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Abstract

In various banking models, banks are viewed as arrangements that insure households against uncertain liquidity needs. However, the exact nature of the liquidity risk faced by households – and hence the insurance function of banks – differs across models. This paper attempts to disentangle the different meanings of the term 'liquidity insurance' in the literature and to clarify what kind of insurance banks provide in which models. The paper also shows under which conditions banking is equivalent to eliminating uncertainty about liquidity needs or letting households trade with each other in an asset market. Special attention is given to the comparison of banking models in the tradition of Diamond and Dybvig (1983) with those based on monetary (notably New Monetarist) frameworks.

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1. Introduction

Ever since the seminal contributions of Bryant (1980) and Diamond and Dybvig (1983), an important strand of the theoretical literature on banking has regarded banks as arrangements that insure their depositors against uncertain liquidity needs. At the core of these insurance-theoretic banking models is the notion that households face uncertainty regarding their future (idiosyncratic) liquidity needs.[1] In models in the tradition of Diamond and Dybvig (1983), this usually means that households want to consume at different points in time in the future, with individual households not knowing beforehand at which point in time they will want to consume. In banking models based on monetary frameworks, it may mean that some households have a desire to purchase a consumption good that can only be bought with money, and again households do not know ex ante whether they will have such a desire.[2]

Generally, banks insure households against liquidity risk in these models by pooling households' assets and then paying out the households according to their (reported) liquidity needs.[3] However, the exact nature of the liquidity risk faced by households – and thus the insurance function of banks – varies across models, and the term 'liquidity insurance' has been used to describe conceptually different kinds of insurance in the literature. This paper attempts to clarify what kinds of liquidity risk are implicit in different banking models and how the insurance function of banks differs across models. Special attention is given to the difference between Diamond-Dybvig type banking models and monetary (especially New Monetarist) models of banking.

To distil the different kinds of liquidity risk present in banking models, I study a model with uncertain liquidity needs where preferences and asset returns are kept sufficiently generic that the model can nest different types of banking models found in the literature. Two assets are available for investment: a 'liquid' low-yielding asset and an 'illiquid' high-yielding asset. If there is no bank and households live in autarky, households first choose how much wealth to accumulate (at some increasing utility cost) and then how to divide

[1] I will limit attention to purely idiosyncratic liquidity risk, meaning that liquidity needs at the level of the bank are known.

[2] Examples of monetary banking models in this vein include Berentsen et al. (2007), Williamson (2012, 2016), Robatto (2019), and Altermatt et al. (2022). Champ et al. (1996), Schreft and Smith (1997), and Smith (2002) study liquidity insurance by banks in monetary models based on the OLG framework.

[3] An exception is Berentsen et al. (2007), where banks do not pool depositors' assets beforehand but channel money from those who do not need it to those who do.

the accumulated wealth between the two assets. That is, in autarky, households first make a *savings choice* followed by a *portfolio choice*. While all households are identical ex ante, ex post they turn out to be either of *type 1* or of *type 2*, where type 1 households represent those experiencing a more urgent 'need for liquidity'. More precisely, if households knew their type before making their portfolio choice, then – keeping wealth fixed – type 1 households would choose a higher portfolio share of the liquid asset than type 2 households.

I first study the benchmark case where households live in autarky, and I show that the risk created by uncertain liquidity needs can be decomposed into three parts, which I call *required portfolio liquidity risk*, *marginal value of wealth risk*, and *type risk*.

Consider first required portfolio liquidity (RPL) risk. It refers to the fact that in autarky, households may choose portfolio shares of the liquid and illiquid asset that are ex post suboptimal, given the realization of their type. For instance, in a monetary model, all households may choose to hold some money to self-insure against the possibility that they will desire to purchase a consumption good with money. Ex post, those households who turn out not to have such a desire wish they had chosen not to hold money in their portfolio, while those who turn out to need money wish they had chosen to hold more of it. Similarly, in variants of the Diamond-Dybvig model with more than one asset, such as the Cooper and Ross (1998) model, 'patient' households without a desire to consume early wish they had invested a smaller share of their wealth in the low-yielding short-term asset, and vice versa for 'impatient' households. Insuring households against RPL risk corresponds to providing them with the same expected utility they would obtain in autarky if they learned their type before making their portfolio choice. By definition, RPL risk does not exist in models with only a single asset to invest in, such as the original Diamond and Dybvig (1983) model or monetary models where money is the only asset.

Consider next marginal value of wealth (MVW) risk. It refers to the fact that in autarky, households may choose to accumulate an ex post suboptimal amount of wealth, given the realization of their type. Consider, for instance, a version of a Diamond-Dybvig model where households first decide how much wealth to accumulate, which they then invest in the single investment technology. If households lived in autarky and knew their type before choosing how much wealth accumulate (i.e., how much to invest), impatient and patient households would generally choose to invest different amounts. The reason is that impatient households, who need to terminate their investment prematurely, earn a lower effective return on their investment than patient households. A similar logic applies to monetary models, where households with a desire to purchase a consumption good with money may have a different marginal value of wealth than those who do not have such a desire. Insuring against MVW risk (which also implies insurance against RPL risk) corresponds to providing households with the same expected utility they would obtain in autarky if they learned their type before choosing how much wealth to accumulate. Different than RPL risk, MVW risk can exist in models with only a single asset to invest in. By definition, MVW risk does not exist in models where households' total investment is given by some exogenous endowment.

Finally, consider type risk. This is the residual risk that agents face in autarky even if they learn their future liquidity needs before making their savings and portfolio choice. Type risk is thus not related to 'regretting' any of the previous choices upon learning one's liquidity needs. One way to see it is that type risk is the risk of simply 'being the wrong type', e.g., the impatient rather than the patient type in a Diamond-Dybvig model.[4] In endowment models with a single asset to invest in, such as the original Diamond and Dybvig (1983) model, by definition, type risk is the only kind of liquidity risk that exists. Note also that insuring households against liquidity risk is equivalent to eliminating uncertainty about liquidity needs if and only if there is no type risk.

Households can insure against (all three kinds of) liquidity risk by forming a banking coalition (or simply 'bank').[5] Each household contributes an identical amount to the bank, which then determines how much to invest in the liquid and illiquid asset and how much to pay out to each household once types are revealed.[6] Since different kinds of liquidity risk are present (or absent) in different models, the exact nature of banks' insurance function differs across models. Therefore, after carving out the three different kinds of risk that may be caused by uncertain liquidity needs, I proceed to analyze which insurance functions of banks are relevant in which type of banking model.

[4]However, the characterization of type risk as the risk of being the wrong type is not helpful in all models (see the discussion in Section 4).

[5]Instead of a coalition of households, one can equivalently view banks as profit-maximizing agents, with the competitive process leading to an outcome where only banks that maximize depositors' expected utility attract depositors.

[6]Types are revealed privately, although this assumption is not important for most of the analysis. I abstract from bank run equilibria where depositors misreport their type. Since there is no aggregate risk, banks could eliminate run equilibria by suspending convertibility after a certain amount of early withdrawals (Diamond and Dybvig (1983)).

One result is that type risk does not exist in models with a linear cost of accumulating wealth. This is, for instance, often the case in New Monetarist models. Therefore, in these models, banks usually do not provide liquidity insurance in the sense of Diamond and Dybvig (1983), where type risk is the only risk that exists. Another result is that MVW risk does not exist in models where in autarky, households of both types would choose to accumulate the same amount of wealth if they knew their type beforehand. This can occur, for instance, if there exists an asset that is sufficiently attractive as a savings instrument for all households. An example would be a New Monetarist model with an asset whose real return equals the rate of time preference. In such cases, RPL risk is the only kind of liquidity risk that exists, since absence of MVW risk implies absence of type risk. Finally, RPL risk exists whenever agents face a nontrivial portfolio choice, i.e., whenever there is more than one asset to invest in and none of the assets dominates.

In addition to examining the insurance function of banks, I also analyze the liquidity insurance provided by an asset market, in which households can exchange liquid and illiquid assets among each other after learning their type. An asset market eliminates RPL risk but does not provide insurance against the other two kinds of liquidity risk. This implies that banking and markets are equivalent if and only if RPL risk is the only kind of liquidity risk that households face.[7] Furthermore, if there exists an asset market and bank depositors have unhindered access to the market, then banks cannot do better than markets, i.e., banks can only provide insurance against RPL risk but not the other risks.

To the best of my knowledge, the only other papers that have decomposed liquidity risk in a similar manner as the present paper are Haubrich and King (1990) and von Thadden (1997, 1999). In particular, these papers have already pointed out the difference between what I call type risk and RPL risk.[8] While Haubrich and King (1990) and von Thadden (1997, 1999) study endowment models (where total investment equals the exogenous endowment), the present paper adds an initial savings (or production) choice, which allows to derive some new insights regarding the insurance function of banks in different models. Furthermore, the present paper concentrates on the question which

[7] Another way to say the same thing is that markets provide optimal liquidity insurance (only) in models where RPL risk is the only kind of liquidity risk.

[8] Somewhat confusingly, the term 'liquidity risk' denotes RPL risk in Haubrich and King (1990) while it denotes type risk in von Thadden (1997, 1999). Haubrich and King (1990) use the term 'income risk' to denote type risk, while von Thadden (1997, 1999) calls insurance against RPL risk 'term structure insurance'.

kinds of liquidity risks are present in different types of banking models, which is not the focus of Haubrich and King (1990) and von Thadden (1997, 1999).

2. The Model

There are three periods, $t = 0, 1, 2$, and a single, nonstorable good that is used for consumption and investment. The good can be invested in two different assets at date 0. First, there is an asset called the 'S-asset', which yields a gross return of $r_S > 0$ units of good at date 1 per unit of good invested at date 0. If the S-asset is let to mature until date 2, then it yields a gross return of $R_S \geqslant r_S$ at date 2 per unit invested at date 0. Second, there is an asset called the 'L-asset', for which the gross returns at date 1 and 2 equal r_L and R_L, respectively, with $R_L \geqslant r_L$. I assume that $r_S \geqslant r_L$ and $R_S \leqslant R_L$, i.e., the S-asset pays a higher short-term return while the L-asset pays a higher long-term return. I denote by q_S and q_L the amount of good invested at date 0 in the S- and the L-asset, respectively. Assets are infinitely divisible, such that any fraction of an asset can be liquidated at date 1 while the remaining fraction is let to mature. I denote by $\lambda_S \in [0, q_S]$ and $\lambda_L \in [0, q_L]$ the amounts of the S- and the L-asset, respectively, liquidated at date 1.

There is a unit measure of households born at date 0 without endowment. At date 0 (and only in this period), each household can produce the good at a utility cost of $z(w)$, where w is the amount of good produced, and $z(w)$ is a $\mathcal{C}^2$ function defined over $w \geqslant 0$ satisfying $z'(w) > 0$, $z''(w) \geqslant 0$, and $z'(0) < \infty$. After producing the good, households can choose how much of it to invest in the S- and the L-asset.

There are two types of households, called 'type 1' and 'type 2'. The utility obtained from consuming at dates 1 and 2 depends on a household's type. Specifically, the payoffs of type 1 and 2 households are given by

$$\text{Type 1:} \quad -z(w) + u_1(c_1) + \rho_2 u_2(c_2) \quad \text{with } 0 \leqslant \rho_2 \leqslant 1, \tag{1}$$

$$\text{Type 2:} \quad -z(w) + \rho_1 u_1(c_1) + u_2(c_2) \quad \text{with } 0 \leqslant \rho_1 < 1. \tag{2}$$

In expressions (1)-(2), c_t denotes consumption at date $t \in \{1, 2\}$, and the utility functions $u_t(c_t)$ are $\mathcal{C}^2$ functions defined over $c_t \geqslant 0$ satisfying $u_t'(c_t) > 0$ for $t \in \{1, 2\}$, $u_1''(c_1) < 0$, and $u_2''(c_2) \leqslant 0$. That is, type 1 households always value consumption at date 1, and depending on the value of ρ_2, they may or may not value date-2 consumption. Conversely, type 2 households always value date-2 consumption, and depending on the value of ρ_1, they may or may not value date-1 consumption. There is no aggregate risk, and the aggregate shares of type 1 and type 2 households are given by π_1 and π_2, respectively, where π_1 and π_2 are two strictly positive constants with $\pi_1 + \pi_2 = 1$. As usual, I assume types are privately revealed to households, although private information is not central to most of what follows.[9]

The existence of two types of households captures the common notion in banking models that some households (here the type 1 households) have a stronger desire to consume early or to purchase a consumption good with a low-yielding asset compared to other households. Applied to Diamond-Dybvig type banking models (which usually assume $\rho_2 = 0$), type 1 households correspond to 'impatient' households (who want to consume early), while type 2 households correspond to 'patient' households (who want to consume late). Alternatively, applied to monetary models of banking, the two types of households represent those who have a desire to buy a consumption good with money (here represented by the S-asset) and those who do not.

I impose the following assumption on parameters, which ensures that in all settings considered below, (i) it is optimal for type 1 households to invest a strictly positive amount in the S-asset, and (ii) if it is optimal for type 2 households to invest a strictly positive amount in the S-asset, then it is also optimal for them to invest a strictly positive amount in the L-asset.

Assumption 1.

(i) $z'(0) < \pi_1 r_S u_1'(0) + \pi_2 R_L u_2'(0)$

(ii) If $u_1'(0)$ *is finite, then* $\dfrac{u_1'(0)}{u_2'(0)} > \max\left\{1, \dfrac{\pi_2}{\pi_1}\right\} \dfrac{R_L}{r_S}$

(iii) If $u_2'(0)$ *is finite, then* $R_L u_2'(0) > \rho_1 r_S u_1'(0)$

In the following sections, I will study the environment laid out above under specific (institutional) settings, namely banking (Section 3), autarky with and without uncertainty (Sections 4-5), and an asset market (Section 6). The main focus will be on comparing the allocation with a banking coalition (which corresponds to a social planner's allocation) to

[9]The assumption of private information is only relevant in Subsection 7.1, which studies coexistence of banks and markets.

the allocation resulting in autarky under different assumptions regarding the point in time when households learn their type. Autarky is the natural benchmark if we assume that households are 'isolated' (as in Wallace (1988)) and cannot interact with each other apart from forming a banking coalition in the initial period. Studying the autarky allocation under different informational assumptions helps disentangle the different kinds of liquidity risk households face in autarky, which in turn helps pin down the exact nature of the insurance function provided by banks.

When studying the allocation under autarky, I will consider three different cases regarding the point in time when households learn their type:

(i) Case 1 (Section 4): Households learn their type before choosing how much wealth w to accumulate.

(ii) Case 2 (Subsection 5.1): Households learn their type after choosing w but before making their portfolio decision, i.e., before deciding how to divide the accumulated wealth between the S- and the L-asset.

(iii) Case 3 (Subsection 5.2): Households learn their type only after making their portfolio decision.

In Section 6, I study the insurance function of an asset market, in which households can trade S- and L-assets with each other after learning their type. Finally, in Subsection 7.1, I study the insurance function of banks when bank depositors have unhindered access to an asset market.

2.1. A Few Examples

I will now provide a few examples showing how the present setup can be related to different banking models encountered in the literature.

Example 1: Diamond-Dybvig model

To replicate a standard Diamond-Dybvig model in the current setup, we can assume constant relative risk aversion (CRRA) utility over consumption, $u_1(\cdot) = u_2(\cdot) = c^{1-\sigma}/(1-\sigma)$, where we would typically assume that $\sigma > 1$. In the Diamond-Dybvig model, there is a single asset to invest in, which pays a gross return of 1 when terminated at date 1 and a gross return of $R > 1$ when allowed to mature. In the present setup, this corresponds to a parametrization with $r_S = r_L = 1$ and $R_S = R_L > 1$. Finally, we have $\rho_2 = 0$, i.e., type 1 households do not value date-2 consumption, and we may also set $\rho_1 = 0$, such that type 2 households do not value date-1 consumption.

Example 2: Cooper-Ross model

The Cooper-Ross model maintains the preference structure of the Diamond-Dybvig model but adds a nontrivial portfolio choice. Specifically, the Cooper-Ross model features a short-term and a long-term asset, where the short-term asset pays a superior date-1 return and the long-term asset pays a superior date-2 return. In the setup at hand, the Cooper-Ross asset structure corresponds to a parametrization with $r_S = R_S = 1$ and $R_L > 1 > r_L \geqslant 0$, while all else remains the same as in the Diamond-Dybvig version.

Example 3: New Monetarist banking model

In monetary models of banking, there are typically some households (here represented by type 1 households) who have a stronger desire to purchase a good with money (here represented by the S-asset) than other households. To capture a New Monetarist model with liquidity risk in the current setup, we can interpret date 0 as the centralized market (CM) in period t and date 2 as the CM in period $t+1$.[10] Date 1 can be interpreted as the decentralized market (DM), where goods are exchanged against money. Disutility of labor $z(w)$ is commonly assumed to be linear, and there is a constant marginal benefit of obtaining the consumption good at date 2, e.g., $u_2(c_2) = \beta c_2$ with $\beta \in (0,1)$. We would also usually assume that $\rho_2 = 1$, i.e., date-2 consumption is valued by all households. The return to the S-asset ('money') equals $r_S = R_S = 1/\Pi$, where Π is the gross inflation rate. To capture a model where money is the only asset, we can set $r_L = r_S$ and $R_L = R_S$. Alternatively, there may be an additional asset used as a store of value. Given linear disutility of labor and linear utility over date-2 consumption, such an asset will only be held in a finite amount if its real date-2 return equals the rate of time preference, e.g., $R_L = 1/\beta$ if $z(w) = w$ and $u_2(c_2) = \beta c_2$; furthermore, we can set $r_L = 0$ to capture the fact that this asset does not serve as means of payment at date 1.

[10]New Monetarist models featuring banks often assume that banks exist for one period only, typically from the CM in period t to the CM in period $t+1$ (e.g., Williamson (2012, 2016) and Altermatt et al. (2022)). Not every New Monetarist banking model fits exactly into the structure presented here. The purpose of these examples is to help think about what kind of liquidity risks are present in what type of model.

Example 4: OLG banking model

New Monetarist models are not the only type of monetary model in which liquidity insurance by banks has been studied. In particular, a number of papers have introduced banks into monetary models based on the overlapping generations (OLG) framework (Champ et al. (1996), Schreft and Smith (1997), Smith (2002)). In these models, all members of a given generation save when they are young and consume when they are old. Liquidity risk stems from the fact that some agents are randomly relocated to another location before they become old, and only fiat money (plus possibly some other types of liquid assets) can be transported to the new location. Such models fit into the current setup if one reinterprets c_1 (c_2) as old-age consumption by relocated (non-relocated) agents, with utility over consumption, $u_1(\cdot) = u_2(\cdot)$, typically being of the CRRA type. The S-asset can again be interpreted as money, with $r_S = R_S = 1/\Pi$, and one can set $r_L = 0$ to capture that relocated agents cannot bring the L-asset to their new location.

3. The Banking Solution

In this section, I assume that households form a banking coalition at date 0, with each household contributing an identical amount of good to the bank. The bank then chooses how much of the collected good to invest in the S- and the L-asset, and it determines how much to pay out to type 1 and 2 households. The bank's objective is to maximize the ex ante expected utility of households, i.e., the bank's solution corresponds to the allocation chosen by a benevolent social planner. I will denote by $c_t(k)$ consumption of households of type $k \in \{1, 2\}$ at date $t \in \{1, 2\}$.

We can start by noting that the bank's total date-0 investment must equal the total amount of good deposited in the bank:[11]

$$w = q_S + q_L. \tag{3}$$

Next, as is not hard to see, we can take it as given without loss of generality that the bank finances date-1 (date-2) payouts solely with the return obtained from the S-asset

[11] Since date-0 production is costly, we can take it as given throughout the paper that no good is wasted, i.e., all of the produced good w is invested.

(L-asset):[12]

$$r_S\, q_S = \pi_1 c_1(1) + \pi_2 c_1(2), \tag{4}$$

$$R_L\, q_L = \pi_1 c_2(1) + \pi_2 c_2(2). \tag{5}$$

Furthermore, the bank's solution must satisfy the incentive compatibility (IC) constraints

$$u_1(c_1(1)) + \rho_2\, u_2(c_2(1)) \geqslant u_1(c_1(2)) + \rho_2 u_2(c_2(2)), \tag{6}$$

$$\rho_1 u_1(c_1(2)) + u_2(c_2(2)) \geqslant \rho_1 u_1(c_1(1)) + u_2(c_2(1)), \tag{7}$$

where (6) says that type 1 households must prefer the payouts intended for them to the payouts intended for 2 households and (7) is the analogous condition for type 2 households. In the canonical banking models, IC constraints are generally slack in the first-best allocation. Unless mentioned otherwise, I will therefore proceed under the assumption that (6) − (7) do not bind in the bank's solution:[13,14]

Assumption 2. *The IC constraints* (6)-(7) *are not binding in the banking coalition's solution.*

Given this, we can express the bank's problem as:

$$\max_{\substack{\{w,q_S,q_L,c_1(1)\\ c_1(2),c_2(1),c_2(2)\}}} -z(w) + \pi_1\left[u_1(c_1(1)) + \rho_2\, u_2(c_2(1))\right] + \pi_2\left[\rho_1 u_1(c_1(2)) + u_2(c_2(2))\right] \tag{8}$$

subject to (3)-(5) and

$$q_S, q_L, c_1(1), c_1(2), c_2(1), c_2(2) \geqslant 0. \tag{9}$$

The first-order conditions of the bank's problem are given by[15]

$$z'(w) = r_S\, u_1'(c_1(1)), \tag{10}$$

$$z'(w) \geqslant \rho_1 r_S\, u_1'(c_1(2)) \quad \text{with equality if } c_1(2) > 0, \tag{11}$$

$$z'(w) \geqslant R_L\, u_2'(c_2(2)) \quad \text{with equality if } c_2(2) > 0, \tag{12}$$

$$z'(w) \geqslant \rho_2 R_L\, u_2'(c_2(1)) \quad \text{with equality if } c_2(1) > 0. \tag{13}$$

[12] Equations (4) and (5) imply that in the bank's solution, $\lambda_S = q_S$ and $\lambda_L = 0$.

[13] I briefly discuss the case with binding IC constraints in Subsection 7.2.

[14] Incentive compatibility usually becomes relevant when studying 'run' equilibria where type 2 households report to be of type 1. I abstract from multiple equilibria in this paper. Note that the bank could always eliminate run equilibria by suspending convertibility after π_1 withdrawals at date 1.

[15] I refer to Appendix A for the details. Assumption 1 ensures that in optimum $c_1(1) > 0$ and hence $q_S > 0$.

Conditions (10)-(13) state that the marginal value of wealth w is set equal to the marginal cost of production and – unless in corner cases where it is optimal that only type 1 households consume – the marginal value of wealth is equalized across types.

Quantities chosen by the bank are denoted by a star. A banking coalition's solution is then given by a vector $(w^*, q_S^*, q_L^*, c_1^*(1), c_1^*(2), c_2^*(1), c_2^*(2))$ satisfying (3)-(5) and (9)-(13).

Welfare under the banking solution, denoted $\mathcal{W}^*$, equals the weighted sum of the payoffs of type 1 and 2 households:

$$\mathcal{W}^* = \pi_1 \left[u_1(c_1^*(1)) + \rho_2 u_2(c_2^*(1))\right] + \pi_2 \left[\rho_1 u(c_1^*(2)) + u_2(c_2^*(2))\right] - z(w^*). \tag{14}$$

4. Autarky With Full Information

In this section, I assume that households live in isolation, and that they know their type before making any decisions. Each household chooses how much wealth to accumulate and in which assets to invest to maximize its own payoff. I denote by $w(k)$ the level of initial wealth accumulation chosen by a household of type $k \in \{1, 2\}$. Similarly, $q_S(k)$ denotes investment in the S-asset by households of type $k \in \{1, 2\}$, and $q_L(k)$ denotes the same for investment in the L-asset. As before, $c_t(k)$ denotes consumption of households of type k at date t.

We begin by noting that a household's total date-0 investment must equal its initial wealth accumulation:

$$w(k) = q_S(k) + q_L(k) \quad \text{for } k \in \{1, 2\}. \tag{15}$$

Next, it is not hard to see that we can take it as given without loss of generality that households finance their date-1 (date-2) consumption solely with the return obtained from the S-asset (L-asset):

$$r_S\, q_S(k) = c_1(k) \quad \text{for } k \in \{1, 2\}, \tag{16}$$

$$R_L\, q_L(k) = c_2(k) \quad \text{for } k \in \{1, 2\}. \tag{17}$$

Furthermore, we have the nonnegativity constraints

$$q_S(k), q_L(k), c_1(k), c_2(k) \geqslant 0 \quad \text{for} \quad k \in \{1, 2\}. \tag{18}$$

A type 1 household's objective is

$$\max_{\{c_1(1),c_2(1),w(1),q_S(1),q_L(1)\}} -z(w(1)) + u_1(c_1(1)) + \rho_2\, u_2(c_2(1)), \tag{19}$$

and a type 2 household's objective is

$$\max_{\{c_1(2),c_2(2),w(2),q_S(2),q_L(2)\}} -z(w(2)) + \rho_1 u_1(c_1(2)) + u_2(c_2(2)). \tag{20}$$

Given Assumption 1, which again ensures that households optimally choose $c_1(1) > 0$ and hence $q_S(1) > 0$, this leads to the first-order conditions[16]

$$z'(w(1)) = r_S\, u_1'(c_1(1)), \tag{21}$$

$$z'(w(1)) \geqslant \rho_2\, R_L\, u_2'(c_2(1)) \quad \text{with equality if } c_2(1) > 0, \tag{22}$$

$$z'(w(2)) \geqslant R_L\, u_2'(c_2(2)) \quad \text{with equality if } c_2(2) > 0, \tag{23}$$

$$z'(w(2)) \geqslant \rho_1 r_S u_1'(c_1(2)) \quad \text{with equality if } c_1(2) > 0. \tag{24}$$

Conditions (21)-(22) state that type 1 households equalize the marginal cost of producing wealth with the marginal benefit of investing in the S-asset; furthermore, if type 1 households invest in both the S- and the L-asset, then they will equalize the marginal benefit of investing in the two assets. Conditions (23)-(24) together with Assumption 1(iii) imply that if type 2 households choose to accumulate a strictly positive amount of wealth, then they will equalize the marginal cost of producing wealth with the marginal benefit of investing in the L-asset; furthermore, if type 2 households choose to invest in both the S- and the L-asset, they will equalize the marginal benefit of investing in either asset.

Different than in the banking coalition's solution, the marginal value of wealth – and hence the optimal wealth accumulation – may differ between type 1 and 2 households in autarky with full information. The reason is that the two types of households will in general choose different asset portfolios and thus enjoy different returns on their investment. For example, in the Diamond-Dybvig model, patient households enjoy a higher return on their investment than impatient households; therefore, if households knew their type before choosing how much to invest, they would in general choose to invest different amounts.

[16] I refer to Appendix B for the details.

I denote the optimal choices of households with an F superscript, where F is mnemonic for 'full information'. An allocation in autarky with full information is then given by two vectors $(w^F(k), q_S^F(k), q_L^F(k), c_1^F(k), c_2^F(k))_{k\in\{1,2\}}$ satisfying (15)-(18) and (21)-(24).

Welfare in autarky with full information, denoted $\mathcal{W}^F$, equals

$$\begin{aligned}\mathcal{W}^F = \pi_1 \left[u_1(c_1^F(1)) + \rho_2\, u_2(c_2^F(1)) - z(w^F(1))\right] \\ + \pi_2 \left[\rho_1 u_1(c_1^F(2)) + u_2(c_2^F(2)) - z(w^F(2))\right].\end{aligned} \tag{25}$$

Since the banking coalition maximizes expected utility of households subject to the resource constraint, welfare under autarky with full information cannot exceed welfare under the bank's solution, i.e., $\mathcal{W}^F \leqslant \mathcal{W}^*$.[17]

Definition 1. ***Type risk*** *is said to exist if and only if* $\mathcal{W}^F < \mathcal{W}^*$*, i.e., if and only if welfare under autarky with full information is strictly below welfare with banking.*

If and only if type risk exists would households find it optimal to form a banking coalition before learning their type even if they know that they will learn their type before having to make any decisions. In this sense, type risk is the risk associated with uncertain liquidity needs that is not related to 'regretting' any decisions upon learning one's type, be it the decision of how to divide one's wealth between the S- and the L- asset or how much wealth to accumulate in the first place. Note that in endowment models with only one asset, e.g., the original Diamond-Dybvig model, by definition, type risk is the only kind of liquidity risk that exists. Note also that banking is equivalent to eliminating uncertainty about types if and only if there is no type risk.

One way to see it is that type risk is the risk of being the 'wrong' type, e.g., the impatient rather than the patient type in a Diamond-Dybvig model. This characterization is not helpful in all contexts, however. The bank's solution is such that marginal values of wealth are equalized across types. In most variants of the Diamond-Dybvig model (including the Cooper-Ross model), this means that the bank effectively redistributes wealth from those who would be better off in full-information autarky (the patient depositors) to those who would be worse off (the impatient depositors). In contrast, in a model where type 1 and 2 households have identical utility over date-2 consumption, type 1 households can be regarded as those having an 'opportunity' to consume at date 1, while type 2 households

[17] This is based on the assumption that the IC constraints (6)-(7) are slack in the bank's solution – see the discussion in Subsection 7.2.

do not have such an opportunity. In this case, it is possible that both the marginal value of wealth and the payoff under full-information autarky are strictly higher for type 1 households; insurance against type risk then implies a redistribution of wealth toward those households (namely, the type 1 households) whose full-information autarky payoff is higher.

We continue with the following result:[18]

Proposition 1. *Type risk does not exist if* $w^F(1) = w^F(2)$.

Proposition 1 states that type risk does not exist if households of both types choose to accumulate the same amount of wealth in autarky with full information.[19] Note in particular that in this case, marginal values of wealth are equalized across types even without a banking coalition.

Next, Proposition 2 shows that type risk does not exist in models with a linear cost of producing wealth at date 0:

Proposition 2. *Type risk does not exist if* $z(w)$ *is linear.*

Proposition 2 implies that in models with a linear cost of producing wealth – which is often the case in New Monetarist models – banking is usually equivalent (in terms of consumption levels and welfare) to assuming that households know their type before making any of the relevant decisions. The liquidity insurance provided by banks in these models is of a different kind than in Diamond and Dybvig (1983), given that type risk is the only kind of liquidity risk present in the Diamond-Dybvig model.[20] Note that with linear production costs $z(w)$, the allocation with banking may not be exactly the same as the full-information autarky allocation. The reason is that households of different types may choose to accumulate different amounts of wealth in autarky (they may 'work different hours'), while all households contribute an identical amount to the banking coalition. However, given linear disutility of accumulating wealth, this difference has no effect on welfare.

It may be useful to compare these results with endowment models where no production takes place at date 0. In endowment models, the marginal values of wealth for given

[18] All proofs can be found in Appendix C.

[19] A well-known case for which this holds is $u_1(\cdot) = u_2(\cdot) = \ln(\cdot)$. See also Proposition 5 further below.

[20] This applies, among others, to the models by Berentsen et al. (2007), Williamson (2012), and Andolfatto et al. (2019).

endowment levels will usually differ between types except for edge cases, e.g., for log-utility in the Diamond-Dybvig model. In models where households can choose how much wealth to accumulate at date 0, the fact that different types of households can choose different levels of wealth allows to bring marginal values of wealth closer together. In the case of linear disutility of accumulating wealth, marginal values of wealth will be equalized across types, whereas with strictly convex disutility, a wedge will usually remain.

5. Autarky With Imperfect Information

In this section, I assume that households need to make decisions before learning their type. In Subsection 5.1, I consider the case where households learn their type after choosing how much wealth w to accumulate but before making their portfolio decision. In Subsection 5.2, I consider the case where households learn their type only after making their portfolio decision.

5.1. Types Revealed Before Portfolio Decision

In this subsection, I assume that households learn their type after choosing how much wealth w to accumulate but before choosing how to divide their wealth between the S- and the L-asset. We can then divide date 0 into two stages: first, a *savings choice stage* where households choose how much wealth w to accumulate without knowing their future type, and second a *portfolio choice stage* where households choose how to allocate their wealth between the S- and the L-asset, knowing their type. We can solve households' problem by proceeding backwards, first considering optimal choices in the portfolio choice stage and then in the savings choice stage.

Step 1: optimal portfolio choice given w

We start by noting that households face the usual budget and nonnegativity constraints in the portfolio choice stage. Since households choose w before learning their type, w will be identical across types. However, different types may divide their wealth differently between the S- and the L-asset:

$$w = q_S(k) + q_L(k) \quad \text{and} \quad q_S(k), q_L(k), c_1(k), c_2(k) \geqslant 0 \quad \text{for} \quad k \in \{1, 2\}. \tag{26}$$

Since households learn their type before making their portfolio decision, households will divide their wealth optimally between the S- and the L-asset given their type. As in the full-information case, we can thus take it as given that conditions (16)-(17) hold in optimum, i.e., date-1 (date-2) consumption will be financed solely with the return from the S-asset (L-asset). Assumption 1 implies that type 1 (type 2) households will invest a strictly positive amount in the S-asset (L-asset). If it is optimal for households of a given type to invest in both assets, then optimal investment levels will be such that the marginal benefit of investing in either asset is equalized:

$$r_S\, u_1'(c_1(1)) \geqslant \rho_2 R_L\, u_2'(c_2(1)) \quad \text{with equality if } c_2(1) > 0, \tag{27}$$

$$R_L u_2'(c_2(2)) \geqslant \rho_1 r_S u_1'(c_1(2)) \quad \text{with equality if } c_1(2) > 0. \tag{28}$$

Next, I denote by $v^k(w)$ the maximum achievable payoff for a household of type $k \in \{1, 2\}$ when entering the portfolio choice stage with wealth w. Since for any given w, type 1 (type 2) households optimally invest a strictly positive amount in the S-asset (L-asset), the envelope condition implies

$$\frac{\partial v^1(w)}{\partial w} = r_S\, u_1'(c_1(1)) \quad \text{and} \quad \frac{\partial v^2(w)}{\partial w} = R_L\, u_2'(c_2(2)). \tag{29}$$

Step 2: optimal savings choice

A household's problem in the savings choice stage can be expressed as

$$\max_w -z(w) + \pi_1 v^1(w) + \pi_2 v^2(w), \tag{30}$$

which leads to the first-order condition[21]

$$\begin{aligned} z'(w) &= \pi_1 \frac{\partial v^1(w)}{\partial w} + \pi_2 \frac{\partial v^2(w)}{\partial w} \\ &= \pi_1 r_S\, u_1'(c_1(1)) + \pi_2 R_L\, u_2'(c_2(2)). \end{aligned} \tag{31}$$

I denote optimal choices in this subsection with an A superscript. An allocation under autarky when types are revealed between the savings and the portfolio choice is then given by a w^A and two vectors $(q_S^A(k), q_L^A(k), c_1^A(k), c_2^A(k))_{k\in\{1,2\}}$ satisfying (16)-(17), (26)-(28), and (31).

[21] Assumption 1(i) rules out a corner solution with $w = 0$.

Welfare under autarky when types are revealed between the savings and the portfolio choice, denoted $\mathcal{W}^A$, is given by

$$\mathcal{W}^A = \pi_1 \left[u_1(c_1^A(1)) + \rho_2\, u_2(c_2^A(1))\right] + \pi_2 \left[\rho_1 u_1(c_1^A(2)) + u_2(c_2^A(2))\right] - z(w^A). \qquad (32)$$

Since households never do worse in autarky when uncertainty about their own type is resolved earlier, we have $\mathcal{W}^A \leqslant \mathcal{W}^F \leqslant \mathcal{W}^*$.

Definition 2. ***Marginal value of wealth (MVW) risk*** *is said to exist if and only if* $\mathcal{W}^A < \mathcal{W}^F$*, i.e., if and only if welfare when types are revealed after the savings choice is strictly below welfare under full information.*

Intuitively, MVW risk exists if and only if households 'regret' having accumulated the wrong amount of wealth upon learning their type. For instance, if $R_L > R_S$ and type 1 (type 2) households only care about date-1 (date-2) consumption, then optimal wealth levels will generally differ across types since type 2 households can profit from a higher-yielding investment technology compared to type 1 agents. Which type of household would then prefer to accumulate more wealth depends on the properties of the utility functions $u_1(\cdot)$ and $u_2(\cdot)$. If $u_1(\cdot) = u_2(\cdot) = u(\cdot)$ is a constant relative risk aversion utility function with a coefficient of relative risk aversion >1 (<1), then type 1 (type 2) households would choose to accumulate more wealth under full information. As another example, consider a New Monetarist model with money as the only asset as described in subsection 2.1: If type 2 households do not value date-1 consumption and the return to money is below the rate of time preference, then type 2 households would choose not to accumulate any wealth under full information.

The following result states (roughly) that MVW risk exists if and only if the marginal value of the S-asset for a type 1 household is different than the marginal value of the L-asset for a type 2 household.

Proposition 3. *MVW risk exists if and only if* $r_S\, u_1'(c_1^A(1)) \neq R_L\, u_2'(c_2^A(2))$.

If $r_S\, u_1'(c_1^A(1)) \neq R_L\, u_2'(c_2^A(2))$, then the marginal value of wealth in the portfolio choice stage is different for type 1 and 2 households, and households wish they had accumulated either more or less wealth upon learning their type. Conversely, if $r_S\, u_1'(c_1^A(1)) = R_L\, u_2'(c_2^A(2))$, households would not revise their savings decision after learning their type, and MVW risk does not exist.

If MVW risk does not exist, then this implies that if households knew their type before making their savings choice, it would be optimal for households of both types to accumulate the same amount of wealth. The following result then follows directly from Proposition 1:

Corollary 1. *If MVW risk does not exist, then type risk does not exist either.*

The next result states that MVW risk (and, as a consequence, type risk) does not exist if (i) all households have identical, linear utility over date-2 consumption, and (ii) it is optimal for type 1 households to consume a strictly positive amount at date 2:

Proposition 4. *Suppose that $u_2(c_2)$ is linear, $\rho_2 = 1$, and $c_2^A(1) > 0$. Then, neither type risk nor MVW risk exists.*

Intuitively, if the conditions in Proposition 4 are met, then it is optimal for households to 'saturate' themselves with wealth in the savings choice stage. In other words, households accumulate enough wealth to buy the (full-information) optimal amount of the S-asset in case they turn out to be of type 1. Since the (ex post) marginal value of wealth for type 2 households is constant (as a result of linear utility over date-2 consumption), households do not regret their savings choice when learning that they are of type 2. Applied to a New Monetarist banking model as outlined in Subsection 2.1, Proposition 4 implies that neither type risk nor MVW risk exists if $R_L = 1/\beta$. Note in particular that with an asset that pays the return of time preference, accumulating wealth in the savings choice stage entails no opportunity cost, which implies that $c_2^A(1) > 0$ is optimal.[22]

Finally, Proposition 5 states that neither type risk nor MVW risk exists in case of Diamond-Dybvig preferences and log-utility over consumption. The reason is that with log-utility, the marginal value of the S-asset for type 1 households equals the marginal value of the L-asset for type 2 households. As a result, in autarky, households do not regret their savings choice upon learning their type, and a bank would not find it optimal to redistribute wealth between households of different types.

Proposition 5. *If $\rho_1 = \rho_2 = 0$ and $u_1(\cdot) = u_2(\cdot) = u(\cdot)$ with $u(\cdot) = A + B\ln(\cdot)$ for some constants A and $B > 0$, then neither type risk nor MVW risk exists.*

[22] In a New Monetarist model as outlined in Subsection 2.1, linear $z(w)$ implies that $c_2^A(1) > 0$ is equivalent to assuming that R_L equals $1/\beta$: If R_L is below $1/\beta$, then $c_2^A(1) = 0$; if $R_L > 1/\beta$, then $c_2^A(1)$ is not defined. Note that the Friedman rule ($R_S = 1/\beta$) is sufficient but not necessary for the elimination of MVW and type risk. Note also that in models with strictly convex $z(w)$, we may have $c_2^A(1) > 0$ for various (sufficiently high) values of R_L.

5.2. Types Revealed After Portfolio Decision

In this subsection, I assume that households learn their type only after deciding how much to invest in the S- and the L-asset. We can again divide date 0 into two stages. In the first stage, households make their savings and portfolio choice before knowing their type. In the second stage, households choose how much to consume at each date, given their type.

Consider first the budget constraints that households face after learning their type, given their asset portfolio (q_S, q_L). It is easy to see that it is never (strictly) optimal for households to invest more in the S-asset at date 0 than what they plan to consume at date 1 as a type 1 household. We can thus take it as given without loss of generality that date-1 consumption of type 1 households will never be lower than the date-1 return obtained from the S-asset, and any consumption of type 1 households at date 2 will be financed with proceeds from the L-asset not liquidated at date 1:[23]

$$r_S\, q_S \leqslant c_1(1) \leqslant r_S\, q_S + r_L\, q_L, \tag{33}$$

$$c_2(1) = R_L \left[q_L - \frac{1}{r_L}(c_1(1) - r_S\, q_S) \right]. \tag{34}$$

Regarding the L-asset, type 1 households may choose to consume its entire return at date 1, consume its entire return at date 2, or consume part of the return at date 1 and part at date 2. In the latter case, the fraction of the L-asset liquidated at date 1 will be such as to equalize the marginal benefit of liquidation (higher date-1 consumption) with the marginal cost (lower date-2 consumption). That is, optimal consumption levels of type 1 households satisfy:

$$\begin{aligned} u_1'(c_1(1))\, r_L &\leqslant \rho_2\, u_2'(c_2(1))\, R_L \qquad \text{if} \quad c_1(1) = r_S q_S \\ u_1'(c_1(1))\, r_L &= \rho_2\, u_2'(c_2(1))\, R_L \qquad \text{if} \quad r_S\, q_S < c_1(1) < r_S\, q_S + r_L\, q_L, \\ u_1'(c_1(1))\, r_L &\geqslant \rho_2\, u_2'(c_2(1))\, R_L \qquad \text{if} \quad c_1(1) = r_S q_S + r_L q_L \end{aligned} \tag{35}$$

Consider next type 2 households. It is again easy to see that it is never (strictly) optimal for households to invest more in the L-asset at date 0 than what they plan to consume at date 2 as a type 2 household. We can thus take it as given without loss of generality that date-2 consumption of type 2 households will never be lower than the date-2 return obtained from the L-asset, and any consumption of type 2 households at date 1 will be

[23] To derive (34), we use $c_2(1) = R_L(q_L - \lambda_L)$ and $c_1(1) = r_S q_S + \lambda_L r_L$.

financed with proceeds from the S-asset not used to finance date-2 consumption:[24]

$$c_1(2) = r_S \left[q_S - \frac{1}{R_S} \left(R_L q_L - c_2(2) \right) \right] \tag{36}$$

$$R_L q_L \leqslant c_2(2) \leqslant R_L q_L + R_S q_S. \tag{37}$$

Regarding the S-asset, type 2 households may choose to consume its entire return at date 1, consume its entire return at date 2, or consume part of the return at date 1 and part at date 2. In the latter case, the fraction of the S-asset liquidated at date 1 will be such as to equalize the marginal benefit of liquidation (higher date-1 consumption) with the marginal cost (lower date-2 consumption). That is, optimal consumption levels of type 2 households satisfy:

$$\begin{aligned} &\rho_1 u_1'(c_1(2))\, r_S \geqslant u_2'(c_2(2))\, R_S && \text{if} \quad c_2(2) = R_L q_L \\ &\rho_1 u_1'(c_1(2))\, r_S = u_2'(c_2(2))\, R_S && \text{if} \quad R_L q_L < c_2(2) < R_L q_L + R_S q_S \\ &\rho_1 u_1'(c_1(2))\, r_S \leqslant u_2'(c_2(2))\, R_S && \text{if} \quad c_2(2) = R_L q_L + R_S q_S \end{aligned} \tag{38}$$

Next, I denote by $\hat{v}_S'(k)$ and $\hat{v}_L'(k)$ the marginal values of an S- and an L-asset, respectively, for a household of type $k \in \{1, 2\}$. Following the discussion above, we get from the envelope condition that:

$$\hat{v}_S'(1) = u_1'(c_1(1))\, r_S, \quad \hat{v}_L'(1) = \max\{u_1'(c_1(1))\, r_L, \rho_2\, u_2'(c_2(1))\, R_L\}, \tag{39}$$

$$\hat{v}_L'(2) = u_2'(c_2(2))\, R_L, \quad \hat{v}_S'(2) = \max\{\rho_1 u_1'(c_1(2))\, r_S,\, u_2'(c_2(2))\, R_S\}. \tag{40}$$

Households' optimal savings choice will be such that the marginal cost of accumulating wealth equals the expected marginal benefit of investing in either asset:[25]

$$z'(w) \geqslant \pi_1 \hat{v}_S'(1) + \pi_2 \hat{v}_S'(2) \quad \text{with equality if } q_S > 0, \tag{41}$$

$$z'(w) \geqslant \pi_1 \hat{v}_L'(1) + \pi_2 \hat{v}_L'(2) \quad \text{with equality if } q_L > 0. \tag{42}$$

Finally, we have the usual budget constraint for date-0 investment and the nonnegativity constraints on investment and consumption levels:

$$w = q_S + q_L \quad \text{and} \quad q_S, q_L, c_1(1), c_1(2), c_2(1), c_2(2) \geqslant 0. \tag{43}$$

[24]To derive (37), we use $c_1(2) = \lambda_S r_S$ and $c_2(2) = R_L q_L + (R_S - \lambda_S) q_S$.

[25]Conditions (41) and (42) are written as inequalities because it may be strictly optimal to invest in only one asset. Assumption 1 again rules out a corner solution with $w^B = 0$, meaning that at least one of the conditions (41) and (42) will hold with equality.

I denote optimal choices in this subsection with a B superscript. An allocation under autarky when types are revealed after the portfolio choice is then given by a vector (w^B, q_S^B, q_L^B) together with two vectors $(c_1^B(k), c_2^B(k))_{k\in\{1,2\}}$ satisfying (33)-(38) and (41)-(43).

Welfare under autarky when types are revealed after the portfolio choice, denoted $\mathcal{W}^B$, is given by

$$\mathcal{W}^B = \pi_1 \left[u_1(c_1^B(1)) + \rho_2\, u_2(c_2^B(1))\right] + \pi_2 \left[\rho_1 u_1(c_1^B(2)) + u_2(c_2^B(2))\right] - z(w^B). \qquad (44)$$

Since households never do worse in autarky when uncertainty about their own type is resolved earlier, we have $\mathcal{W}^B \leqslant \mathcal{W}^A \leqslant \mathcal{W}^F \leqslant \mathcal{W}^*$.

Definition 3. ***Required portfolio liquidity (RPL) risk*** *is said to exist if and only if $\mathcal{W}^B < \mathcal{W}^A$, i.e., if and only if welfare when types are revealed after the portfolio choice is strictly below welfare when types are revealed before the portfolio choice.*

Intuitively, RPL risk exists if and only if households 'regret' not having divided their wealth differently between the S- and the L-asset upon learning their type. The following result states that RPL risk exists whenever households face a nontrivial portfolio choice, i.e., whenever neither of the two assets dominates:

Proposition 6. *RPL risk exists if and only if $r_S \neq r_L$ and $R_S \neq R_L$.*

6. The Insurance Function of an Asset Market

In this section, I maintain the assumption from Subsection 5.2 that households learn their type only after choosing their asset portfolio. However, at the beginning of date 1 after learning their type, households can now trade S- and L-assets with each other in a competitive asset market. I denote p_S and p_L as the market prices (in terms of consumption good) of one unit of the S- and the L-asset, respectively, where 'one unit' refers to one unit of date-0 investment. As before, households can liquidate assets at fixed technological returns r_S and r_L.[26] Since trade is only relevant if no asset dominates, I assume in this section that $r_S > r_L$ and $R_S < R_L$.

From a few simple no-arbitrage conditions described below, we have that equilibrium asset prices satisfy $p_S = p_L = r_S$, which I will take as given henceforth. To see why

[26]Note the difference between liquidating assets (at technologically fixed returns r_S and r_L) and selling them (at prices p_S and p_L).

we must have $p_S = p_L$ in equilibrium note that, from the perspective of price-taking households, assets become perfect substitutes when they can be exchanged at a fixed relative price. The only prices consistent with equilibrium are then such that households are just indifferent between investing in the two assets at date 0. If $p_S > p_L$, then households would choose to invest only in the S-asset and sell it against the L-asset in the asset market if needed, which violates market clearing; similarly, if $p_S < p_L$, households would invest only in the L-asset. Furthermore, $p_S = r_S$ is the only price at which households are just indifferent between liquidating the S-asset at date 1 and selling it on the market. If $p_S < r_S$, households could liquidate the S-asset for r_S units of good, repurchase the S-asset at price p_S and liquidating it again; repeating this process would yield an infinitely large amount of good, violating market clearing. Conversely, if $p_S > r_S$, selling the S-asset would be strictly more profitable than liquidating it. Since liquidating the L-asset is not profitable either, no household would choose to liquidate assets at date 1, such that no consumption good is available against which to sell assets, which is also inconsistent with market clearing.[27]

Since $p_S = p_L$, and since the S-asset (L-asset) pays a strictly higher date-1 (date-2) return, we can assume without loss of generality that households finance all their date-1 (date-2) consumption with the return obtained from their (post-trading) holdings of the S-asset (L-asset). Households' budget constraint in the trading stage is[28]

$$\frac{c_1(k)}{r_S} + \frac{c_2(k)}{R_L} = w \quad \text{for } k \in \{1, 2\}, \tag{45}$$

and market clearing requires

$$\pi_1 \, c_1(1) + \pi_2 c_1(2) = r_S \, q_S \quad \text{and} \quad \pi_1 \, c_2(1) + \pi_2 \, c_2(2) = R_L q_L, \tag{46}$$

where q_S and q_L are the amounts invested by households in the S- and the L-asset, respectively, at date 0.

As in the previous sections, Assumption 1 ensures that type 1 (type 2) households optimally consume a strictly positive amount at date 1 (date 2). If $\rho_2 = 0$ ($\rho_1 = 0$), then type 1 (type 2) households will only consume at date 1 (date 2) and sell all their

[27] Note that $p_S = p_L > r_L$ implies that households strictly prefer selling the L-asset to liquidating it, which is consistent with equilibrium.

[28] To derive (45), we can denote $z_S(k)$ and $z_L(k)$ as the post-trading holdings of the S- and the L-asset, respectively, of a household of type $k \in \{1, 2\}$. We have $c_1(k) = z_S(k) r_S$ and $c_2(k) = z_L(k) R_L$. Given $p_S = p_L$, we have $z_L(k) = q_S + q_L - z_S(k) = w - z_S(k)$, which leads to equation (45).

holdings of the L-asset (S-asset) in the asset market. If $\rho_2 > 0$ ($\rho_1 > 0$), then type 1 (type 2) households may choose to consume at both dates, in which case they will not sell their entire holdings of the L-asset (S-asset). In this case, the amounts traded in the asset market will be such as to equalize the marginal benefit of holding either asset. This means that, just as under autarky when types are revealed between the savings and the portfolio choice, equilibrium consumption levels of households satisfy conditions (27)-(28).

Next, analogous to Subsection 5.2, I denote by $\tilde{v}_S'(k)$ and $\tilde{v}_L'(k)$ the marginal values of carrying an S- and an L-asset, respectively, into the asset market for a household of type $k \in \{1, 2\}$. Given that assets can be exchanged at a relative price p_S/p_L, we have $(p_L/p_S)\, \tilde{v}_S'(k) = \tilde{v}_L'(k)$. Since $p_S/p_L = 1$ in equilibrium, we thus have $\tilde{v}_S'(k) = \tilde{v}_L'(k)$. Note that the envelope condition implies $\tilde{v}_S'(1) = r_S\, u_1'(c_1(1))$ and $\tilde{v}_L'(2) = R_L\, u_2'(c_2(2))$.

As in Subsection 5.2 (see conditions (41) and (42)), households' optimal choice of w will be such that the marginal cost of accumulating wealth equals the marginal benefit of investing in either asset. Since $\tilde{v}_S'(k) = \tilde{v}_L'(k)$ in equilibrium, this now means that the equilibrium savings choice satisfies[29]

$$\begin{aligned} z'(w) &= \pi_1 \tilde{v}_S'(1) + \pi_2 \tilde{v}_L'(2) \\ &= \pi_1 r_S\, u_1'(c_1(1)) + \pi_2 R_L\, u_2'(c_2(2)). \end{aligned} \tag{47}$$

Equilibrium quantities with an asset market are denoted by an M superscript. An asset market equilibrium is then given by a vector $(w^M, q_S^M, q_L^M, c_1^M(1), c_1^M(2), c_2^M(1), c_2^M(2))$ satisfying conditions (27)-(28), (43), and (45)-(47).

Welfare with an asset market, denoted $\mathcal{W}^M$, is given by

$$\mathcal{W}^M = \pi_1 \left[u_1(c_1^M(1)) + \rho_2\, u_2(c_2^M(1))\right] + \pi_2 \left[\rho_1 u_1(c_1^M(2)) + u_2(c_2^M(2))\right] - z(w^M). \tag{48}$$

We continue with the following result:

Proposition 7. *We have* $\mathcal{W}^M = \mathcal{W}^A, w^M = w^A$, *and* $c_t^M(k) = c_t^A(k)$ *for* $t \in \{1, 2\}$ *and* $k \in \{1, 2\}$.

Proposition 7 states that consumption and welfare with an asset market are the same as under autarky when types are revealed before the portfolio choice but after the savings

[29]Since Assumption 1(i) rules out a corner solution with $w^M = 0$, we can write (47) as an equality.

choice. In other words, markets eliminate RPL risk, but they do not insure against any of the other kinds of liquidity risk.[30]

The next result follows directly from Proposition 7 and Corollary 1:[31]

Corollary 2. *In models without MVW risk, welfare with an asset market is the same as with a banking coalition.*

To see how the result in Corollary 2 can be applied to existing models, consider a New Monetarist model with uncertain liquidity needs as outlined in Subsection 2.1. From Proposition 4, we know that MVW risk does not exist in such models if there is an asset whose real return equals the rate of time preference. Corollary 2 tells us that in this case, banking leads to the same outcome as an asset market.[32] The reason is that RPL risk – which is the risk eliminated by asset markets – is the only kind of liquidity risk faced by households. Conversely, in a New Monetarist model without an asset that pays the return of time preference, an asset market will usually not lead to the same outcome as a banking coalition. In this case, even if households hold the optimal asset portfolio given their type and wealth when leaving the asset market, the wealth they have accumulated before entering the asset market may be suboptimal given their type. By requiring an identical contribution from all participating households, a banking coalition essentially insures them against the risk of accumulating a (ex post) suboptimal amount of wealth.

Although their model cannot be mapped one-to-one into the current paper's setup, Corollary 2 is closely related to the result in Andolfatto et al. (2019), who study a Diamond-Dybvig economy with money as the short-term asset and find that markets implement first-best risk sharing if monetary policy is at the Friedman rule. In the terminology of the current paper, there is no MVW risk if monetary policy is at the Friedman rule. The reason is that under the Friedman rule, there is no opportunity cost for investors to accumulate an amount of real money balances that is sufficient to obtain the first-best consumption level in case they turn out to be impatient. In particular, patient investors do

[30] The result that markets eliminate RPL risk (there called 'liquidity risk' and 'maturity risk', respectively) is also found in Haubrich and King (1990) and von Thadden (1999). These models do not feature MVW risk.

[31] Corollary 2 applies to the case where a banking coalition and an asset market are considered in isolation, i.e., there is no interaction between the two. I discuss coexistence of banks and markets in Subsection 7.1.

[32] From Proposition 5, we know that the same is true in a Diamond-Dybvig economy with log-preferences. It follows that markets provide optimal liquidity insurance in a Diamond-Dybvig economy with log-preferences, as also shown in Allen and Gale (2007, Section 3.3).

not regret having accumulated 'too much' money since carrying unspent money balances across periods entails no cost.

7. Discussion

7.1. Coexistence of a Bank With an Asset Market

As has been well known since Jacklin (1987), the presence of an asset market may severely limit a bank's ability to provide liquidity insurance. To examine this issue in the current setup, I consider in this subsection the case where a banking coalition coexists with an asset market. I follow Andolfatto et al. (2019) in assuming that there is a large exogenous mass of households who do not deposit in the bank and trade S- and L-assets in an asset market at date 1. A banking coalition acts as a price taker, and as in Section 6, no-arbitrage implies that asset prices satisfy $p_S = p_L = r_S$, which I will take as given in the remainder of this subsection.

Since households can always choose not to join the banking coalition at date 0, the expected payoff provided by the banking coalition needs to be at least as high as that obtained by participating in the asset market. Furthermore, the households depositing in the bank at date 0 can still participate in the asset market at date 1. That is, after learning their type at the start of date 1, bank depositors can buy assets on the market either by using consumption good withdrawn from the bank at date 1 or by pledging the date-2 income from their bank deposit.[33]

As shown in Farhi et al. (2009) (see also Zannini (2020)), giving bank depositors the option to trade in an asset market corresponds to modifying the IC constraints that the bank faces when setting payouts. Specifically, if bank depositors have the option to trade in an asset market, the present value (evaluated at market prices) of the payouts offered to all depositors must be identical. Otherwise, all depositors could select the option with the higher market value and then – if needed – trade on the market to achieve the desired consumption allocation. This means that with the asset market, the bank faces the additional IC constraint

$$c_1(1) + \frac{p_L}{R_L} c_2(1) = c_1(2) + \frac{p_L}{R_L} c_2(2) \;\Leftrightarrow\; \frac{c_1(1)}{r_S} + \frac{c_2(1)}{R_L} = \frac{c_1(2)}{r_S} + \frac{c_2(2)}{R_L}, \tag{49}$$

[33] Further below, I discuss the case where depositors can only trade on a spot market at date 1 and cannot pledge their date-2 income received from the bank.

where we have used the fact that equilibrium market prices satisfy $p_L = r_S$.

Except for the additional constraint (49), the bank's problem is the same as in Section 3. The quantities chosen by the bank in the presence of an asset market are denoted by a C superscript, where C is mnemonic for 'coexistence'. A banking coalition's solution in the presence of an asset market is given by a vector $(w^C, q_S^C, q_L^C, c_1^C(1), c_1^C(2), c_2^C(1), c_2^C(2))$ that maximizes (8) subject to (3)-(5), (9), and (49).[34] Depositor welfare in the presence of an asset market, denoted $\mathcal{W}^C$, is given by

$$\mathcal{W}^C = \pi_1 \left[u_1(c_1^C(1)) + \rho_2\, u_2(c_2^C(1))\right] + \pi_2 \left[\rho_1 u_1(c_1^C(2)) + u_2(c_2^C(2))\right] - z(w^C). \tag{50}$$

Note that we can combine the date-0 budget constraint (3) with (4)-(5) to obtain

$$w^C = \pi_1 \left[\frac{c_1^C(1)}{r_S} + \frac{c_2^C(1)}{R_L}\right] + \pi_2 \left[\frac{c_1^C(2)}{r_S} + \frac{c_2^C(2)}{R_L}\right]. \tag{51}$$

From constraints (49) and (51), we immediately get that the bank's solution needs to satisfy (45), i.e., the bank's payouts to each individual depositor are subject to the same budget constraint that households face when trading in the asset market. This results from the fact that the presence of an asset market precludes the bank from redistributing funds among depositors. Note also that constraints (4)-(5) are the same as the market clearing conditions (46). Therefore, with an asset market, the banking coalition's solution is subject to the same constraints that are present in a market equilibrium without a bank. It is then not too surprising that the banking coalition cannot provide depositors with a higher welfare than what households would achieve by trading directly on the market:[35]

Proposition 8. *We have $\mathcal{W}^C = \mathcal{W}^M, w^C = w^M$, and $c_t^C(k) = c_t^M(k)$ for $t \in \{1, 2\}$ and $k \in \{1, 2\}$.*

That banks cannot provide better liquidity insurance than markets if all agents have unhindered market access is a relatively robust result in banking theory (e.g., Jacklin (1987), Haubrich and King (1990), Hellwig (1994), von Thadden (1999), Andolfatto et al. (2019)) and is confirmed in this model.[36] This suggests that to the extent that banks

[34] I refer to Appendix C.8 for the derivation of the first-order conditions. As in Section 3, I take it as given that constraints (6)-(7) are slack in the bank's solution. (The result in Proposition 8 implies that (6)-(7) will indeed not be binding in optimum, for all parameters.)

[35] I show in Appendix C.8 that solving the banking coalition's problem in presence of an asset market is isomorphic to solving for the market equilibrium of Section 6.

[36] When market access is limited, banks usually still have a role in providing liquidity insurance (Diamond (1997), Andolfatto et al. (2019)).

have a role in providing liquidity insurance, it is related to frictions that make it costly for households to trade in markets.

Thus far, I have assumed that depositors can pledge the date-2 income from their bank deposit to purchase assets at date 1. The result of Proposition 8 would arguably not be materially affected if one assumed instead that depositors can only trade on spot markets, i.e., that they can only buy assets with consumption good received at date 1 from the bank. In this case, the only relevant deviation is that type 2 depositors may pretend to be of type 1, withdraw at date 1, and purchase L-assets on the market. Constraint (49) then becomes an inequality, where the left-hand side needs to be weakly lower than the right-hand side. In most banking models, the bank wishes to redistribute from type 2 to type 1 households, in which case constraint (49) still binds, and the result of Proposition 8 stands.[37]

7.2. What if the IC Constraints Bind in the Bank's Solution?

Suppose now that, contrary to what is assumed in the rest of the paper, one of the IC constraints (6)-(7) binds in the banking coalition's solution, meaning that consumption levels in the banking coalition's solution are different than in an identical setup where the bank observes households' types.[38] It is straightforward that consumption levels in the economy of Subsection 5.1, where households live in autarky and learn their type before the portfolio but after the savings choice, satisfy the IC constraints (6)-(7). As shown in Section 6, this allocation is identical to that with an asset market. Since a banking coalition can always replicate this allocation, welfare with a banking coalition will be weakly higher than $\mathcal{W}^A = \mathcal{W}^M$.[39] However, if one of the IC constraints (6)-(7) binds in the bank's solution, then there is no guarantee that the bank can provide its depositors with a welfare level as high as the one resulting in autarky when households face no uncertainty about their own type ($\mathcal{W}^F$). Providing depositors with a welfare level equal to (or higher than) $\mathcal{W}^F$ generally entails a redistribution between depositors of

[37] An exception is Andolfatto (2020), where it is optimal to shift consumption from early to late consumers, and banks can provide better risk-sharing than (spot) asset markets.

[38] An important special case is the one with linear date-2 utility $u_2(c_2)$. Ignoring constraints (6)-(7), optimal date-2 consumption levels $c_2(1)$ and $c_2(2)$ may then not be pinned down uniquely. Some of the optimal date-2 consumption levels may be such that constraints (6)-(7) are satisfied while others are not. In this case, incorporating constraints (6)-(7) changes the set of optimal consumption levels but has no effect on welfare.

[39] Apart from edge cases, welfare under the banking coalition will be strictly higher than $\mathcal{W}^A = \mathcal{W}^M$.

different types. If one of the IC constraints (6)-(7) binds in the bank's solution, it means that the bank cannot implement this redistribution to the extent it would if types were observable. Welfare with a banking coalition may then be above or below $\mathcal{W}^F$, depending on parameters.

8. Conclusion

This paper compares the insurance function of banks in different models and attempts to resolve the various meanings of the term 'liquidity insurance' in the literature. Even if many banking models featuring households with uncertain liquidity needs have a 'Diamond-Dybvig flavor' to them, the exact nature of the liquidity risk faced by households may be quite different than in the original Diamond-Dybvig model. Identifying what kind of liquidity risk exists in which models also helps clarify the role of banks in different models. For instance, it allows to determine whether introducing banks is equivalent to eliminating uncertainty about liquidity needs and whether banking leads to the same outcome as an asset market.

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Appendix

A. The Banking Coalition's Problem

Inserting constraint (3) into the objective function, the Lagrangian of the bank's problem writes:

$$\begin{aligned}\mathcal{L} = &- z(q_S + q_L) + \pi_1 \left[u_1(c_1(1)) + \rho_2 u_2(c_2(1))\right] + \pi_2 \left[\rho_1 u_1(c_1(2)) + u_2(c_2(2))\right] \\ &+ \mu_1[r_S\, q_S - \pi_1 c_1(1) - \pi_2 c_1(2)] + \mu_2[R_L\, q_L - \pi_1 c_2(1) - \pi_2 c_2(2)].\end{aligned} \tag{52}$$

With the nonnegativity constraints (9), the first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial q_S} : \quad \mu_1 \leqslant \frac{z'(w)}{r_S} \qquad \text{with equality if } q_S > 0, \tag{53}$$

$$\frac{\partial \mathcal{L}}{\partial q_L} : \quad \mu_2 \leqslant \frac{z'(w)}{R_L} \qquad \text{with equality if } q_L > 0, \tag{54}$$

$$\frac{\partial \mathcal{L}}{\partial c_1(1)} : \quad \mu_1 \geqslant u_1'(c_1(1)) \qquad \text{with equality if } c_1(1) > 0, \tag{55}$$

$$\frac{\partial \mathcal{L}}{\partial c_1(2)} : \quad \mu_1 \geqslant \rho_1 u_1'(c_1(2)) \qquad \text{with equality if } c_1(2) > 0, \tag{56}$$

$$\frac{\partial \mathcal{L}}{\partial c_2(1)} : \quad \mu_2 \geqslant \rho_2 u_2'(c_2(1)) \qquad \text{with equality if } c_2(1) > 0, \tag{57}$$

$$\frac{\partial \mathcal{L}}{\partial c_2(2)} : \quad \mu_2 \geqslant u_2'(c_2(2)) \qquad \text{with equality if } c_2(2) > 0. \tag{58}$$

We can first show that $w > 0$ with a proof by contradiction. Suppose that $w = 0$ such that, by (3), we have $q_S = q_L = 0$, which, by (4), implies $c_1(1) = 0$. From conditions (53) and (55), we then get $r_S u_1'(0) \leqslant z'(0)$. This violates Assumption 1(i)-(ii), which says that $r_S u_1'(0) > z'(0)$.

Next, we can show that $q_S > 0$ with a proof by contradiction. Suppose that $q_S = 0$. Given $w > 0$, this implies that $q_L > 0$ (see (3)). Conditions (57)-(58) imply that $c_2(1) > 0$ can only be optimal if $c_2(2) > 0$ is optimal; therefore, by (5), $q_L > 0$ implies $c_2(2) > 0$. From (54) and (58), we then get $z'(w) = R_L u_2'(c_2(2)) \leqslant R_L u_2'(0)$. Since $q_S = 0$ implies $c_1(1) = 0$ (see (4)), we obtain from (53) and (55) that $z'(w) \geqslant r_S u_1'(0)$. We thus get that $R_L u_2'(0) \geqslant r_S u_1'(0)$, which violates Assumption 1(ii).

Finally, conditions (55)-(56) imply that $c_1(2) > 0$ can only be optimal if $c_1(1) > 0$ is optimal. From (4), we thus get that $q_S > 0$ implies $c_1(1) > 0$. As a result, conditions (53) and (55) hold with equality in optimum, and we can write the first-order conditions as in (10)-(13).

B. Households' Problem in Autarky with Full Information

Consider the optimization problem of a type 1 household. Inserting constraints (15)-(17) into the objective function (19), the Lagrangian writes:

$$\mathcal{L} = -z(q_S(1) + q_L(1)) + u_1(r_S q_S(1)) + \rho_2 u_2(R_L q_L(1)). \tag{59}$$

With the nonnegativity constraints (18), the first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial q_S(1)} : \quad z'(w(1)) \geqslant r_S u_1'(c_1(1)) \qquad \text{with equality if } c_1(1) > 0, \tag{60}$$

$$\frac{\partial \mathcal{L}}{\partial q_L(1)} : \quad z'(w(1)) \geqslant \rho_2 R_L u_2'(c_2(1)) \quad \text{with equality if } c_2(1) > 0. \tag{61}$$

It is straightforward to show that Assumption 1 implies that $c_1(1) > 0$ in optimum, such that the first-order conditions can be expressed as (21)-(22).

The derivation of the first-order conditions of type 2 households' problem follows the same steps as for type 1 households and is omitted.

C. Proofs

C.1. Proof of Proposition 1

With slight abuse of notation, denote by $q_S^F \equiv \pi_1 q_S^F(1) + \pi_2 q_S^F(2)$ aggregate investment in the S-asset in autarky with full information. Similarly, aggregate investment in the L-asset is denoted $q_L^F \equiv \pi_1 q_L^F(1) + \pi_2 q_L^F(2)$. Combining this with optimality conditions (16)-(17), we get that:

$$r_S q_S^F = \pi_1 c_1^F(1) + \pi_2 c_1^F(2), \tag{62}$$

$$R_L q_L^F = \pi_1 c_2^F(1) + \pi_2 c_2^F(2). \tag{63}$$

Consider now a solution where both types of households accumulate an identical amount of wealth w^F, i.e., $w^F(1) = w^F(2) = w^F$. It is straightforward that, from (15), we have

$$w^F = q_S^F + q_L^F. \tag{64}$$

Furthermore, the first-order conditions (21)-(24) become

$$z'(w^F) = r_S\, u_1'(c_1^F(1)), \tag{65}$$

$$z'(w^F) \geqslant \rho_2\, R_L\, u_2'(c_2^F(1)) \quad \text{with equality if } c_2^F(1) > 0, \tag{66}$$

$$z'(w^F) \geqslant R_L\, u_2'(c_2^F(2)) \qquad \text{with equality if } c_2^F(2) > 0, \tag{67}$$

$$z'(w^F) \geqslant \rho_1 r_S u_1'(c_1^F(2)) \qquad \text{with equality if } c_1^F(2) > 0. \tag{68}$$

A solution for autarky with full information with $w^F(1) = w^F(2) = w^F$ is thus sufficiently characterized by a vector $(c_1^F(1), c_1^F(2), c_2^F(1), c_2^F(2), w^F, q_S^F, q_L^F)$ satisfying conditions (62)-(68). The optimality conditions (62)-(68) are identical to the optimality conditions of the banking coalition's problem (conditions (3)-(5) and (10)-(13)), from which it follows that $w^F = w^*$ and $c_t^F(k) = c_t^*(k)$ for $t \in \{1,2\}$ and $k \in \{1,2\}$. This implies that $\mathcal{W}^F = \mathcal{W}^*$ such that, by Definition 1, type risk does not exist. □

C.2. Proof of Proposition 2

If $z(\cdot)$ is linear, then $z'(\cdot)$ is a constant, which implies that conditions (10)-(13) fully determine the set of optimal consumption levels in the banking coalition's solutions. In the full-information autarky solution, optimal consumption levels are fully determined by conditions (21)-(23), which are identical to those for the banking coalition. We have therefore shown that optimal consumption levels with the banking coalition are the same as in the full-information autarky case, i.e., $c_t^F(k) = c_t^*(k)$ for $t \in \{1,2\}$ and $k \in \{1,2\}$. Since in both cases, aggregate date-1 consumption is fully financed with the return from the S-asset, while aggregate date-2 consumption is fully financed with the return from the L-asset, the optimal aggregate date-0 investment in the S- and the L-asset is the same in both cases, i.e., $q_S^* = q_S^F(1)+q_S^F(2)$ and $q_L^* = q_L^F(1)+q_L^F(2)$. This in turn implies that the optimal aggregate wealth accumulation is the same, i.e., $w^* = w^F(1) + w^F(2)$. With a linear $z(w)$, welfare is fully determined by consumption levels and the aggregate amount of accumulated wealth, which implies that $\mathcal{W}^F = \mathcal{W}^*$. □

C.3. Proof of Proposition 3

I will first show that MVW risk exists if $r_S\, u_1'(c_1^A(1)) \neq R_L\, u_2'(c_2^A(2))$. To see this, note first that the set of feasible allocations when types are revealed after the savings but before the portfolio choice is a strict subset of the set of feasible allocations under full information, namely the subset where households of both types are restricted to accumulate the same amount of wealth ($w(1) = w(2)$). This means that to achieve welfare level $\mathcal{W}^F$, the allocation given by w^A and $(c_1^A(k), c_2^A(k))_{k\in\{1,2\}}$ needs to satisfy conditions (21)-(24), with $w(1) = w(2) = w^A$. Assumptions 1(i) and (iii) jointly ensure that $w^A > 0$ and $c_2^A(2) > 0$. From (21) and (23), we then get that a necessary condition to achieve welfare level $\mathcal{W}^F$ is

$$z'(w^A) = r_S u_1'(c_1^A(1)) = R_L u_2'(c_2^A(2)). \tag{69}$$

If $r_S\, u_1'(c_1^A(1)) \neq R_L\, u_2'(c_2^A(2))$, then (69) cannot be fulfilled, from which it follows that $\mathcal{W}^A < \mathcal{W}^F$.

Next, I will show that MVW risk does not exist if $r_S\, u_1'(c_1^A(1)) = R_L\, u_2'(c_2^A(2))$. To see this, note that the probabilities of turning out to be a given type, π_1 and π_2, enter the optimality conditions only through condition (31). This means that if there exists a solution with $r_S\, u_1'(c_1^A(1)) = R_L\, u_2'(c_2^A(2))$, then this must solve a household's problem for any (π_1, π_2), including $(\pi_1, \pi_2) = (1, 0)$ and $(\pi_1, \pi_2) = (0, 1)$. This implies that the allocation also solves households' problem in an identical economy with full information, from which it follows that $\mathcal{W}^A = \mathcal{W}^F$. □

C.4. Proof of Proposition 4

If $u_2(c_2)$ is linear, $\rho_2 = 1$, and $c_2^A(1) > 0$, then we get from (27) that

$$r_S u_1'(c_1^A(1)) = R_L u_2'(c_2^A(1)) = R_L u_2'(c_2^A(2)),$$

where the second equality follows from the linearity of $u_2(c_2)$. By Proposition 3, MVW risk then does not exist, and, by Corollary 1, type risk does not exist either. □

C.5. Proof of Proposition 5

Consider households' problem under full information. If $\rho_1 = \rho_2 = 0$, then type 1 households only invest in the S-asset while type 2 households only invest in the L-asset. We thus have $q_L^F(1) = q_S^F(2) = 0$, and hence, by (15), $w^F(1) = q_S^F(1)$ and $w^F(2) = q_L^F(2)$. Inserting (16) and (17) into optimality conditions (21) and (23), and using $u_1(\cdot) = u_2(\cdot) = A + B\ln(\cdot)$, we obtain[40]

$$z'(w^F(1)) = \frac{B}{q_S^F(1)} = \frac{B}{w^F(1)}, \tag{70}$$
$$z'(w^F(2)) = \frac{B}{q_L^F(2)} = \frac{B}{w^F(2)}. \tag{71}$$

There is a unique $w^F = w^F(1) = w^F(2)$ solving (70)-(71), which in turns pins down investment levels $q_S^F(1) = q_L^F(2) = w^F$ and consumption levels $c_1^F(1) = r_S w^F$ and $c_2^F(2) = R_L w^F$.

From the fact that optimal wealth levels $w^F(1)$ and $w^F(2)$ under full information are identical for the two types, it follows from Proposition 1 that type risk does not exist. Since households can choose this common optimal wealth level before knowing their type, MVW risk does not exist either . □

[40] The fact that $z'(0) < \infty$ implies $c_2^F(2) > 0$, such that condition (23) holds with equality.

C.6. Proof of Proposition 6

First, if $r_S = r_L$, then the L-asset dominates since it pays the same date-1 return and a higher date-2 return; similarly, if $R_S = R_L$, the S-asset dominates. It is clear that RPL risk does not exist in this case, since investing the entire wealth in the asset whose return dominates is always ex post optimal, independent of what type a household turns out to be.

Next, I will show that if $r_S > r_L$ and $R_S < R_L$, then $\mathcal{W}^B < \mathcal{W}^A$. If $r_S > r_L$ and $R_S < R_L$, then a necessary condition for an allocation when types are revealed after the portfolio choice (i.e., an allocation given by (w^B, q_S^B, q_L^B) and $(c_1^B(k), c_2^B(k))_{k\in\{1,2\}})$ to achieve welfare level $\mathcal{W}^A$ is that households of both types finance their date-1 (date-2) consumption solely with the return from their investment in the S-asset (L-asset). To see this, note that if households of a given type finance part of their date-1 (date-2) consumption with their holdings of the L-asset (S-asset), then these households could achieve a strictly higher payoff if they knew their type before making their portfolio choice by dividing their accumulated wealth w^B differently between the two assets.

Furthermore, we also know from Subsection 5.1 that, as a consequence of Assumption 1, an allocation where households of both types consume zero at one of the dates (i.e., an allocation where either $c_1(1) = c_1(2) = 0$ or $c_2(1) = c_2(2) = 0$) cannot achieve welfare level $\mathcal{W}^A$.

It follows that if $r_S > r_L$ and $R_S < R_L$, then an allocation with types revealed after the portfolio choice can achieve welfare level $\mathcal{W}^A$ only if households of both types consume identical, strictly positive, amounts at both dates, i.e., only if $c_1^B(1) = c_1^B(2) = r_S q_S > 0$ and $c_2^B(1) = c_2^B(2) = R_L q_L > 0$. This can only be utility-maximizing if the marginal rate of substitution between consumption at date 1 and 2 evaluated at these common consumption levels is the same for households of both types, i.e., only if

$$\frac{u_1'(c_1^B(1))}{\rho_2 u_2'(c_2^B(1))} = \frac{\rho_1 u_1'(c_1^B(1))}{u_2'(c_2^B(1))} \quad \Leftrightarrow \quad \rho_1\rho_2 = 1, \tag{72}$$

which is violated since $\rho_1 \in [0,1)$ and $\rho_2 \in [0,1]$. This completes the proof that $\mathcal{W}^B < \mathcal{W}^A$ when $r_S > r_L$ and $R_S < R_L$. □

C.7. Proof of Proposition 7

Consider any $(w^M, q_S^M, q_L^M, c_1^M(1), c_1^M(2), c_2^M(1), c_2^M(2))$ that satisfies conditions (27)-(28), (43), and (45)-(47), i.e., it solves households' problem with an asset market. It is then not hard to verify that $w^A = w^M$, $(c_1^A(k), c_2^A(k))_{k\in\{1,2\}} = (c_1(k)^M, c_2(k)^M)_{k\in\{1,2\}}$, and

$$\begin{aligned} q_S^A(1) &= \frac{1}{\pi_1}\left[q_S^M - \frac{\pi_2}{r_S}c_1^M(2)\right], \quad q_L^A(1) = \frac{1}{\pi_1}\left[q_L^M - \frac{\pi_2}{R_L}c_2^M(2)\right] \\ q_S^A(2) &= \frac{1}{\pi_2}\left[q_S^M - \frac{\pi_1}{r_S}c_1^M(1)\right], \quad q_L^A(2) = \frac{1}{\pi_2}\left[q_L^M - \frac{\pi_1}{R_L}c_2^M(1)\right] \end{aligned}$$

satisfy conditions (16)-(17), (26)-(28), and (31), i.e., they solve households' problem in an identical economy where households live in autarky and types are revealed after the savings but before the portfolio choice. Since households' production and consumption levels are identical in both cases, this implies $\mathcal{W}^A = \mathcal{W}^M$. □

C.8. Proof of Proposition 8

Inserting constraint (3) into the objective function, the Lagrangian of the bank's problem writes:

$$\begin{aligned} \mathcal{L} = &- z(q_S + q_L) + \pi_1\left[u_1(c_1(1)) + \rho_2 u_2(c_2(1))\right] + \pi_2\left[\rho_1 u_1(c_1(2)) + u_2(c_2(2))\right] \\ &+ \mu_1[r_S\, q_S - \pi_1 c_1(1) - \pi_2 c_1(2)] + \mu_2[R_L\, q_L - \pi_1 c_2(1) - \pi_2 c_2(2)] \\ &+ \gamma\left[\frac{c_1(1)}{r_S} + \frac{c_2(1)}{R_L} - \frac{c_1(2)}{r_S} - \frac{c_2(2)}{R_L}\right]. \end{aligned} \tag{73}$$

With the nonnegativity constraints (9), the first-order conditions are:[41]

$$\frac{\partial \mathcal{L}}{\partial q_S}: \quad \mu_1 \leqslant \frac{z'(w)}{r_S} \qquad \text{with equality if } q_S > 0, \tag{74}$$

$$\frac{\partial \mathcal{L}}{\partial q_L}: \quad \mu_2 \leqslant \frac{z'(w)}{R_L} \qquad \text{with equality if } q_L > 0, \tag{75}$$

$$\frac{\partial \mathcal{L}}{\partial c_1(1)}: \quad \mu_1 \geqslant u_1'(c_1(1)) + \frac{\gamma}{\pi_1 r_S} \qquad \text{with equality if } c_1(1) > 0, \tag{76}$$

$$\frac{\partial \mathcal{L}}{\partial c_1(2)}: \quad \mu_1 \geqslant \rho_1 u_1'(c_1(2)) - \frac{\gamma}{\pi_2 r_S} \qquad \text{with equality if } c_1(2) > 0, \tag{77}$$

$$\frac{\partial \mathcal{L}}{\partial c_2(1)}: \quad \mu_2 \geqslant \rho_2 u_2'(c_2(1)) + \frac{\gamma}{\pi_1 R_L} \qquad \text{with equality if } c_2(1) > 0, \tag{78}$$

$$\frac{\partial \mathcal{L}}{\partial c_2(2)}: \quad \mu_2 \geqslant u_2'(c_2(2)) - \frac{\gamma}{\pi_2 R_L} \qquad \text{with equality if } c_2(2) > 0. \tag{79}$$

To prove Proposition 8, I will proceed in several steps.

Step 1: show that $w > 0$. We can show this with a proof by contradiction. Suppose that $w = 0$. Since no investment takes place, we then have $c_1(1) = 0$ and $c_2(2) = 0$. From (74) and (76), we obtain $\gamma \leqslant \pi_1[z'(0) - r_S u_1'(0)]$. Furthermore, from (75) and (79), we obtain $\gamma \geqslant \pi_2[R_L u_2'(0) - z'(0)]$. These two inequalities can only be jointly fulfilled if $\pi_2[R_L u_2'(0) - z'(0)] \leqslant \pi_1[z'(0) - r_S u_1'(0)]$. Rearranging this and using $\pi_1 + \pi_2 = 1$ yields $z'(0) \geqslant \pi_1 r_S u_1'(0) + \pi_2 R_L u_2'(0)$, which violates Assumption 1(i) and thus leads to a contradiction.

[41] To reduce notational clutter, I will omit the C superscripts throughout.

Step 2: show that $c_1(2) > 0$ *implies* $c_2(2) > 0$. To show this, we proceed with a proof by contradiction. Suppose $c_1(2) > 0$ and $c_2(2) = 0$. By (4), $c_1(2) > 0$ implies $q_S > 0$. From (74), we then have $r_S\mu_1 = z'(w)$. From (77), we obtain $\gamma = \pi_2[r_S\rho_1 u_1'(c_1(2)) - z'(w)]$. Next, from (75) and (79), we have $\gamma \geqslant \pi_2[R_L u_2'(0) - z'(w)]$. Combining these two conditions yields $R_L u_2'(0) \leqslant \rho_1 r_S u_1'(c_1(2))$, which violates Assumption 1(iii).

Step 3: show that $c_2(1) > 0$ *implies* $c_1(1) > 0$. We proceed again with a proof by contradiction. Suppose that $c_2(1) > 0$ and $c_1(1) = 0$. By (5), $c(1) > 0$ implies $q_L > 0$. From (75), we then have $R_L\mu_2 = z'(w)$. From (78), we obtain $\gamma = \pi_1[z'(w) - R_L\rho_2 u_2'(c_2(1))]$. Inserting this into (76) and combining with (74) yields $r_S u_1'(0) \leqslant \rho_2 R_L u_2'(c_2(1))$, which violates Assumption 1(ii).

Step 4: show that $c_1(1) > 0$ *and* $c_2(2) > 0$. We can show that $c_1(1) > 0$ with a proof by contradiction. Suppose that $c_1(1) = 0$. By the result of step 3, this implies that $c_2(1) = 0$. By (49), this implies that $c_1(2) = c_2(2) = 0$ and thus, by (3), $w = 0$, which violates our result from step 1. The proof that $c_2(2) > 0$ proceeds in the same way and is omitted.

By (4) and (5), the result in step 4 implies that $q_S > 0$ and $q_L > 0$. Using the fact that (74)-(76) and (79) all hold with equality, we obtain

$$\pi_1[z'(w) - r_S u_1'(c_1(1))] = \pi_2[R_L u_2'(c_2(2)) - z'(w)] = \gamma. \tag{80}$$

Rearranging this and using $\pi_1 + \pi_2 = 1$ yields

$$z'(w) = \pi_1 r_S u_1'(c_1(1)) + \pi_2 R_L u_2'(c_2(2)). \tag{81}$$

Next, inserting the expressions for γ, μ_1, and μ_2 into (77) and (78) and rearranging terms yields:

$$\rho_1 r_S u_1'(c_1(2)) \leqslant R_L u_2'(c_2(2)) \quad \text{with equality if } c_1(2) > 0, \tag{82}$$

$$\rho_2 R_L u_2'(c_2(1)) \leqslant r_S u_1'(c_1(1)) \quad \text{with equality if } c_2(1) > 0. \tag{83}$$

Notice that conditions (81)-(83) are the same as the optimality conditions (**??**)-(47) in the asset market equilibrium.

The bank's solution in the presence of an asset market is thus given by a vector $(w^C, q_S^C, q_L^C, c_1^C(1), c_1^C(2), c_2^C(1), c_2^C(2))$ satisfying (81)-(83) as well as constraints (3)-(5), (9) and (49). As shown in the main text, the bank's constraints are equivalent to the constraints that households face in the asset market equilibrium, namely (43) and (45), plus the market clearing condition (46). Therefore, the optimality conditions of the bank's problem in the presence of an asset market are the same as the equilibrium conditions of the asset market equilibrium in

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