storing another contract. In fact, agent j could compensate i and still be better off. Such an outcome could be achieved if the CCP would, instead of facing the bidders with a tear-up, consider covering the losses by the other recovery measures described in section 8 (e.g., cash-calls, initial margin haircuts).

		Agent j	
		Pay	Not Pay
Agent i	Pay	$-1, -2$	$-2, 0$
	Not Pay	$0, -3$	$-1, -2$

Table 4: Payoff with concrete values

In case of a full tear-up, all bidders would lose their positions, including those who were not affected by the default of the clearing member. The payoff in equation (19) now applies to all bidders (i.e., including the brackets), and it is easy to show that in this case, none of the bidders have an incentive to bid higher and avoid a full tear-up. The following proposition, therefore, holds for any type of tear-up.

Proposition 7 *Tear-ups are more costly on an aggregate basis compared to other recovery measures. Although agents would weakly prefer to share the losses instead of avoid having contracts torn-up, an auction cannot achieve the desirable outcome.*

9 Conclusions

In this paper, I formalize the incentives of a CCP and the surviving clearing agents in an auction conducted by the CCP when one agent defaults. I show that incentives (for example simple forms of juniorisation of default fund contributions) increase the overall bidding price but that this does not affect either the CCP's or the surviving agent's profits. In other words, even though clearing agents offer higher prices, less of their own default fund contributions will be deducted. The overall effect nets to zero.

A complete loss allocation can incentivise CCPs in certain instances to wait rather than quickly conduct an auction. This for example the case in situations, where all sizes of losses are covered by the surviving agents by way of cash calls, use of initial margins, or variation margin haircuts. This can inflict costs on the financial system, and has implications for the governance structure of a CCP. Far reaching decisions by a CCP, for example, the time of an auction, should incorporate the interests of the clearing agents.

It is a standard result from auction theory, that having more agents participating in an auction increases competition and bidding prices. Conducting CCP auctions with a subset of agents has distributional implications too, where the invited agents are better off than those not invited to the auction. This applies, for example, to clients who might be subject to the loss allocation in the case of variation margin haircuts. Having more agents participate in an auction will not only raise the average winning bid but alleviate distributional issues.

Finally, I show, that tearing-up of contracts is an expensive recovery measure compared to other alternatives, and that the threat of a tear-up does not coordinate agents sufficiently to bid higher prices to avoid the tear-up in the first place. CCP's should not rely on tear-ups as an incentive but use it only as a measure of last resort.

In this paper I have taken the size of the financial resources (e.g., default fund, initial margins, CCP's equity) as well as the default waterfall as given. Further research could build on this and frame the auction as a broader design problem that includes the optimal size of the financial resources provided by the clearing agents, the CCP as well as the order of the financial resources to be used.

Proofs

Optimal Bidding: Somebody Else Pays

Suppose that bidders $j \neq i$ follow the symmetric, increasing, and differentiable equilibrium bidding strategy β and that bidder i receives a signal that its private value is $x_i = v_A - c_i$ and bids b. In the following, I want to determine the optimal b.

The upper and lower limits of the optimal bid can be defined as follows. First, it is never optimal to bid $b > \beta(v_A)$ since bidder i would definitely win and can always do better by reducing its bid, and I will consider only bids $b \leq \beta(v_A)$. Second, a bidder with value $x = v_A - 1$ would never submit a bit that is higher than its private value since she would take a loss if she wins. If she bids lower, then she will definitely lose, and the profit is zero. Thus, it is a weakly dominant strategy to bid $\beta(v_A - 1) = v_A - 1$.

Bidder i wins the auction whenever she submits the highest bid $b_i > \max_{j \neq i} \beta(x_i)$. Since β is increasing, $\max_{j \neq i} \beta(x_i) = \beta(\max_{j \neq i} x_i)$ and, thus, bidder i wins whenever $b > \beta(\max_{j \neq i} x_i)$, or whenever $\beta^{-1}(b) > \max_{j \neq i} x_i$. The expected profit of a bidder is therefore as follows:

$$\pi(x, b) = G(\beta^{-1}(b))(x_i - b)$$

Maximizing w.r.t. b yields the following first-order condition where $G' = g$ is the density of the highest value, as follows:

$$\frac{g(\beta^{-1}(b)}{\beta'(\beta^{-1}(b))}(x - b) - G(\beta^{-1}(b)) = 0$$

At a symmetric equilibrium $b = \beta(x)$ and the equation above can be simplified as follows (the second expression is equivalent to the first one):

$$G(x)\beta'(x) + g(x)\beta(x) = xg(x)$$
$$\frac{d}{dx}(G(x)\beta(x)) = xg(x)$$

and since $G(v_A - 1) = 0$, we have the following:

$$\beta(x) = \frac{1}{G(x)} \int_{v_A - 1}^{x} yg(y)dy$$

Finally, by integrating by parts this can be rewritten as follows:

$$\beta(x) = x - \int_{v_A - 1}^{x} \frac{G(y)}{G(x)} dy = \frac{N-1}{N} x_i + \frac{1}{N}(v_A - 1)$$

The first-order condition is a necessary but not sufficient condition. I refer to Krishna (2010) for the last part of the proof.

Optimal Bidding: Equal Loss Allocation

Inserting the risk-sharing function $f_e(\cdot) = f(\cdot) = h(\cdot)$ into equation (3) I obtain the following:

$$\pi_e(x_i, b) = G(\beta_e^{-1}(b))(x_i - b - f_e(b)) - \int_{\beta^{-1}(b)}^{v_A} f_e(\beta(y))dG(y)$$

Maximizing w.r.t. b and by using the Leibnitz Rule for the integral yields the following first-order condition:

$$\frac{g(\beta_e^{-1}(b)}{\beta_e'(\beta_e^{-1}(b))}(x_i - b - f_e(b)) - G(\beta_e^{-1}(b))(1 + f_e'(b)) + \frac{g(\beta_e^{-1}(b)}{\beta_e'(\beta_e^{-1}(b))} f_e(b) = 0$$

At a symmetric equilibrium $b = \beta_e(x_i)$ the equation can be written as follows:

$$G(x_i)\beta'(x_i)(1 + f_e'(\beta_e(x_i))) + g(x_i)\beta(x) = x_i g(x_i)$$

After entering the respective values for $G(x_i) = (x_i - (v_A - 1))^{N-1}$ and $g(x_i) = (N-1)(x_i - (v_A - 1))^{N-2}$, the above expression can be simplified into $(x_i - (v_A - 1))\beta'(x_i)\frac{1+f'_e}{N-1} + \beta(x_i) = x_i$, for which the solution is as follows:

$$\beta_e(x) = \frac{N-1}{N+f'_e}x + \frac{1+f'_e}{N+f'_e}(v_A - 1)$$

There is no constant since it is weakly optimal for the bidder with private value $v_A - 1$ to bid its own value. For a concrete risk-sharing function $f'_e = -\frac{1}{N}$ I obtain the following:

$$\beta_e(x) = \frac{N}{N+1}x + \frac{1}{N+1}(v_A - 1)$$

Optimal Bidding: Losers Loss Allocation

Inserting the risk-sharing function $f_w(\cdot) = 0$ and $h_w(\cdot)$ into equation (3) I obtain the following:

$$\pi_w(x_i, b) = G(\beta_w^{-1}(b))(x_i - b) - \int_{\beta^{-1}(b)}^{v_A} h_w(\beta(y))dG(y)$$

Maximizing w.r.t. b and by using the Leibnitz Rule for the integral, I obtain the following first-order condition:

$$\frac{g(\beta^{-1}(b)}{\beta'_w(\beta_w^{-1}(b))}(x_i - b) - G(\beta_w^{-1}(b)) + \frac{g(\beta_w^{-1}(b)}{\beta'_w(\beta_w^{-1}(b))}h_w(b) = 0$$

At a symmetric equilibrium $b = \beta_w(x)$ and by simplifying $h_w = h_w(\beta_w(x))$, the above equation can be expressed as follows (the second line is a reformulation of the first):

$$G(x_i)\beta'_w(x_i) + g(x_i)(\beta_w(x_i) - h_w) = x_i g(x_i)$$
$$\frac{d}{dx}G(x_i)(\beta_w(x_i) - h_w) = x_i g(x_i) - G(x_i)h'_w$$

Since $G(v_A - 1) = 0$, the second line can be formulated as follows:

$$G(x)(\beta_w(x) - h_w) = \int_{v_A-1}^{x} yg(y)dy - \int_{v_A-1}^{x} G(y)h'_w dy$$

Integrating by parts (where $\int uv' = [uv] - \int u'v$) and noting that $\int_{v_A-1}^{x} G(y)dy = \frac{1}{N}G(x_i)(x_i - (v_A - 1))$, as well as $h''_w = 0$, I obtain the following:

$$G(x_i)(\beta_w(x_i) - h_w) = x_i G(x_i) - \frac{1}{N}G(x_i)(x_i - (v_A - 1)) - h'_w\frac{1}{N}G(x_i)(x_i - (v_A - 1))$$

Finally, by re-arranging the expression, I obtain the following:

$$\beta_w(x) = \frac{N-1-h'_w}{N}x + \frac{1+h'_w}{N}(v_A - 1) + h_w$$

Note that this time, it is not weakly dominant for a bidder with private value $v_A - 1$ to bid its own value. Imagine a bidder with private value $v_A - 1 + \epsilon$, where ϵ is very small. If it wins, it does not have to pay h_w, but if it loses (which is very likely), if will have to pay h_w anyway. If it bids its own value, then another bidder close but with a slightly lower private value could increase its chances of winning by bidding slightly more than its own private value. Therefore, it cannot be an equilibrium.
For a concrete risk-sharing function $h_w(\beta_w(x_i)) = \frac{(v_D - \beta_w(x_i)) - (\gamma + \delta)}{N-1}$ I obtain the following:

$$\beta_w(x_i) = \frac{N-1}{N+1}x_i + \frac{N-1}{N(N+1)}(v_A - 1) + \frac{v_D - (\gamma + \delta)}{N}$$

Optimal Bidding: Overlapping Losses

The bidder whose bid is above the critical value $b \geq \sigma$ knows that the default fund will not be used and has to solve a maximization problem expressed by equation (15). The first-order condition can be written as follows:

$$\frac{d}{dx}(G(x)\beta_D(x)) = xg(x)$$

Since $\beta_D(\zeta) = \sigma$, I have that for any optimal bid above σ (i.e., where only the defaulter's collateral is used) is as follows:

$$\beta_D(x_i) = \sigma\frac{G(\zeta)}{G(x_i)} + \frac{1}{G(x)}\int_{\zeta}^{x} yg(y)dy$$

Integrating by parts yields the following:

$$\beta_D(x_i) = \sigma\frac{G(\zeta)}{G(x)} + \frac{1}{G(x)}\Big([yG(y)]_{\zeta}^{x} - \int_{\zeta}^{x} G(y)dy\Big)$$

$$= \sigma\frac{G(\zeta)}{G(x_i)} + x - \frac{G(\zeta)}{G(x_i)}\Big(\zeta - \frac{1}{N}(\zeta - (v_A - 1))\Big) - \frac{1}{N}(x_i - (v_A - 1))$$

$$\beta_D(x_i) = \frac{N-1}{N}x_i + \frac{1}{N}(v_A - 1) + \frac{1}{N(N+1)}\frac{(\zeta - (v_A - 1))^N}{(x_i - (v_A - 1))^{N-1}}$$

Comparison of the Loss Allocation Arrangements

Somebody Else Pays

The expected profit of a bidder with private value x_i in the case where someone else pays, i.e., with the optimal bidding strategy $\beta(x_i) = \frac{N-1}{N}x_i + \frac{1}{N}(v_A - 1)$ and where $l(x_i) = 0$ is as follows:

$$\pi(x_i) = G(x_i)(x_i - \beta(x_i)) = (x_i - (v_A - 1))^{N-1}(x_i - \frac{N-1}{N}x_i - \frac{1}{N}(v_A - 1))$$

$$= \frac{1}{N}(x_i - (v_A - 1))^N$$

Equal Loss Allocation

The expected payoff of a bidder with private value x_i in the case where some bidders share the losses, i.e., with the optimal bidding strategy $\beta_e(x) = \frac{N}{N+1}x + \frac{1}{N+1}(v_A - 1)$ and the respective loss allocation function is:

$$\pi_e(x_i) = G(x_i)(x_i - \beta_e(x_i) - f_e(\beta_e(x_i))) - \int_{x_i}^{v_A} f_e(\beta_e(y))dG(y)$$

where $f_e(\beta_e(x_i)) = \frac{v_D - \beta_e(x_i) - (\gamma+\delta)}{N}$. The expression is more complicated, and I will calculate both parts separately. The first part can be calculated in two different ways, as follows:

$$G(x_i)(x_i - \beta_e(x_i) - f_e(\beta_e(x_i))$$

$$= \frac{1}{N+1}\Big(x_i - (v_A - 1)\Big)^N - \Big(x_i - (v_A - 1)\Big)^{N-1}\frac{v_D - \beta_e(x) - (\gamma + \delta)}{N}$$

$$= (x_i - (v_A - 1))^{N-1}\frac{1}{N}\Big(\frac{2N}{N+1}x_i - \frac{N-1}{(N+1)}(v_A + 1) - v_D + (\gamma + \delta)\Big) = \Upsilon$$

The second part can be calculated in the following way:

$$\int_{x_i}^{v_A} f_e(\beta_e(y))dG(y) = \int_{x_i}^{v_A} \frac{v_D - \beta_e(y) - (\gamma + \delta)}{N}dG(y)$$

$$= \frac{N-1}{N}\int_{x_i}^{v_A}[v_D - (\gamma + \delta) - (\frac{N}{N+1}x_i + \frac{1}{N+1}(v_A - 1))](x_i - (v_A - 1))^{N-2}$$

$$= \frac{N-1}{N}\Big[\Big(y - (v_A - 1)\Big)^{N-1}\Big(\frac{v_D - (\gamma + \delta)}{N-1} - 2\frac{v_A - 1}{N^2 - 1} - \frac{y}{N+1}\Big)\Big]_{x_i}^{v_A}$$

$$= \frac{1}{N}\Big[\Big(y - (v_A - 1)\Big)^{N-1}\Big(v_D - (\gamma + \delta) - 2\frac{v_A - 1}{N+1} - y\frac{N-1}{N+1}\Big)\Big]_{x_i}^{v_A}$$

$$= \frac{1}{N}\Big(v_D - (\gamma + \delta) - 2\frac{v_A - 1}{N+1} - v_A\frac{N-1}{N+1}\Big) - \Psi$$

$$= \frac{1}{N}\Big((v_D - v_A) - (\gamma + \delta) + \frac{2}{N+1}\Big) - \Psi$$

where $\Psi = \Big(x_i - (v_A - 1)\Big)^{N-1} \frac{1}{N}\Big(v_D - (\gamma+\delta) - 2\frac{v_A-1}{N+1} - x_i\frac{N-1}{N+1}\Big)$. After combining both parts, I obtain the following result:

$$\Upsilon - \frac{1}{N}\Big((v_D - v_A) - (\gamma+\delta) + \frac{2}{N+1}\Big) + \Psi$$

where $\Upsilon + \Psi = \Big(x_i - (v_A - 1)\Big)^{N-1} \frac{1}{N}\Big(x_i(\frac{2N}{N+1} - \frac{N-1}{N+1}) - (v_A-1)(\frac{2}{N+1} + \frac{N-1}{N+1})\Big) = \frac{1}{N}\Big(x_i - (v_A-1)\Big)^N$ so that finally the expected profit is as follows:

$$\pi_e(x_i) = \frac{1}{N}\Big(x_i - (v_A - 1)\Big)^N - \frac{1}{N}\Big((v_D - v_A) - (\gamma+\delta) + \frac{2}{N+1}\Big)$$

Losers Loss Allocation

The expected payoff of a bidder with private value x_i in the case where default fund contributions are seniorized and, thus, with the optimal bidding strategy $\beta_w(x) = \frac{N-1}{N+1}x + \frac{N-1}{N(N+1)}(v_A - 1) + \frac{v_D-(\gamma+\delta)}{N}$ is as follows:

$$\pi_w(x_i) = G(x_i)(x_i - \beta_w(x_i)) - \int_{x_i}^{v_A} h(\beta_w(y))dG(y) \tag{20}$$

where $h(\beta_w(y)) = \frac{v_D - \beta_w(y) - (\gamma+\delta)}{N-1}$. It can be shown that $\pi_w(x_i) = \pi_e(x_i), \forall x_i$, i.e., that the expected profit is the same for participants for both the equal sharing and the seniorization of the default fund.

Proof of Propositions 1 and 2

First, I will show that the expected as well as the ex-ante profit of the bidders is not affected by the loss allocation when bidders have to pay. In the second part, I will show that the CCP's expected profit is not affected either.

Independence of the Bidders' Profit

Define $Z(b_i) = b_i + f(b_i)$ as the payment that bidder i providing bid b_i would need to pay to the CCP if she wins. Then, her pay-off, as defined in equation (2), can be re-written as follows:

$$u_i = \begin{cases} x_i - Z(b_i) & \text{if } b_i > b_{-i} = \max b_{j\neq i} \\ -h(b_{-i}) & \text{if } b_i < b_{-i} \end{cases}$$

Since the budget constraint in equation (5) must be satisfied for all possible winning bids, I can define the payment that bidder i would need to make in case she loses as follows:

$$h(b_{-i}) = \frac{1}{N-1}\Big(v_D - (\gamma+\delta+\epsilon) - Z(b_{-i})\Big)$$

Therefore, the payoff to bidder i providing bid b_i can be expressed as follows:

$$u_i = \begin{cases} x_i - Z(b_i) & \text{if } b_i > b_{-i} = \max b_{j\neq i} \\ -\frac{1}{N-1}\Big(v_D - (\gamma+\delta+\epsilon) - Z(b_{-i})\Big) & \text{if } b_i < b_{-i} \end{cases}$$

Given that the other bidders follow the symmetric, increasing, and differentiable equilibrium strategy β, bidder i with the above payoff maximizes the following expected profit by choosing the optimal bid b_i, as follows:

$$\pi(x_i, b_i) = G(\beta^{-1}(b_i))(x_i - Z(b_i)) - \frac{1}{N-1}\int_{\beta^{-1}(b_i)}^{v_A} \Big(v_D - (\gamma+\delta+\epsilon) - Z(\beta(y))\Big)dG(y)$$

where $G(y) = F(y)^{N-1}$ denotes the distribution of the highest order statistics of the remaining $N-1$ bidders that bidder i is competing against.

Differentiating w.r.t. b_i (using the Leibnitz Rule for the integral) yields the following first-order condition, where $G' = g$ is the density of the highest order statistics (note also that $dG(\beta^{-1}(y)) = \frac{g(\beta^{-1}(y))}{\beta'(\beta^{-1}(y))}dy$), as follows:

$$\frac{g(\beta^{-1}(b_i))}{\beta'(\beta^{-1}(b_i))}(x_i - Z(b_i)) - G(\beta^{-1}(b_i))Z'(b_i) + \frac{1}{N-1}\frac{g(\beta^{-1}(b_i))}{\beta'(\beta^{-1}(b_i))}(v_D - (\gamma + \delta + \epsilon) - Z(b_i)) = 0$$

which can be simplified as follows (note, that $Z'(b_i) = (1 + f'(b_i))$:

$$\frac{N}{N-1}g(\beta^{-1}(b_i))Z(b_i)) + G(\beta^{-1}(b_i))\beta'(\beta^{-1}(b_i))(1 + f'(b_i)) = g(\beta^{-1}(b_i))(x_i + \frac{v_D - (\gamma + \delta + \epsilon)}{N-1})$$

At symmetric equilibrium $b_i = \beta(x_i)$, the first-order differential equation can be written as follows (note that $Z'(\beta(x_i)) = \beta'(x_i)(1 + f'(\beta(x_i)))$):

$$\frac{N}{N-1}g(x_i)Z(\beta(x_i))) + G(x_i)Z'(\beta(x_i)) = g(x_i)(x_i + \frac{v_D - (\gamma + \delta + \epsilon)}{N-1})$$

Since $Z(\beta(x_i))) = \beta(x_i) + f(\beta(x_i))$ only depends on x_i, I can replace it with $\hat{Z}(x_i) = Z(\beta(x_i)))$, as follows:

$$\frac{N}{N-1}g(x_i)\hat{Z}(x_i) + G(x_i)\hat{Z}'(x_i) = g(x_i)(x_i + \frac{v_D - (\gamma + \delta + \epsilon)}{N-1})$$

The solution to this first-order differential equation is as follows:

$$\hat{Z}(x_i) = c_1(v_A - 1 - x_i)^{-N} + \frac{N-1}{N+1}x_i + \frac{N-1}{N(N+1)}(v_A - 1) + \frac{v_D - (\gamma + \delta + \epsilon)}{N}, \text{ where } c_1 = 0$$

which proves that the payment that the winning bidder pays to the CCP is independent of the loss allocation arrangement.

Finally, the expected profit $\pi(x_i)$ as well as the ex post payment can be expressed in terms of $\hat{Z}(x_i)$, which finalizes the proof.

Independence of the CCP's Profit

Take the expected loss of a CCP as expressed in equation (1). The first two cases are not interesting since the expected loss is either zero or the complete share of equity ϵ will be lost. Therefore, I will show that in the third case, where $L_{CCP} \in (0, \epsilon)$, the loss allocation cannot affect the expected loss of a CCP.

Consider any possible loss allocation function $\langle \hat{\mathcal{W}}, \hat{\mathcal{L}} \rangle$ leading to the optimal bidding strategy $\hat{\beta}$, and where $\hat{\beta}(\zeta) = v_D - (\gamma + \delta + \epsilon)$ with $\zeta \in (v_A - 1, v_A)$, i.e., there is a bid that exactly exhausts all the defaulting agent's collateral as well as the CCP's share of equity. According to the contractual arrangements for any loss allocation measures, it must be that $\hat{\mathcal{W}}(\zeta) = \hat{\mathcal{L}}(\zeta) = 0$, and so inserting this into the above solution for $\hat{Z}(\zeta)$ leads to the following:

$$\hat{Z}(\zeta) = \hat{\beta}(\zeta) = \frac{N-1}{N+1}x_i + \frac{N-1}{N(N+1)}(v_A - 1) + \frac{\hat{\beta}(\zeta)}{N}$$

Solving for $\hat{\beta}(\zeta)$ I obtain the following:

$$\hat{\beta}(\zeta) = \frac{N}{N+1}\zeta + \frac{1}{N+1}(v_A - 1)$$

which is the solution for the optimal bidding strategy in the case of equal loss allocation, i.e., all loss allocation functions converge to point A in figure (2). Any bidding above $\hat{\beta}(\zeta)$ follows the same pattern, independent of the loss allocation function. From that, it follows that the expected loss L_{CCP} cannot be affected by the loss allocation function.

Proof of Proposition 3

First, I will show that including agents not subject to the loss allocation increases the optimal bidding strategy for both, including for those subject to the loss allocation. Second, I will show that this reduces the CCP's expected losses and, finally, I show that the losers' ex ante losses will also be lower as a result.

Adding agents not subject to the loss allocation

Add to the N agents subject to the loss allocation $\langle \mathcal{W}(\cdot), \mathcal{L}(\cdot) \rangle$ S agents who are not. Given winning bid β^*, the CCP's budget condition must satisfy one of two cases, as follows: either an agent N wins or where an agent S wins. In the case of an equal sharing function $\mathcal{W}_e(\cdot)$, the budget constraint will always be the same, as follows:

$$(v_D - \beta^*) - (\gamma + \delta + \epsilon) = N\mathcal{W}_e(\beta^*)$$

Given an equal sharing loss $W_e(\cdot)$, the first group of N agents choose b_N to maximize the following expected profit:

$$\pi_N(x_i, b_N) = F(\beta_N^{-1}(b_N))^{(N-1)} F(\beta_S^{-1}(b_N))^S (x_i - b_N - \mathcal{W}_e(b_N))$$
$$- \int_{b_N}^{\beta_S(v_A)} \mathcal{W}_e(y) d\Big(F(\beta_N^{-1}(y))^{(N-1)} F(\beta_S^{-1}(y))^S\Big)$$

where $F(y) = (y - (v_A - 1))$.
The second group of S agents maximize the following expected profit:

$$\pi_S(x_i, b_S) = F(\beta_N^{-1}(b_S))^N F(\beta_S^{-1}(b_S))^{(S-1)} (x_i - b_S)$$

The first-order condition of the N agent subject to loss allocation can be written as follows(second equation assumes that $b_N = \beta_N(x_i')$):

$$\left(\frac{(N-1)F(\beta_N^{-1}(b_N))^{(N-2)} F(\beta_S^{-1}(b_N))^S}{\beta_N'(\beta_N^{-1}(b_N))} + \frac{SF(\beta_N^{-1}(b_N))^{(N-1)} F(\beta_S^{-1}(b_N))^{(S-1)}}{\beta_S'(\beta_S^{-1}(b_N))} \right) (x_i' - b_N)$$
$$= F(\beta_N^{-1}(b_N))^{(N-1)} F(\beta_S^{-1}(b_N))^S (1 + \mathcal{W}_e'(b_N))$$
$$\left(\frac{(N-1)F(x_i')^{(N-2)} F(\beta_S^{-1}(\beta_N(x_i')))^S}{\beta_N'(x_i')} + \frac{SF(x_i')^{(N-1)} F(\beta_S^{-1}(\beta_N(x_i')))^{(S-1)}}{\beta_S'(\beta_S^{-1}(\beta_N(x_i')))} \right) (x_i' - \beta_N(x_i'))$$
$$= F(x_i')^{(N-1)} F(\beta_S^{-1}(\beta_N(x_i')))^S (1 + \mathcal{W}_e'(\beta_N(x_i')))$$

Additionally, the first-order condition of the S agents not subject to the loss allocation can be written as follows (second equation inserts $b_S = \beta_S(x_i)$):

$$\left(\frac{NF(\beta_N^{-1}(b_S))^{(N-1)} F(\beta_S^{-1}(b_S))^{(S-1)}}{\beta_N'(\beta_N^{-1}(b_S))} + \frac{(S-1)F(\beta_N^{-1}(b_S))^N F(\beta_S^{-1}(b_S))^{(S-2)}}{\beta_S'(\beta_S^{-1}(b_S))} \right) (x_i - b_S)$$
$$= F(\beta_N^{-1}(b_S))^N F(\beta_S^{-1}(b_S))^{(S-1)}$$
$$\left(\frac{NF(\beta_N^{-1}(\beta_S(x_i)))^{(N-1)} F(x_i)^{(S-1)}}{\beta_N'(\beta_N^{-1}(\beta_S(x_i)))} + \frac{(S-1)F(\beta_N^{-1}(\beta_S(x_i)))^N F(x_i)^{(S-2)}}{\beta_S'(x_i)} \right) (x_i - \beta_S(x_i))$$
$$= F(\beta_N^{-1}(\beta_S(x_i)))^N F(x_i)^{(S-1)}$$

Assuming that $\beta_N(x_i) \geq \beta_N(x_i)$, I define $x_i' = \beta_N^{-1}(\beta_S(x_i))$ so that $\beta_N(x_i') = \beta_S(x_i)$ and so that it follows $x_i' \leq x_i$. The first-order conditions of both types can be rewritten as follows:

$$\left(\frac{(N-1)F(x_i')^{(N-2)} F(x_i)^S}{\beta_N'(x_i')} + \frac{SF(x_i')^{(N-1)} F(x_i)^{(S-1)}}{\beta_S'(x_i)} \right) (x_i' - \beta_N(x_i')) = F(x_i')^{(N-1)} F(x_i)^S (1 + \mathcal{W}_e'(\beta_N(x_i')))$$
$$\left(N\frac{F(x_i')^{(N-1)} F(x_i)^{(S-1)}}{\beta_N'(x_i')} + (S-1)\frac{F(x_i')^N F(x_i)^{(S-2)}}{\beta_S'(x_i)} \right) (x_i - \beta_S(x_i)) = F(x_i')^N F(x_i)^{(S-1)}$$

And after some further reformulations I obtain the following:

$$\begin{aligned}\Big(S\frac{F(x_i')^{(N-1)}}{F(x_i)}\frac{\beta_N'(x_i')}{\beta_S'(x_i)}+(N-1)F(x_i')^{(N-2)}\Big)(x_i'-\beta_N(x_i'))=F(x_i')^{(N-1)}\beta_N'(x_i')\frac{N-1}{N}\\ \Big(N\frac{F(x_i)^{(S-1)}}{F(x_i')}\frac{\beta_S'(x_i)}{\beta_N'(x_i')}+(S-1)F(x_i)^{(S-2)}\Big)(x_i-\beta_S(x_i))=F(x_i)^{(S-1)}\beta_S'(x_i)\end{aligned}\tag{21}$$

Setting $N = S = A$ and taking a ration of these two FOCs and after some reformulations I obtain the following:

$$\frac{A-1}{A}(x_i-\beta_S(x_i))=(x_i'-\beta_N(x_i'))$$

Which proves that $\beta_S(x_i) \neq \beta_N(x_i)$ and, for an increasing equilibrium bidding strategy, that $\beta_N(x_i) > \beta_S(x_i)$.
A comparison of the first condition in (21) to the first-order condition when N agents subject to the equal loss allocation are bidding on their own in equation (10) reveals that the agent is subject to a loss allocation bid at least as high when the group which is not subject to loss allocation is added.

Reduce the CCP's expected losses

Strategy of proof: just show for one example how losses will be lower, concentrate thereby on the β where all do not expect to pay.

Lower ex ante losses

Proof-Strategy: There are two possible outcomes. Either someone subject to the loss allocation wins or someone not subject to the loss allocation wins. Simply show for both cases that the ex-ante loss of the losers will be lower in both cases compared to where only agents subject to the loss allocation participate in the auction.

Proof of Proposition 4

I need to show that the participation constraint is always satisfied and that the termination constraint is always satisfied whenever the payment of the loser is lower than the maximal losses the CCP can inflict on the leaving agent.

Participation constraint

The expected loss when not participating in equation (6) can be calculated for all loss allocation arrangements based on equation (17) as follows:

$$\begin{aligned}\int_{v_A-1}^{v_A}\mathcal{L}(\beta(y))dG(y)&=\int_{v_A-1}^{v_A}\Big(-\frac{1}{N+1}y-\frac{1}{N(N+1)}(v_A-1)+\frac{v_D-(\gamma+\delta+\epsilon)}{N}\Big)dG(y)\\ &=-\frac{1}{N+1}\int_{v_A-1}^{v_A}ydG(y)+\int_{v_A-1}^{v_A}\Big(-\frac{1}{N(N+1)}(v_A-1)+\frac{v_D-(\gamma+\delta+\epsilon)}{N}\Big)dG(y)\\ &=-\frac{N-1}{N+1}(\frac{v_A-1}{N-1}+\frac{1}{N})-\frac{1}{N(N+1)}(v_A-1)+\frac{v_D-(\gamma+\delta+\epsilon)}{N}\\ &=\frac{1}{N}\big((v_D-v_A)-(\gamma+\delta+\epsilon)+\frac{2}{N+1}\big)\end{aligned}$$

The expected profit for any loss allocation arrangement in equation (17) is replicated here again as follows:

$$\pi(x_i)=\frac{(1-c_i)^N}{N}-\frac{1}{N}\big((v_D-v_A)-(\gamma+\delta+\epsilon)+\frac{2}{N+1}\big)\geq-\int_{v_A-1}^{v_A}\mathcal{L}(\beta(y))dG(y)$$

so that it is easy to show that the inequality always holds since the expected profit for any loss allocation is at least as large as the losses that the CCP can inflict on the non-participating agent.

Termination constraint

For which bidder / loser combination is the termination constraint (7) most limiting?
First, given the winning bid $\beta^*(x_i)$, then the termination constraint is most limiting where the losers have private costs very close to the winning bidder, i.e., $\lim c_{-i} \to c_i$.
Second, given that $c_{-i} = c_i$, then the termination constraint is most limiting where the winner has a private cost of zero, i.e., $c_i = 0$ because

$$\mathcal{L}(\beta(x_i)) - c_i = -\frac{1}{N+1}x_i - \frac{1}{N(N+1)}(v_A - 1) + \frac{v_D - (\gamma + \delta + \epsilon)}{N} - c_i$$

is highest for $c_i = 0$.
Therefore, the TC will never be violated when I can show that $\mathcal{L}\beta(v_A) \leq l$.

Proof of Proposition 6

For $N_A < N$ participating in the auction but all N agents being subject to the loss allocation, the budget constraint of the CCP is as follows:

$$(v_D - \beta^*_{N_A}) - (\gamma + \delta + \epsilon) = (N-1)\mathcal{L}_{N_A}(\beta^*_{N_A}) + \mathcal{W}_{N_A}(\beta^*_{N_A})$$

The first-order conditions can be easily derived from equation (4), as follows:

$$\frac{N}{N-1} g_{N_A}(x_i) Z_{N_A}(x_i) + G_{N_A}(x_i) Z'_{N_A}(x_i) = g_{N_A}(x_i)\Big(x_i + \frac{v_D - (\gamma + \delta + \epsilon)}{N-1}\Big)$$

with $G_{N_A}(x_i) = (x_i - (v_A - 1))^{N_A - 1}$ and $Z_{N_A}(x_i) = \beta_{N_A}(x_i) + \mathcal{W}_{N_A}(\beta_{N_A}(x_i))$ leading to the solution

$$\begin{aligned} Z_{N_A}(x_i) &= \frac{N-1}{N + \frac{N-1}{N_A - 1}} x_i + \frac{N-1}{N(\frac{N(N_A-1)}{N-1} + 1)}(v_A - 1) + \frac{v_D - (\gamma + \delta + \epsilon)}{N} \\ \mathcal{L}_{N_A}(x_i) &= -\frac{1}{N + \frac{N-1}{N_A - 1}} x_i - \frac{1}{N(\frac{N(N_A-1)}{N-1} + 1)}(v_A - 1) + \frac{v_D - (\gamma + \delta + \epsilon)}{N} \end{aligned}$$

Relating this to equation (17), it can be shown that $Z_{N_A}(x_i) < Z(x_i)$ and $\mathcal{L}_{N_A}(x_i) > \mathcal{L}(x_i)$ for $N_A < N$, so the profit of the invited agents is always larger:

$$\underbrace{\pi_{N_A}(x_i)}_{\text{expected profit of invited agents}} \quad > \quad \underbrace{-\int_{v_A - 1}^{v_A} \mathcal{L}(\beta_{N_A}(y)) dG(y),}_{\text{expected profit of non-invited agents}} \quad \text{for } x_i > v_A - 1$$

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